Economic Growth

Lecture 4: Solow model, part three

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Solow Growth Model Meets the Data

- Examine cross-country data through lens of the Solow model.

- Throughout, a key question is how much observed variation in cross-country output per person (either growth or in levels) is accounted for by *inputs* and how much by *productivity*.

- **Background:**

  - motivation for 1980s-1990s literature on *endogenous growth* was a view, forcefully articulated by P. Romer (1986), that standard growth models of the time were fundamentally incapable of explaining the cross-country data

  - this view was challenged by Mankiw, D. Romer and Weil (1992), who argued that an *augmented Solow model* with *human capital* could explain a surprisingly large amount of the cross-country data

  - Mankiw, Romer and Weil's criticised on grounds of (i) implausible identifying assumptions and (ii) implausibly large returns to human capital

  - turn to *development accounting*, stronger economic assumptions

Outline

1. **Growth accounting: traditional approach**

2. **Augmented Solow model with human capital**
   Mankiw, Romer and Weil
   Problems with MRW

3. **Growth accounting: modern approach**
   Development accounting

Growth Accounting

- Solow (1957) growth accounting approach.

- Aggregate production function in continuous time

$$Y(t) = F(K(t), L(t), A(t))$$

- Taking log derivatives

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{F_K(t)K(t)}{Y(t)}\frac{\dot{K}(t)}{K(t)} + \frac{F_L(t)L(t)}{Y(t)}\frac{\dot{L}(t)}{L(t)} + \frac{F_A(t)A(t)}{Y(t)}\frac{\dot{A}(t)}{A(t)}$$

- Let $\alpha_K(t) = R(t)K(t)/Y(t)$ and $\alpha_L(t) = w(t)L(t)/Y(t)$ denote the factor shares. With competitive factor markets $F_K(t) = R(t)$ & $F_L(t) = w(t)$ so

$$\alpha_K(t) = \frac{F_K(t)K(t)}{Y(t)}, \qquad \text{and} \qquad \alpha_L(t) = \frac{F_L(t)L(t)}{L(t)}$$

Growth Accounting: Traditional Approach

- Let $g_Y(t)$, $g_K(t)$ etc denote the instantaneous growth rates.

- Let $x(t)$ denote the contribution of productivity $A(t)$ to growth

$$x(t) \equiv \frac{F_A(t)A(t)}{Y(t)} \frac{\dot{A}(t)}{A(t)}$$

- We then infer $x(t)$ from

$$x(t) = g_Y(t) - \alpha_K(t)g_K(t) - \alpha_L(t)g_L(t)$$

- Solow (1957) applied this to US data 1909–1949 and attributed '87.5%' of observed growth in output per worker to $A(t)$.

- A 'measure of our ignorance' (Abramowitz 1956) aka the 'Solow residual'.

Key Problems

- *Omitted inputs/misspecification.*
  - if true data generating process includes inputs omitted from assumed production function, estimates of $x(t)$ will be *overstated*
- *Measurement*
  - capital stock measured as *capital expenditure*, conflates changes in amount of capital with its price, systematic decline in relative price of capital will lead to underestimate of capital's contribution, again $x(t)$ *overstated*
- *Structural interpretation*
  - ironically, makes no use of the structure of the Solow model
  - in that model, $g_K(t)$ is *endogenous* to $g_A(t)$ and *all trend growth* in $Y(t)/L(t)$ is attributable to $A(t)$
  - so $x(t)$ may be *understated*

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Human Capital

- In Solow model, implicitly all workers have the same skills (and hours).
- But what matters is the *efficiency units* of labor supplied.
- And the efficiency units a worker can supply can be built up over time, by investments in education and training (and learning by doing).
- Following Shultz (1965), Becker (1965) and Mincer (1974), stock of efficiency units a worker can supply known as the worker's *human capital*.
- A model that omits human capital implicitly attributes variation in these efficiency units (over time or across countries) to the productivity residual.

Augmented Solow Model

- Aggregate production function with stock of human capital $H(t)$

$$Y(t) = F(K(t), H(t), A(t)L(t))$$

with usual curvature properties and with CRS in the three inputs.

- Exogenous productivity and labor force

$$\dot{A}(t) = gA(t), \qquad \dot{L}(t) = nL(t)$$

- Constant savings rates s_k, s_h and depreciation rates δ_k, δ_h in physical and human capital so that

$$\dot{K}(t) = s_k Y(t) - \delta_k K(t)$$

$$\dot{H}(t) = s_h Y(t) - \delta_h H(t)$$

- Write output *per effective worker*, physical capital *per effective worker* etc

$$y(t) \equiv \frac{Y(t)}{A(t)L(t)}, \qquad k(t) \equiv \frac{K(t)}{A(t)L(t)} \qquad h(t) \equiv \frac{H(t)}{A(t)L(t)}$$

Augmented Solow Model

- Following usual steps, physical capital per effective worker and human capital per effective worker are given by

$$\dot{k}(t) = s_k f(k(t), h(t)) - (\delta_k + g + n)k(t)$$
$$\dot{h}(t) = s_h f(k(t), h(t)) - (\delta_h + g + n)h(t)$$

where $f(k, h) \equiv F(k, h, 1)$.

- Steady state k^*, h^* pinned down by two equations

$$s_k f(k^*, h^*) = (\delta_k + g + n)k^*$$
$$s_h f(k^*, h^*) = (\delta_h + g + n)h^*$$

- Notice that in steady state,

$$\frac{k^*}{y^*} = \frac{s_k}{\delta_k + g + n}, \qquad \frac{h^*}{y^*} = \frac{s_h}{\delta_h + g + n}, \qquad \frac{k^*}{h^*} = \frac{\frac{s_k}{\delta_k+g+n}}{\frac{s_h}{\delta_h+g+n}}$$

- Pin down levels by substituting back into one steady-state condition.

Cobb-Douglas

- Example. Suppose the aggregate production function is

$$Y = K^{\alpha} H^{\beta} (AL)^{1-\alpha-\beta}, \qquad 0 < \alpha, \beta < 1$$

so that output per effective worker is $y = f(k,h) = k^{\alpha} h^{\beta}$.

- Then the steady state levels evaluate to

$$k^* = \left(\left(\frac{s_k}{\delta_k + g + n} \right)^{1-\beta} \left(\frac{s_h}{\delta_h + g + n} \right)^{\beta} \right)^{\frac{1}{1-\alpha-\beta}}$$

$$h^* = \left(\left(\frac{s_k}{\delta_k + g + n} \right)^{\alpha} \left(\frac{s_h}{\delta_h + g + n} \right)^{1-\alpha} \right)^{\frac{1}{1-\alpha-\beta}}$$

- Collecting terms we then get output per effective worker

$$y^* = \Bigg(\underbrace{\frac{s_k}{\delta_k + g + n}}_{=k^*/y^* \text{ ratio}} \Bigg)^{\frac{\alpha}{1-\alpha-\beta}} \Bigg(\underbrace{\frac{s_h}{\delta_h + g + n}}_{=h^*/y^* \text{ ratio}} \Bigg)^{\frac{\beta}{1-\alpha-\beta}}$$

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A World of Augmented Solow Economies

- Mankiw, Romer and Weil (1992) take this model to cross-country data.
- Countries $j = 1, \ldots, N$ each 'as an island' [no explicit interactions].
- Each has a Cobb-Douglas production function

$$Y_j = K_j^{\alpha} H_j^{\beta} (A_j L_j)^{1-\alpha-\beta}, \qquad 0 < \alpha, \beta < 1$$

- Countries differ in savings rates $s_{k,j}$, $s_{h,j}$, labor force growth n_j.
- **Key assumptions:** (i) *common productivity growth* $g_j = g$ across countries, and (ii) *independent productivity levels* across countries

$$A_j(t) = e^{gt} A_j(0), \qquad \text{and} \qquad A_j(0) = e^{\varepsilon_j} \bar{A}$$

where the ε_j are IID draws from some fixed distribution, independent of j.

- Countries differ in their long run productivity levels $A_j(0)$, but, by assumption, these differences are independent of country j outcomes.
- Also assume $\delta_{k,j} = \delta_{h,j} = \delta$ all j, but less important.

Mankiw, Romer and Weil

- In steady state

$$k_j^* = \left(\left(\frac{s_{k,j}}{\delta + g + n_j} \right)^{1-\beta} \left(\frac{s_{h,j}}{\delta + g + n_j} \right)^{\beta} \right)^{\frac{1}{1-\alpha-\beta}}$$

$$h_j^* = \left(\left(\frac{s_{k,j}}{\delta + g + n_j} \right)^{\alpha} \left(\frac{s_{h,j}}{\delta + g + n_j} \right)^{1-\alpha} \right)^{\frac{1}{1-\alpha-\beta}}$$

- So, *along a balanced growth path*, output per worker is given by

$$\frac{Y_j(t)}{L_j(t)} = A_j(t) \times \Big(\underbrace{\frac{s_{k,j}}{\delta + g + n_j}}_{=k_j^*/y_j^* \text{ ratio}} \Big)^{\frac{\alpha}{1-\alpha-\beta}} \Big(\underbrace{\frac{s_{h,j}}{\delta + g + n_j}}_{=h_j^*/y_j^* \text{ ratio}} \Big)^{\frac{\beta}{1-\alpha-\beta}}$$

Mankiw, Romer and Weil

- Or in logs

$$\log \frac{Y_j(t)}{L_j(t)} = \log \bar{A} + gt + \frac{\alpha}{1-\alpha-\beta} \log \frac{s_{k,j}}{\delta+g+n_j} + \frac{\beta}{1-\alpha-\beta} \log \frac{s_{h,j}}{\delta+g+n_j} + \varepsilon_j$$

- Mankiw, Romer and Weil estimate the parameters α, β etc using cross-country variation at a given point in time
  - measure $s_{k,j}$ by average *investment rates*, $s_{k,j} = I_{k,j}/Y_{k,j}$
  - measure $s_{h,j}$ by average fraction of the working age population enrolled in *secondary school* [this is important]
  - measure n_j by average growth of working age population
  - assign $\delta = 0.03$ and $g = 0.02$
- Begin with restricted $\beta = 0$ version [textbook Solow model].

MRW: Textbook Solow Model [$\beta = 0$]

ESTIMATION OF THE TEXTBOOK SOLOW MODEL

Dependent variable: log GDP per working-age person in 1985			
Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	5.48	5.36	7.97
	(1.59)	(1.55)	(2.48)
ln(I/GDP)	1.42	1.31	0.50
	(0.14)	(0.17)	(0.43)
$\ln(n + g + \delta)$	−1.97	−2.01	−0.76
	(0.56)	(0.53)	(0.84)
$\overline{R}^2$	0.59	0.59	0.01
s.e.e.	0.69	0.61	0.38
Restricted regression:			
CONSTANT	6.87	7.10	8.62
	(0.12)	(0.15)	(0.53)
$\ln(I/GDP) - \ln(n + g + \delta)$	1.48	1.43	0.56
	(0.12)	(0.14)	(0.36)
$\overline{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.37
Test of restriction:			
p-value	0.38	0.26	0.79
Implied α	0.60	0.59	0.36
	(0.02)	(0.02)	(0.15)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05.

MRW: Textbook Solow Model [$\beta = 0$]

- Regression coefficients have predicted signs.

- Large fraction of the cross-country variation in output per worker accounted for by variation in $s_{k,j}$ and n_j.

- But estimated coefficient $\alpha/(1-\alpha) \approx 1.5$ implies $\alpha \approx 0.6$, much higher than capital share of $\approx 1/3$.

- Empirically, $s_{h,j}$ positively correlated with $s_{k,j}$ (and negatively with n_j), suggests including $s_{h,j}$ will reduce estimated α.

MRW: Augmented Solow Model

Estimation of the Augmented Solow Model

Dependent variable: log GDP per working-age person in 1985

Sample:	Non-oil	Intermediate	OECD
Observations:	98	75	22
CONSTANT	6.89	7.81	8.63
	(1.17)	(1.19)	(2.19)
ln(I/GDP)	0.69	0.70	0.28
	(0.13)	(0.15)	(0.39)
$\ln(n + g + \delta)$	−1.73	−1.50	−1.07
	(0.41)	(0.40)	(0.75)
ln(SCHOOL)	0.66	0.73	0.76
	(0.07)	(0.10)	(0.29)
$\overline{R}^2$	0.78	0.77	0.24
s.e.e.	0.51	0.45	0.33
Restricted regression:			
CONSTANT	7.86	7.97	8.71
	(0.14)	(0.15)	(0.47)
$\ln(I/GDP) - \ln(n + g + \delta)$	0.73	0.71	0.29
	(0.12)	(0.14)	(0.33)
$\ln(SCHOOL) - \ln(n + g + \delta)$	0.67	0.74	0.76
	(0.07)	(0.09)	(0.28)
$\overline{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.32
Test of restriction:			
p-value	0.41	0.89	0.97
Implied α	0.31	0.29	0.14
	(0.04)	(0.05)	(0.15)
Implied β	0.28	0.30	0.37
	(0.03)	(0.04)	(0.12)

Note. Standard errors are in parentheses. The investment and population growth rates are averages for the period 1960–1985. $(g + \delta)$ is assumed to be 0.05. SCHOOL is the average percentage of the working-age population in secondary school for the period 1960–1985.

MRW: Augmented Solow Model

- Taken at face-value, seems like success.

- Estimated $\alpha = 0.3$ in line with capital share of $\approx 1/3$.

- Large fraction of the cross-country variation in output per worker accounted for by variation in $s_{k,j}, s_{h,j}$ and n_j.

- Suggests while Solow model might not be a satisfactory theory of economic *growth*, since g is exogenous, when augmented this way it might provide a good account of the *cross section*.

- Remark. Just because $R^2 = 0.78$ doesn't mean residual productivity differences are not playing a large role. Still have std dev log productivity $= \sqrt{0.22} = 0.47$, say $\approx 1/2$ std dev log output per worker.

- But there are bigger problems to discuss.

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Problems with MRW

- **Assumption that A_j is independent of $s_{k,j}, s_{h,j}$.**

- Two main reasons to think they may not be independent

  (i) *omitted variable bias*

  e.g., some omitted Z_j drives both A_j and $s_{k,j}, s_{h,j}$

  seems plausible that countries that have invested less in physical and human capital for various reasons will also have lower productivity

  (ii) *endogeneity bias*

  seems plausible that countries with high A_j will find it optimal to invest more in physical and human capital, increasing $s_{k,j}, s_{h,j}$

- If $s_{k,j}, s_{h,j}$ are positively correlated with A_j, estimates of α, β biased up.

- Moreover independent reasons to think β in particular is too large.

Problems with MRW

- **Estimated β seems too large relative to micro evidence**.

- Recall that $s_{h,j}$ is measured as average fraction of the working age population enrolled in *secondary-school.*

- Other things equal, how much variation in output per worker is attributable to this variation in schooling?

  – this measure of $s_{h,j}$ ranges from 0.004 to 0.12 in the data

  – predicted log difference

$$\frac{\beta}{1-\alpha-\beta}(\log(0.12)-\log(0.004)) = 0.66 \times 3.40 = 2.24$$

  – country with schooling 0.12 should be on the order of $e^{2.24} = 9.4$ times richer than one with schooling 0.004, other things equal

- Much larger than implied by micro evidence on returns to schooling.

Returns to Schooling

- Traditional approach to estimate 'returns to schooling' is the *Mincer regression*, say

$$\log w_i = \boldsymbol{x}_i' \boldsymbol{\gamma} + \phi s_i$$

- Standard estimates between $\phi = 0.06$ and $\phi = 0.1$.

- If this applied uniformly, and *assuming away human capital externalities*, how much variation in income per person does this imply?

- A country with 12 more years schooling should have between $e^{(0.06)(12)} = 2.05$ and $e^{(0.1)(12)} = 3.32$ times the income per person as a country with zero years of schooling.

- There is much less than 12 years difference in schooling in the MRW data, so this is a generous upper bound.

Returns to Schooling

- In short, micro evidence suggests variation in schooling can account for 2-3 fold differences in income per person.

- But MRW cross-country estimates suggest variation in schooling can account for 9+ fold differences in income per person.

- In this sense, estimated MRW β seems implausibly high.

- As discussed, plausible to think that $s_{k,j}, s_{h,j}$ are positively correlated with ε_j, estimates of β biased up.

- Moreover using *primary and secondary* schooling as measure of $s_{h,j}$ reduces estimated β, less cross-country variation in primary schooling.

- See Klenow and Rodíguez-Clare (1997) for extensive discussion.

Alternative Approach

- Estimation approach, inherits usual difficulties in estimating production functions compounded with difficulties of cross-country regression.

- A popular alternative approach is to *impose* a lot more structure.

- Advantage of being simple and transparent, but at cost of assumptions that may be hard to swallow.

- A form of 'growth accounting' [that can be run in differences or levels].

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Growth Accounting: Modern Approach

- To streamline exposition, consider production function

$$Y = F(K, AH) = K^{\alpha} (AH)^{1-\alpha}$$

- Standard procedure is to proxy human capital as in a Mincer regression

$$H = e^{\phi(S)} L$$

where S is average years of schooling, or perhaps better

$$H = \sum_{s} e^{\phi(s)} L(s)$$

where $L(s)$ is the labor input of workers with s years of schooling.

- For the simple example, $e^{\phi(S)}$ is the efficiency units of labor for worker with S years of schooling relative to worker with $S = 0$ years of schooling.

- Return to schooling in a Mincer regression is $\phi'(S)$, use to calibrate model.

- Nests model with homogeneous labor by setting $\phi(S) = 0$.

Growth Accounting: Modern Approach

- **A better way to do growth accounting.**

- Divide both sides by Y^{α} and solve for Y to get

$$Y = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} AH$$

  so in per worker terms

$$\frac{Y}{L} = \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} \frac{H}{L} A$$

- Remarks. The advantage of writing things this way is that in most growth models, including the Solow model, the long-run K/Y ratio is independent of A while the K/L ratio is not.

- But as in traditional growth accounting, no stand taken on what generates K/Y and H/L, taken straight from the data.

- Can run the accounting in differences or levels.

Growth Accounting for the United States.

		Contributions from		
Period	**Output per hour**	***K/Y***	**Labor composition**	**Labor-Aug. TFP**
1948–2013	**2.5**	**0.1**	**0.3**	**2.0**
1948–1973	3.3	−0.2	0.3	3.2
1973–1990	1.6	0.5	0.3	0.8
1990–1995	1.6	0.2	0.7	0.7
1995–2000	3.0	0.3	0.3	2.3
2000–2007	2.7	0.2	0.3	2.2
2007–2013	1.7	0.1	0.5	1.1

Source: Jones (2016). Here labor input L is measured as total hours worked so Y/L is output per hour. Since different workers have different amounts of human capital, H/L also captures composition effects.

Over whole sample, A accounts for about 80% of growth (2% out of 2.5%), though its relative contribution has fallen somewhat in recent years.

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Development Accounting

- Can also run the same approach in *levels*.

- This is often known as *development accounting* (King and Levine 1994, Klenow and Rodriguez-Clare 1997, Hall and Jones 1999).

- Suppose output per worker in country $j = 1, 2, \ldots$ is given by

$$\frac{Y_j}{L_j} = \left(\frac{K_j}{Y_j}\right)^{\frac{\alpha}{1-\alpha}} \frac{H_j}{L_j} A_j$$

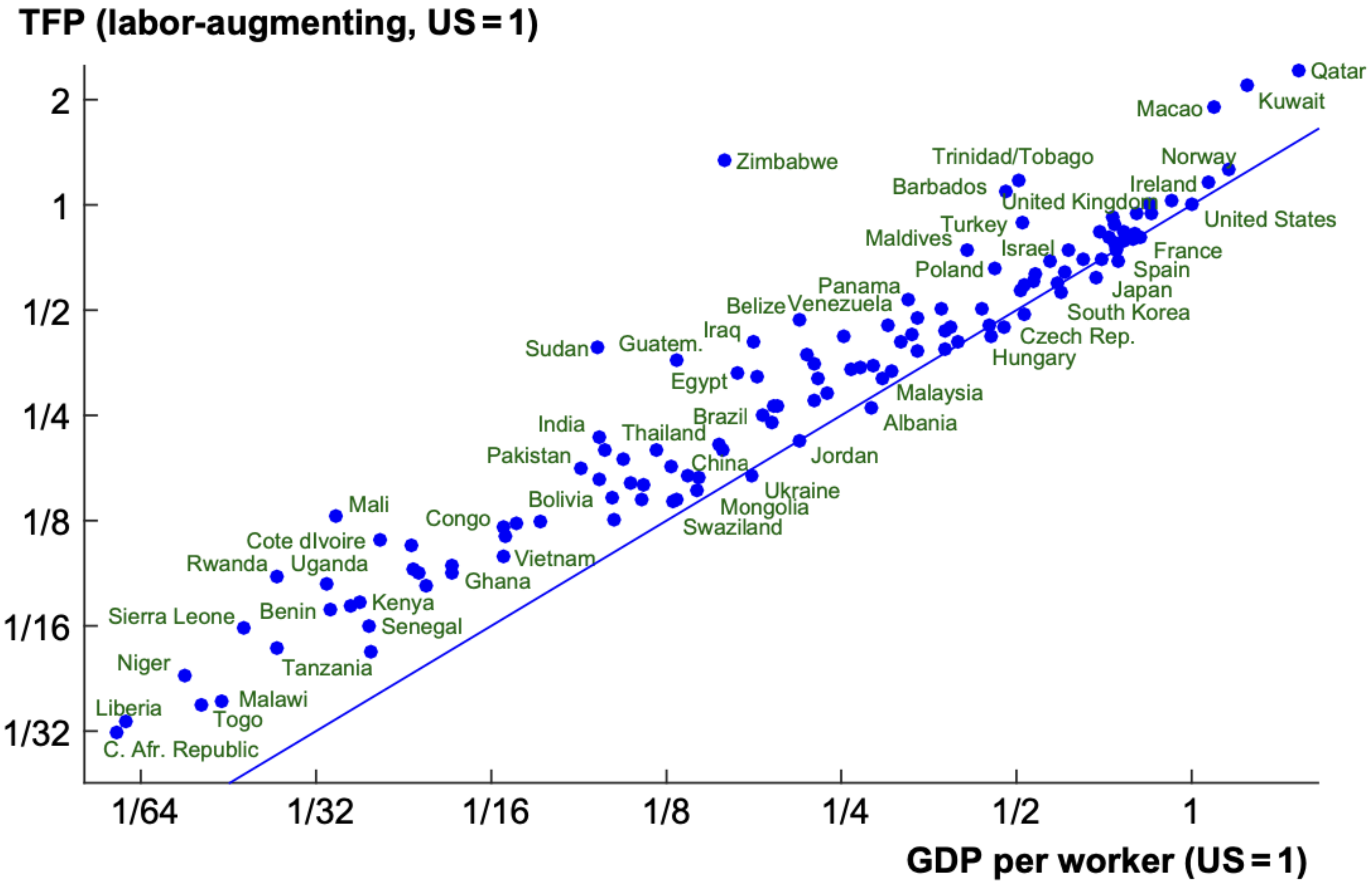
- Infer A_j levels that rationalize observed Y_j/L_j given K_j/Y_j and H_j/L_j.

- **Key assumption.** That α is the same across countries [more generally, production function is the same across countries up to A_j differences].

Development Accounting

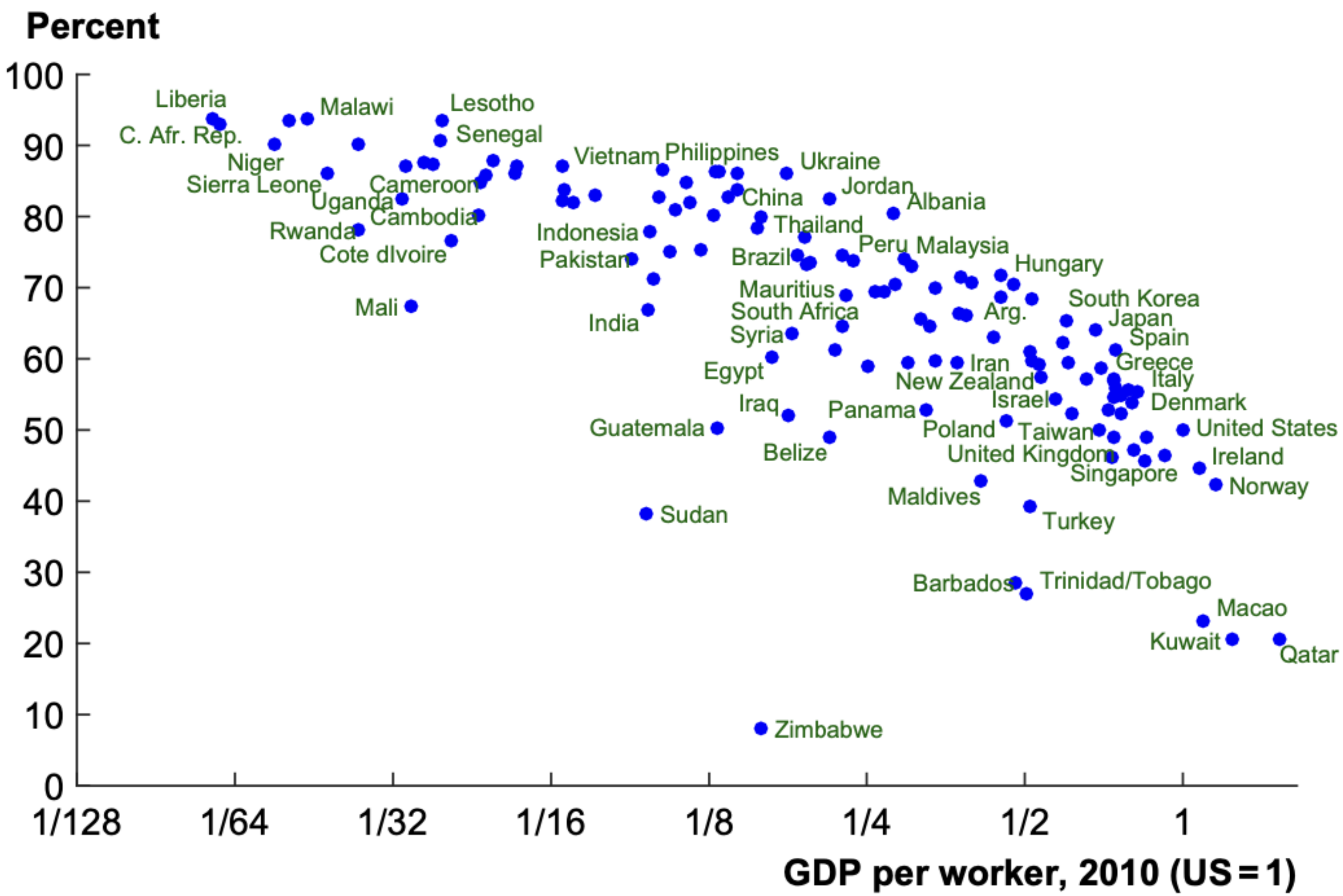
	GDP per worker, y	Capital/GDP $(K/Y)^{\alpha/(1-\alpha)}$	Human capital, h	TFP	Share due to TFP
United States	1.000	1.000	1.000	1.000	–
Hong Kong	0.854	1.086	0.833	0.944	48.9%
Singapore	0.845	1.105	0.764	1.001	45.8%
France	0.790	1.184	0.840	0.795	55.6%
Germany	0.740	1.078	0.918	0.748	57.0%
United Kingdom	0.733	1.015	0.780	0.925	46.1%
Japan	0.683	1.218	0.903	0.620	63.9%
South Korea	0.598	1.146	0.925	0.564	65.3%
Argentina	0.376	1.109	0.779	0.435	66.5%
Mexico	0.338	0.931	0.760	0.477	59.7%
Botswana	0.236	1.034	0.786	0.291	73.7%
South Africa	0.225	0.877	0.731	0.351	64.6%
Brazil	0.183	1.084	0.676	0.250	74.5%
Thailand	0.154	1.125	0.667	0.206	78.5%
China	0.136	1.137	0.713	0.168	82.9%
Indonesia	0.096	1.014	0.575	0.165	77.9%
India	0.096	0.827	0.533	0.217	67.0%
Kenya	0.037	0.819	0.618	0.073	87.3%
Malawi	0.021	1.107	0.507	0.038	93.6%
Average	0.212	0.979	0.705	0.307	63.8%
1/Average	4.720	1.021	1.418	3.260	69.2%

Bulk of Variation in Y_j/L_j Attributed to A_j



Inferred A_j given $\alpha = 1/3$ against Y_j/L_j. Bulk of observed variation in Y_j/L_j is attributed to variation in A_j, with variation in K_j/Y_j playing a minor role. Source: Jones (2016).

Share Attributed to A_j



Share of observed variation in Y_j/L_j attributed to A_j negatively correlated with level of Y_j/L_j. Source: Jones (2016).

Ongoing Concerns

- Same issues with omitted inputs as traditional growth accounting
- Stronger functional form assumption.
- In practice, common α — in principle could relax this, but suitable data is limited for many countries.
- Proxy measures for physical capital — results sensitive if e.g., price of investment goods vary systematically across countries.
- Proxy measures for human capital — is there more to human capital than schooling? what about human capital externalities?

Mixed Results

- Solow model not without empirical content, despite its simplicity it is rich enough to take to the data in various ways (growth accounting, cross-country regressions, calibration exercises).

- Each approach has strengths and weakenesses.

- Subject of ongoing debate, but standard view is that cross-country variation in output per worker due in large part to cross-country variation in productivity — not just differences in physical and human capital.

  [even MRW implies large productivity variation, their point was that physical and human capital seems to explain a 'surprisingly large' amount]

- These differences in productivity are probably not just (or even mainly) technological in the narrow sense.

- Unsettling, because Solow model takes all this as exogenous.

Disclaimer

- Focus here is on *proximate* causes of variation in output per worker.
- Should keep in mind that these may also reflect deeper underlying causes
  - institutions
  - geography
  - luck/historical contingency (e.g., multiple equilibria)

Next Class

- Back to theoretical foundations.
- The neoclassical growth model with endogenous saving.
- Ramsey (1928), Cass (1965), and Koopmans (1965).
- Starting point for a vast array of applied macroeconomic models.

Homework

- Consider the Solow model in continuous time with trend growth and production function $Y = K^\alpha (AL)^{1-\alpha}$. Suppose the parameter values $\alpha = 1/3$ and that $g = 0.02$, $n = 0.01$, $\delta = 0.06$ per year.

- Check. Show that the 'speed of convergence' to the balanced growth path is 0.06 per year. Is this fast or slow?

- Consider the augmented Solow model and suppose $\beta = 1 - \alpha$ so that the production function is $Y = K^\alpha H^{1-\alpha}$. Suppose also $\delta_k = \delta_h = \delta$.

- Check. Show that this economy is 'asymptotically AK'. Characterize the dynamics of $K(t)$ and $H(t)$ in the short run and the long run.

- Hint. Consider the dynamics of the ratio $K(t)/H(t)$.