MIT Economics

14.452
Fall 2021

Economic Growth: Problem Set #2

Due Monday November 22

Please only hand in Questions 1 and 3, which will be graded. The rest will be reviewed in the recitation but should not be handed in.

1. **Development accounting.** For this exercise, use the Penn World Tables (Version 10.0)

   https://www.rug.nl/ggdc/productivity/pwt/

   Consider the cross-section of countries in the year 2017. Suppose all countries have the Cobb-Douglas production function

   $$Y_i = A_i K_i^{\alpha} H_i^{1-\alpha}, \qquad 0 < \alpha < 1$$

   To be specific, let Y_i denote 'output-side real GDP at chained PPPs (in mil. 2017US$)', K_i denote the 'capital stock at current PPPs (in mil. 2017US$)', L_i denote 'number of persons engaged [employed] (in millions)', $H_i = h_i L_i$**, and** h_i **denote the PWT 'index of human capital per person, based on years of schooling and returns to education'.**

   Fixing the value $\alpha = 1/3$, decompose the observed levels of Y_i/L_i into terms reflecting the K_i/Y_i ratio, H_i/L_i, and a productivity residual. Express each of these variables relative to the United States (normalized to one), and do your best to replicate Table 6, Figure 29, and Figure 30 in Jones (2016) 'Facts of Economic Growth' *Handbook of Macroeconomics*, as discussed in class.

2. **Government purchases in the neoclassical growth model.** Consider a neoclassical growth model in continuous time. The planner chooses private consumption per worker $c(t) \geq 0$ and government purchases per worker $g(t) \geq 0$ to maximize

   $$\int_0^{\infty} e^{-\rho t}\, u(c(t), g(t))\, dt, \qquad \rho > 0$$

   subject to the flow resource constraint

   $$\dot{k}(t) = f(k(t)) - \delta k(t) - c(t) - g(t), \qquad 0 < \delta < 1$$

   given initial capital per worker $k(0) > 0$.

   (a) Setup up a current-value Hamiltonian and derive the key optimality conditions for $c(t)$, $g(t)$ and $k(t)$. Explain clearly how the time-paths for $c(t)$, $g(t)$ and $k(t)$ are determined.

(b) Derive expressions characterizing the steady-state values c^*, g^*, k^*. Do these steady-state values depend on the instantaneous utility function $u(c, g)$? Explain.

Now suppose that the instantaneous utility function is

$$u(c, g) = (1 - \gamma) \log(c) + \gamma \log(g), \qquad 0 < \gamma < 1$$

and that the production function is $y = f(k) = k^\alpha$ for $0 < \alpha < 1$.

(c) Explain how the steady-state capital k^* and steady-state shares c^*/y^* and g^*/y^* depend on the parameter γ.

(d) Use a phase diagram to explain the transitional dynamics of $c(t)$, $g(t)$ and $k(t)$ around the steady state. [*Hint*: you can reduce this to a system in $c(t)$ and $k(t)$].

(e) Suppose the economy is in steady state when suddenly the preference for government purchases γ increases permanently to $\gamma' > \gamma$. Explain the transitional dynamics of $c(t)$, $g(t)$ and $k(t)$ as the economy adjusts to this change. Give intuition for your answers.

3. **Capital accumulation and financial frictions.** Consider a neoclassical growth model in continuous time. The planner chooses consumption per worker $c(t) \geq 0$ to maximize

$$\int_0^\infty e^{-\rho t} u(c(t))\, dt, \qquad \rho > 0$$

subject to the flow resource constraint

$$\dot{k}(t) = f(k(t)) - c(t) - \delta k(t), \qquad 0 < \delta < 1$$

given initial capital per worker $k(0) > 0$. Suppose that the period utility function is

$$u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}, \qquad \theta > 0$$

and that the production function is $y = f(k) = k^\alpha$ for $0 < \alpha < 1$.

(a) Setup up a current-value Hamiltonian and derive the key optimality conditions for $c(t)$ and $k(t)$. Explain clearly how the time-paths for $c(t)$ and $k(t)$ are determined.

(b) Derive expressions characterizing the steady-state values c^* and k^*. Do these steady-state values depend on the instantaneous utility function $u(c)$? Explain. Use a phase diagram to explain the transitional dynamics of $c(t)$ and $k(t)$ around the steady state.

Now suppose that the economy is less efficient at turning household savings into investment. These *financial frictions* are captured by a parameter ϕ such that for every 1 unit saved only $1 - \phi$ units augment the capital stock. In short, capital accumulation is now given by

$$\dot{k}(t) = (1 - \phi)[f(k(t)) - c(t)] - \delta k(t), \qquad 0 \leq \phi < 1$$

If $\phi = 0$ this reduces to the frictionless case above.

(c) How, if at all, do these financial frictions change your answers to parts (a) and (b) above? Suppose we observed a range of countries $i = 1, 2, ...$ differing only in ϕ_i, explain how these countries would differ in terms of long-run levels of capital and consumption. Would these countries be predicted to differ in terms of their long-run savings rates?

(d) Now consider one country and suppose it is in steady state c^*, k^* with $\phi > 0$. Consider a local approximation around the steady state. How if at all do the local dynamics depend on the magnitude of ϕ? Does ϕ affect the speed of convergence to steady state? Does ϕ affect the slope of the stable arm of the saddle path? Explain.

(e) Starting in steady state c^*, k^* with $\phi > 0$, suppose that suddenly there is crisis that permanently increases the level of financial frictions to $\phi' > \phi$. Explain the transitional dynamics of $c(t)$ and $k(t)$ as the economy adjusts to this shock.

4. **Investment-specific productivity growth in a two-sector model.** We will now consider a *two-sector* growth model where the consumption good and the capital good are physically distinct. Given a constant labor force $L > 0$, the planner seeks to maximize

$$\int_0^\infty e^{-\rho t} \log\left(\frac{C(t)}{L}\right)L, \qquad \rho > 0$$

The production function for the consumption good is

$$Y(t) = K(t)^\alpha L^{1-\alpha}, \qquad 0 < \alpha < 1$$

Consumption goods can either be consumed or transformed into capital goods. In particular, one unit of the consumption good can be transformed into $A(t) > 0$ units of the capital good. So we now have the resource constraints

$$C(t) + I(t) = Y(t)$$

and

$$\dot{K}(t) = A(t)I(t) - \delta K(t), \qquad 0 < \delta < 1$$

where $K(0) > 0$ is given and where $I(t)$ denotes units of the consumption good that turns into $A(t)I(t)$ units of new capital goods to augment the existing capital stock (which depreciates at rate δ). Suppose that the productivity of investment in new capital goods grows at rate g

$$\dot{A}(t) = gA(t)$$

(a) Derive the key optimality conditions for the levels of $C(t)$ and $K(t)$ [note these are the raw levels, not their detrended values].

(b) Let g_C, g_K, g_Y denote the long-run growth rates of consumption $C(t)$, capital $K(t)$, and output $Y(t)$ along a balanced growth path. Let $R(t)$ and $w(t)$ denote the marginal products of labor and capital along a balanced growth path. Derive expressions for g_C, g_K, g_Y and for $R(t), w(t)$ in terms of the investment-specific productivity growth g and other parameters.

(c) Let $R(t)K(t)/Y(t)$ denote the capital income share. Is the capital income share in this model constant along a balanced growth path? Why or why not? How if at all is this different to a standard model with labor-augmenting productivity growth?

(d) What is the connection between the investment-specific productivity here and the parameter ϕ in Question 3 above?