# Economic Growth

## Lecture 13: Firm dynamics, misallocation, and growth

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# Outline

1. **Benchmark quality-ladder model**

2. **Firm dynamics in a quality-ladder model: Klette-Kortum (2004).**

3. **Static misallocation: Hsieh-Klenow (2009).**

4. **Dynamic misallocation in a quality-ladder model: Peters (2020).**

# Quality Ladder Model

- Continuous time $t \geq 0$.
- Representative household

$$U = \int_0^\infty e^{-\rho t} \log C(t)\, dt, \qquad \rho > 0$$

- Aggregate consumption $C(t)$ depends on
  - $j \in [0, 1]$ continuum horizontally differentiated varieties
  - $k \in \{0, 1, \dots, J(j, t)\}$ discrete vertically differentiated vintages of $j$
  - *state-of-the-art* vintage $J(j, t)$ for each horizontal variety $j$
- Let $z_k(j)$ denote *quality* and $x_k(j, t)$ denote *quantity* of variety $j, k$.

# Aggregate Consumption

- Instantaneous utility

$$\log C(t) = \int_0^1 \log \left[ \sum_{k=0}^{J(j,t)} z_k(j) x_k(j,t) \right] dj$$

- Note: *imperfect* horizontal differentiation (elasticity of subs. $=1$) but *perfect* vertical differentiation (elasticity of subs. $=\infty$).

- Let $q > 1$ denote the size of the *quality step*, i.e., for each $j$

$$z_k(j) = q\, z_{k-1}(j) \qquad k = 1, 2, \ldots, J(j,t)$$

- Choose physical units for each variety so that $z_0(j) = 1$ for all $j$. Then simply $z_k(j) = q^k$ for all $j$.

# Expenditure

- Let $P(t)$ denote aggregate price index associated with $C(t)$ and let $E(t) = P(t)C(t)$ denote aggregate expenditure.

- Let $k^*(j,t)$ denote variety that charges lowest price per unit quality.

- Demand for variety $j,k$ is then

$$x_k(j,t) = \begin{cases} \dfrac{E(t)}{p_k(j,t)} & \text{if } k = k^*(j,t) \\ 0 & \text{otherwise} \end{cases}$$

- Aggregate expenditure satisfies the intertemporal Euler equation

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho, \qquad \Leftrightarrow \qquad \frac{\dot{C}(t)}{C(t)} = r(t) - \rho - \frac{\dot{P}(t)}{P(t)}$$

# Expenditure

- Let $E(t) = 1$ be the numeraire. Then from the Euler equation

$$r(t) = \rho \qquad \Leftrightarrow \qquad \frac{\dot{C}(t)}{C(t)} = -\frac{\dot{P}(t)}{P(t)}$$

- And expenditure on variety $j, k$ is

$$p_k(j,t)x_k(j,t) = \begin{cases} 1 & \text{if } k = k^*(j,t) \\ 0 & \text{otherwise} \end{cases}$$

# Production

- Wage rate $w(t)$ per unit labor engaged in production.

- Flow profits from production of $j, k$

$$\pi_k(j,t) = (p_k(j,t) - w(t))x_k(j,t)$$

(i.e., it takes one unit of labor to produce one unit of output, $x = l$)

- Inelastic aggregate labor supply $L$. Labor may be employed in *goods production* $L_X(t)$ or in *research* $L_R(t)$

$$L_X(t) + L_R(t) = L$$

# Pricing

- Consider leader firm with state-of-the-art quality and its closest follower, one step behind.

- Leader has quality advantage $q > 1$ over follower.

- Leader charges *limit price* $p_k(j,t) = qw(t)$ to prevent entry.

- In symmetric equilibrium only the state-of-the-art quality is sold $k^*(j,t) = J(j,t)$, and all leaders have flow profits

$$\pi_{J(j,t)}(j,t) = (p_{J(j,t)}(j,t) - w(t)) \times x_{J(j,t)}(j,t) = \frac{q-1}{q} =: \pi$$

# Innovation and Entry

- Any firm can target any product line in an attempt to improve state-of-the-art.

- **Technology for innovation:** Choose Poisson arrival rate $\lambda \geq 0$ for new quality step at cost $c\lambda$ in units of labor.

- Let $V(t)$ denote the value of an incumbent firm, to be determined.

- Free-entry complementary slackness condition

$$V(t) \leq cw(t), \qquad \text{and} \qquad \Big[V(t) - cw(t)\Big]\lambda = 0$$

  so that $V(t) = cw(t)$ whenever $\lambda > 0$.

- Remark. In this simple model, will turn out that, in equilibrium, incumbent firms will not innovate to get more than 1 quality step ahead. But this is not generally true, as we will see.

# Bellman Equation for Incumbents

- Flow profits $\pi$.

- Lose incumbency to successful innovator with arrival rate $\lambda$.

- Value $V(t)$ satisfies continuous time Bellman equation

$$(r + \lambda)V(t) = \pi + \dot{V}(t)$$

- Remark. No aggregate risk, all idiosyncratic risk perfectly diversified. Also implicitly assuming, as will be true in equilibrium, that $\lambda$ is constant.

# Balanced Growth Path

- Focus on a balanced growth path, characterized by constants

$$(V^*,\, \lambda^*,\, w^*)$$

- Value for incumbents, from steady-state of Bellman equation with $r = \rho$

$$V = \frac{\pi}{\rho + \lambda}, \qquad \pi = \frac{q-1}{q}$$

for some $\lambda$ to be determined.

- Labor market clearing

$$L_X + L_R = L$$

with total labor employed in each sector

$$L_X = \frac{1}{qw}, \qquad \text{and} \qquad L_R = \lambda c$$

# Free Entry Condition $V \leq wc$

- CASE 1: $\lambda > 0$ (INNOVATION). Then $V = wc$ and we can write the labor market clearing condition

$$\frac{(\rho + \lambda)c}{q\pi} + \lambda c = L$$

or

$$\lambda^* = \pi \frac{L}{c} - \frac{\rho}{q}$$

from which we can then recover $V^* = \pi/(\rho + \lambda^*)$ and $w^* = V^*/c$.

- CASE 2: $\lambda = 0$ (NO INNOVATION). Then $L_R = 0$, $L_X = L$ and so $w^* = 1/qL$, $V^* = \pi/\rho$.

- REMARK. Balanced growth path with innovation exists if $q$-steps

$$q > 1 + \rho \frac{c}{L}$$

(i.e., if large population, low discount rate, or low innovation cost etc)

# Aggregate Consumption

- Recall aggregate consumption index

$$\log C(t) = \int_0^1 \log \Big[ \sum_{k=0}^{J(j,t)} q^k \, x_k(j,t) \Big] \, dj$$

  using $z_k(j,t) = q^k$ for all $j, t$.

- In equilibrium only latest vintage sold, i.e., $x_k(j,t) = 1/qw(t)$ for $k = J(j,t)$ and zero otherwise. Hence

$$\log C(t) = (\log q) \int_0^1 J(j,t) \, dj - \log(q \, w(t))$$

# Aggregate Growth

- Use LLN to calculate cross-sectional average

$$\int_0^1 J(j,t)\,dj = \mathbb{E}[J(t)]$$

where $J(t)$ is a Poisson process with intensity $\lambda^*$, so

$$\mathbb{E}[J(t)] = \lambda^* t$$

- Hence along a balanced growth path with $w(t) = w^*$, aggregate growth is

$$g^* \equiv \frac{\dot{C}(t)}{C(t)} = (\log q)\lambda^*$$

# Real Wage

- Along balanced growth path, wage $w^*$ is a constant.

- But *real wage* $w^*/P(t)$ is growing. Recall that

$$1 = E(t) = P(t)C(t)$$

so

$$-\frac{\dot{P}(t)}{P(t)} = \frac{\dot{C}(t)}{C(t)} = g^*$$

- Hence real wage is growing at $g^*$ too.

- REMARK. All growth is due to quality upgrading.

# Aggregate Growth

- If interior equilibrium

$$g^* = (\log q)\lambda^* \qquad \text{where} \qquad \lambda^* = \pi\frac{L}{c} - \frac{\rho}{q}$$

- So in this case $g^*$ is

  - increasing in $q$ (directly, and indirectly via $\lambda^*$)
  - increasing in $L$ (scale effect)
  - increasing in $\pi$ (monopoly profits from successful innovation)

  - decreasing in $c$ (barrier to entry/cost of innovation)
  - decreasing in $\rho$ (greater impatience)

- Otherwise, namely if $q < 1 + \rho c/L$, then corner equilibrium with $\lambda^* = 0$ and hence $g^* = 0$ etc.

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# Klette/Kortum: Stylized Facts

1. Productivity and R&D positively correlated across firms, but productivity growth not strongly correlated with firm R&D.
2. Patents and R&D positively correlated, both in the cross-section of firms and over-time for a given firm.
3. R&D intensity uncorrelated with firm size.
4. R&D intensity is highly skewed across firms; many firms do zero R&D.
5. Differences in R&D intensity across firms very persistent.
6. Firm-level R&D investment follows geometric random walk.
7. Size distribution also highly skewed.
8. Smaller firms have low survival probability, but those that do survive grow faster than large firms. Among large firms, growth independent of firm size.
9. Variance of growth rates higher for smaller firms.
10. Younger firms are small, have low survival probability, but those that survive grow faster than older firms. Market share of a cohort declines with age.

# Klette/Kortum: Model Overview

- Continuous time $t \geq 0$.
- Firm size $n$ follows discrete stochastic *birth/death process*
  - births: new products added when innovation is successful
  - deaths: products lost when competing firms innovate
- No natural size of a firm.
- Firms can grow unboundedly large, but takes time and luck.
- Firms that hit a string of bad luck exit.

# Innovation

- Innovation production function

$$I = G(R, n)$$

where $I$ is innovation rate, $R$ is R&D effort, $n$ current size.

- Innovation technology $G(R, n)$ is

  – strictly increasing in $R$ and $n$
  (existing knowledge capital facilitates innovation)

  – strictly concave in $R$

  – homogenous degree one in $R$ and $n$
  (neutralizes effect of firm size on innovation)

- Use homogeneity to write as

$$R = nc(\lambda)$$

where $\lambda \equiv I/n$ is *innovation intensity* (cf., quality ladders).

# Value of a Firm

- Product line gives constant profit flow $\pi \in (0, 1)$.
- Let $V_n$ denote value of firm with $n$ products, $V_0 = 0$ (exit).
- Bellman equation for firm with $n > 0$ products, *on balanced growth path*

$$rV_n = \max_{\lambda} \left[ \pi n - c(\lambda)n + \lambda n(V_{n+1} - V_n) - \mu n(V_n - V_{n-1}) \right]$$

with interest rate $r > 0$ and product destruction rate $\mu > 0$

- Value is linear in $n$, $V_n = vn$, for some $v > 0$ to be determined

$$(r + \mu)v = \max_{\lambda} \left[ \pi - c(\lambda) + \lambda v \right]$$

with $c'(\lambda) = v$ for $\lambda > 0$ [or $c'(0) > v$ and $\lambda = 0$]. Innovation intensity *independent of firm size*, increasing in $\pi$, decreasing in $r, \mu$.

# Firm Dynamics and Life-Cycle

- Let $p_n(t)$ denote prob. firm is size $n$ at $t$ given size $n_0 = 1$ at $t = 0$.

- Law of motion for $n \geq 1$ products

$$\begin{aligned}\dot{p}_n(t) &= (n-1)\lambda p_{n-1}(t) + (n+1)\mu p_{n+1}(t) \\ &\quad - n(\lambda + \mu)p_n(t)\end{aligned}$$

- Firms with no products exit, $n = 0$ is an absorbing state

$$\dot{p}_0(t) = \mu p_1(t)$$

# Firm Dynamics and Life-Cycle

- Solving the system of differential equations gives

$$p_0(t) = \frac{\mu}{\lambda}\gamma(t), \qquad \gamma(t) \equiv \frac{\lambda - \lambda e^{-(\mu-\lambda)t}}{\mu - \lambda e^{-(\mu-\lambda)t}}$$

  and

$$p_1(t) = [1 - p_0(t)][1 - \gamma(t)], \quad p_n(t) = p_{n-1}(t)\gamma(t) \quad \text{for } n = 2, 3, ...$$

- *Geometric distribution* conditional on survival

$$\frac{p_n(t)}{1 - p_0(t)} = [1 - \gamma(t)]\gamma(t)^{n-1}, \qquad n = 1, 2, ...$$

# Firm Dynamics and Life-Cycle

- Firms eventually exit, $\lim_{t\to\infty} p_0(t) = 1$.
- Geometric distribution with parameter $\gamma(t)$ increasing in $t$
  - distribution grows stochastically over time
  - conditional on survival, mean and variance of size increase with $t$
- Firm with $n$ products at $t$ behaves as if $n$ independent firms each of size 1.
- Larger firms have smaller exit hazard.

# Aggregation

- Let $M_n(t)$ denote measure of size $n$ firms at date $t$ and let

$$M(t) \equiv \sum_{n=1}^{\infty} M_n(t)$$

- Unit mass of products, each product produced by exactly one firm

$$1 = \sum_{n=1}^{\infty} n M_n(t)$$

- Total innovation rate by incumbents

$$\sum_{n=1}^{\infty} I(n) M_n(t) = \sum_{n=1}^{\infty} \lambda n M_n(t) = \lambda$$

independent of size distribution of firms

# Industry Equilibrium

- Unlimited potential entrants. If entrants have innovation rate $\eta$, total *product destruction rate* is

$$\mu = \lambda + \eta$$

- Pay sunk cost $k_e > 0$ to enter, gives Poisson arrival rate 1 of entering with $n = 1$ products. Free-entry complementary slackness condition

$$v \le k_e, \qquad \text{and} \qquad \left[v - k_e\right]\eta = 0$$

- Recall incumbents' first order condition $c'(\lambda) = v$, so whenever $\eta > 0$ this pins down R&D intensity, $\lambda^*$ that solves

$$c'(\lambda^*) = v = k_e$$

- Then from the incumbent's Bellman equation

$$(r + \mu)v = (\pi - c(\lambda) + v\lambda) \qquad \Rightarrow \qquad \eta^* = \frac{\pi - c(\lambda^*)}{k_e} - r$$

(or $\eta^* = 0$ if the last is negative, in which case $v < k_e$)

# Size Distribution

- Given these solutions for $\lambda$ and $\eta$, $\mu = \lambda + \eta$ we can compute the size distribution as follows.

- Law of motion is, for $n = 1$,

$$\dot{M}_1(t) = \eta + 2\mu M_2(t) - (\lambda + \mu)M_1(t)$$

- Similarly for $n = 2, 3, ...$

$$\dot{M}_n(t) = (n-1)\lambda M_{n-1}(t) + (n+1)\mu M_{n+1}(t) - n(\lambda + \mu)M_n(t)$$

- And, by our adding up condition, the total measure $M(t)$ follows

$$\dot{M}(t) = \eta - \mu M_1(t)$$

# Size Distribution

- Find *stationary distribution*, by setting time derivatives to zero.

- From the adding up condition

$$M_1 = \eta/\mu$$

- Plugging into the law of motion for $n = 1$ and solving for $M_2$

$$M_2 = ((\lambda + \mu)M_1 - \eta)/2\mu = \lambda\eta/(2\mu^2)$$

- And so on, by induction

$$M_n = \frac{\lambda^{n-1}\eta}{n\mu^n} = \frac{\theta}{n}\left(\frac{1}{1+\theta}\right)^n, \qquad \theta \equiv \eta/\lambda$$

(for $\lambda > 0, \eta > 0$)

# Size Distribution

- Total mass of firms

$$M = \sum_{n=1}^{\infty} M_n = \sum_{n=1}^{\infty} \frac{\theta}{n} \left( \frac{1}{1+\theta} \right)^n = \theta \log \left( \frac{1+\theta}{\theta} \right)$$

- So finally, size distribution $P_n \equiv M_n/M$ is given by

$$P_n = \frac{(1/(1+\theta))^n}{n \log((1+\theta)/\theta)}$$

the *logarithmic* or *log-series* distribution with parameter $1/(1+\theta)$.

- Endogenously skewed size distribution. Mean given by

$$\sum_{n=1}^{\infty} n P_n = \frac{1/\theta}{\log((1+\theta)/\theta)}$$

which is decreasing in $\theta \equiv \eta/\lambda$

  - when $\theta$ small, some firms have time to get very large
  - when $\theta$ large, entry dominates and there are many $n = 1$ firms

# General Equilibrium

- Horizontal varieties $j \in [0, 1]$.

- Inelastic supply of aggregate labor

$$L = L_X + L_S + L_R$$

  $L_X$ producing goods, $L_S$ in research at 'startups' trying to enter, $L_R$ in research at incumbent firms.

- Labor requirements for research

$l_S$ researchers for size 0 firm (entrant) to innovate at rate 1
(i.e., sunk entry cost is $k_e = wl_S$ for $w$ to be determined)

$l_R(\lambda)$ researchers for size 1 firm (incumbent) to innovate at rate $\lambda$
(i.e., innovation cost function is $c(\lambda) = wl_R(\lambda)$ for each $n$)

assumed strictly increasing, strictly convex in $\lambda$

# Stochastic Quality Ladders

- Each innovation (by new or incumbent) is *quality improvement* to randomly drawn variety $j \in [0, 1]$.

- Improvements arrive with endogenous Poisson intensity $\mu$.

- Let $J(j, t)$ denote *number of improvements* that have hit $j$ at time $t$, this is Poisson with intensity $\mu t$.

- Let $z_k(j)$ denote the *quality* of the $k$'th vintage of variety $j$

$$1 \equiv z_0(j) < z_1(j) < \cdots < z_k(j) < \cdots < z_{J(j,t)}(j)$$

- Quality step is *random* (not constant)

$$q_k(j) \equiv \frac{z_k(j)}{z_{k-1}(j)} > 1, \qquad q \sim \text{IID}\,\Psi(q)$$

# Preferences and Expenditure

- Representative household

$$U = \int_0^\infty e^{-\rho t} \log C(t)\, dt$$

- As before, varieties $j \in [0,1]$ are imperfect (Cobb-Douglas) substitutes while vintages $k \in \{0, \ldots, J(j,t)\}$ are perfect substitutes

$$\log C(t) = \int_0^1 \log \Big[ \sum_{k=0}^{J(j,t)} z_k(j) x_k(j,t) \Big]\, dj$$

- In equilibrium only highest quality vintage is sold, limit price

$$p(j,t) = wq(j,t), \qquad q(j,t) \equiv q_{J(j,t)}(j)$$

- Take aggregate expenditure as numeraire, $P(t)C(t) = E(t) = 1$. Then expenditure on each variety is

$$1 = p(j,t)x(j,t) \quad \Rightarrow \quad x(j,t) \equiv x(J(j,t),t) = \frac{1}{wq(j,t)}$$

# Profits

- Flow profit per variety $j$ is then

$$\pi(j,t) = (p(j,t) - w)x(j,t) = \frac{q(j,t) - 1}{q(j,t)}$$

- Quality step distribution $\Psi(q)$ implies profit distribution $\Psi(q^{-1}(\pi))$.

- Previous analysis goes through replacing $\pi$ with average profits

$$\bar{\pi} = \int_0^1 \left[1 - q(j,t)^{-1}\right] dj = 1 - \int_1^\infty q^{-1}\, d\Psi(q)$$

# Aggregate Growth

- Aggregate rate of innovation

$$\mu^* = \lambda^* + \eta^*$$

- Aggregate growth rate of consumption (and real wage etc)

$$g^* = \mu^* \log \bar{q}, \qquad \log \bar{q} \equiv \int_1^\infty \log q \, d\Psi(q)$$

- Simple comparative statics
  - increasing in labor force $L$ and in average profits $\bar{\pi}$
  - decreasing in impatience $\rho$ and in entry labor requirement $l_S$

(here presuming an 'interior' steady state with entry, $L_S > 0$, etc)

- Tractable merger of quality-ladder model with non-trivial firm dynamics.

# Misallocation

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# Hsieh/Klenow: Overview

- Background: large aggregate TFP differences across countries
  - US manufacturing TFP 2.3$\times$ China (in 1996)
  - US manufacturing TFP 2.6$\times$ India
- Why is aggregate TFP so low compared to the US?
  - traditional explanations focus on barriers to technology diffusion
  - misallocation explanation focuses on *inefficient use of technologies* (e.g., licensing regulations, size-dependent policies, SOEs, markups)
- Main findings
  - quantify misallocation from gaps in marginal products
  - larger gaps in China and India than US
  - can account for about half of aggregate TFP differences
  - shrinking gaps in China but not India
  - large plants have large marginal products in China and India

# Model

- Final output $Y$ a Cobb-Douglas aggregate of industry output

$$\log Y = \sum_{s=1}^{S} \theta_s \log Y_s$$

- Industry output a CES aggregate of $M_s$ differentiated products

$$Y_s = \left( \sum_{i=1}^{M_s} Y_{is}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1$$

- Firms produce with a Cobb-Douglas aggregate of capital and labor

$$Y_{is} = A_{is} K_{is}^{\alpha_s} L_{is}^{1-\alpha_s}, \qquad 0 < \alpha_s < 1 \qquad \text{for each } s = 1, ..., S$$

# Distortions

- Firm-specific (idiosyncratic) distortions.

- Individual firm faces two types of distortions

  **(i)** $\tau_{Y,is}$ distortions to marginal product of capital *and* labor

  **(ii)** $\tau_{K,is}$ distortions to marginal product of capital *relative to* labor

- Profits for an individual firm

$$\pi_{is} = (1 - \tau_{Y,is})\, P_{is} Y_{is} - w L_{is} - (1 + \tau_{K,is}) r K_{is}$$

- Distortions to labor can be obtained as combinations of $\tau_{Y,is}$ and $\tau_{K,is}$.

- Let $c(r, w, \alpha)$ denote the frictionless Cobb-Douglas factor price index

$$c(r, w, \alpha) = \left(\frac{r}{\alpha}\right)^{\alpha} \left(\frac{w}{1-\alpha}\right)^{1-\alpha}$$

- With distortions, equilibrium is given by

$$P_{is} = \frac{\sigma}{\sigma - 1} \frac{c(r, w, \alpha_s)}{A_{is}} \frac{(1 + \tau_{K,is})^{\alpha_s}}{(1 - \tau_{Y,is})}$$

$$Y_{is} = \left(\frac{P_{is}}{P_s}\right)^{-\sigma} Y_s$$

$$(1 + \tau_{K,is}) r K_{is} = \alpha_s \frac{c(r, w, \alpha_s)}{A_{is}} (1 + \tau_{K,is})^{\alpha_s} Y_{is}$$

$$w L_{is} = (1 - \alpha_s) \frac{c(r, w, \alpha_s)}{A_{is}} (1 + \tau_{K,is})^{\alpha_s} Y_{is}$$

- Goal is to use this structure to *infer* distortions from micro data.

# Key to Inference

- Focus on variation over $i$ within $s$

  **(i)** variation in observed capital/labor ratio reveals $\tau_{K,is}$

$$1 + \tau_{K,is} = \frac{\alpha_s}{1-\alpha_s} \times \frac{wL_{is}}{rK_{is}}$$

  **(ii)** variation in labor share reveals $\tau_{Y,is}$

$$1 - \tau_{Y,is} = \frac{\sigma}{\sigma-1}\left(\frac{1}{1-\alpha_s}\right) \times \frac{wL_{is}}{P_{is}Y_{is}}$$

- Residual demand $Y_{is} = (P_{is}/P_s)^{-\sigma} Y_s$ so revenue share

$$\frac{P_{is}Y_{is}}{P_sY_s} = \left(\frac{Y_{is}}{Y_s}\right)^{\frac{\sigma-1}{\sigma}}$$

- Given an estimate of elasticity $\sigma$, can infer quantities from revenues.

# TFPQ vs. TFPR

- We are interested in physical productivity $A_{is}$ but we can typically only measure *revenue productivity*

- Let TFPQ denote physical productivity and TFPR denote revenue productivity. Define them as follows

$$\text{TFPQ}_{is} \equiv \frac{Y_{is}}{K_{is}^{\alpha_s} L_{is}^{1-\alpha_s}} = A_{is}$$

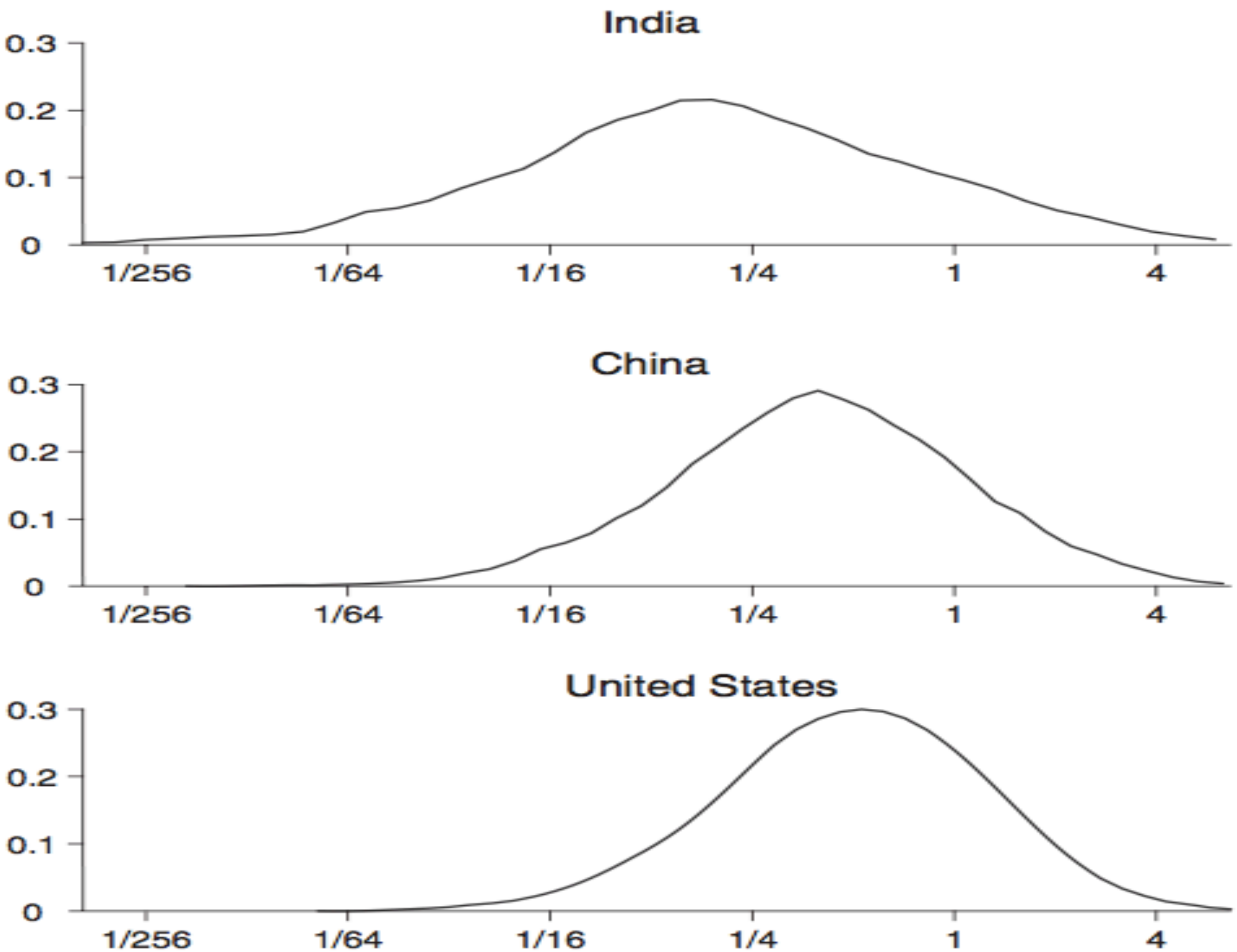
$$\text{TFPR}_{is} \equiv \frac{P_{is} Y_{is}}{K_{is}^{\alpha_s} L_{is}^{1-\alpha_s}} = P_{is} A_{is}$$

- In the efficient benchmark, TFPQ naturally varies across firms with $A_{is}$ but TFPR would be constant across firms (higher productivity firms charging proportionately lower prices).

- With distortions, firm-level TFPR is

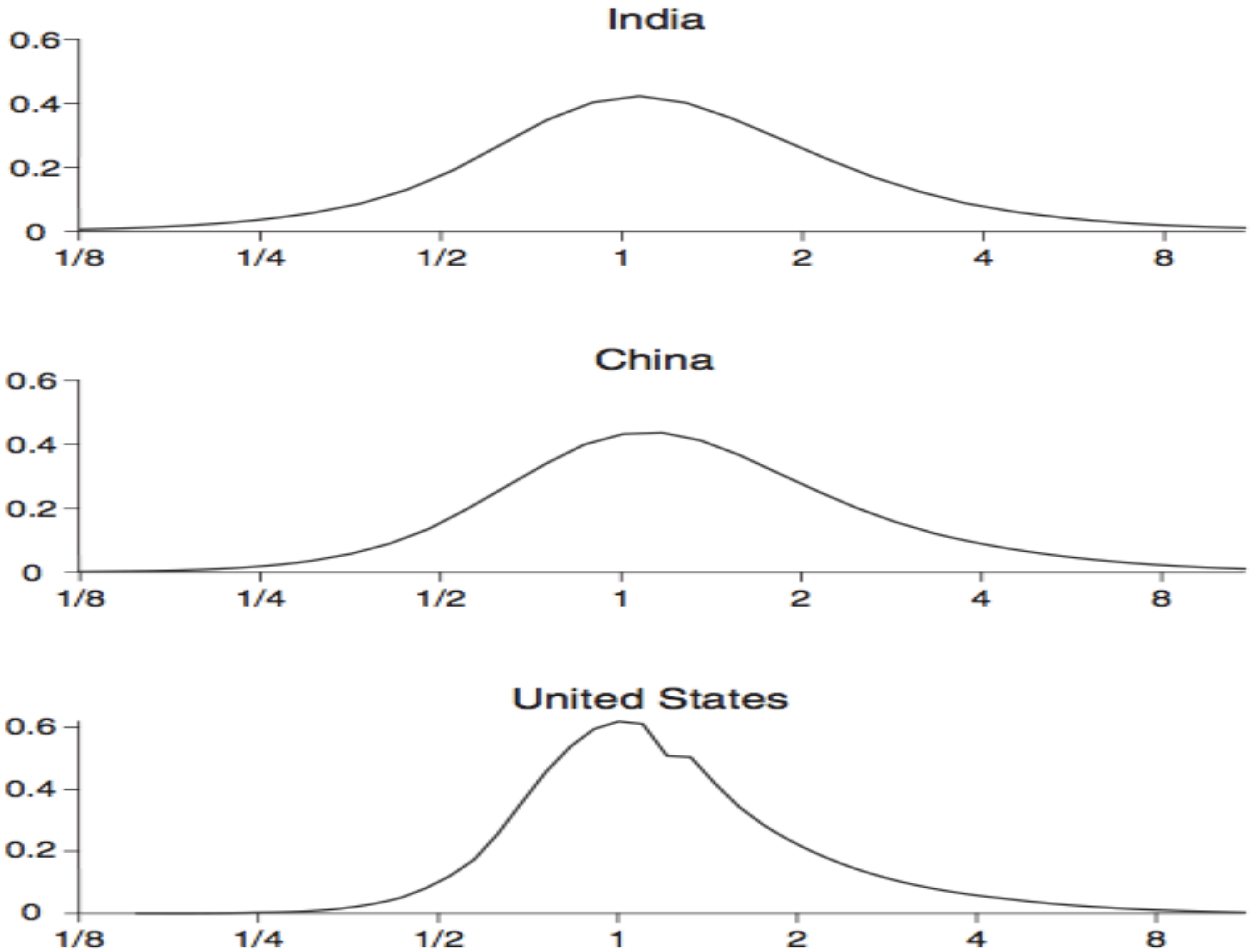
$$P_{is} A_{is} = \frac{\sigma}{\sigma - 1} c(r, w, \alpha_s) \frac{(1 + \tau_{K,is})^{\alpha}}{1 - \tau_{Y,is}}$$

# Distribution of TFPQ ($= A_{is}$)




Distributions for most recent year. Small firms underreported in Chinese data so US and India better comparison. Many more small plants in India.

# Distribution of TFPR ($= P_{is}A_{is}$)




All expressed relative to aggregate TFPR ($= P_s A_s$). Suggestive of larger distortions in India and China as compared to US.

# Sources of TFPR Variation Within Industries

| | Ownership | Age | Size | Region |
|---|---|---|---|---|
| India | 0.58 | 1.33 | 3.85 | 4.71 |
| China | 5.25 | 6.23 | 8.44 | 10.01 |

*Notes.* Entries are the cumulative percent of within-industry TFPR variance explained by dummies for ownership (state ownership categories), age (quartiles), size (quartiles), and region (provinces or states). The results are cumulative in that "age" includes dummies for both ownership and age, and so on.

For example, ownership accounts for only 0.6% of the variance in India but about 5% in China. Ownership *and* age account for 1.3% in India and 6.2% in China, etc.

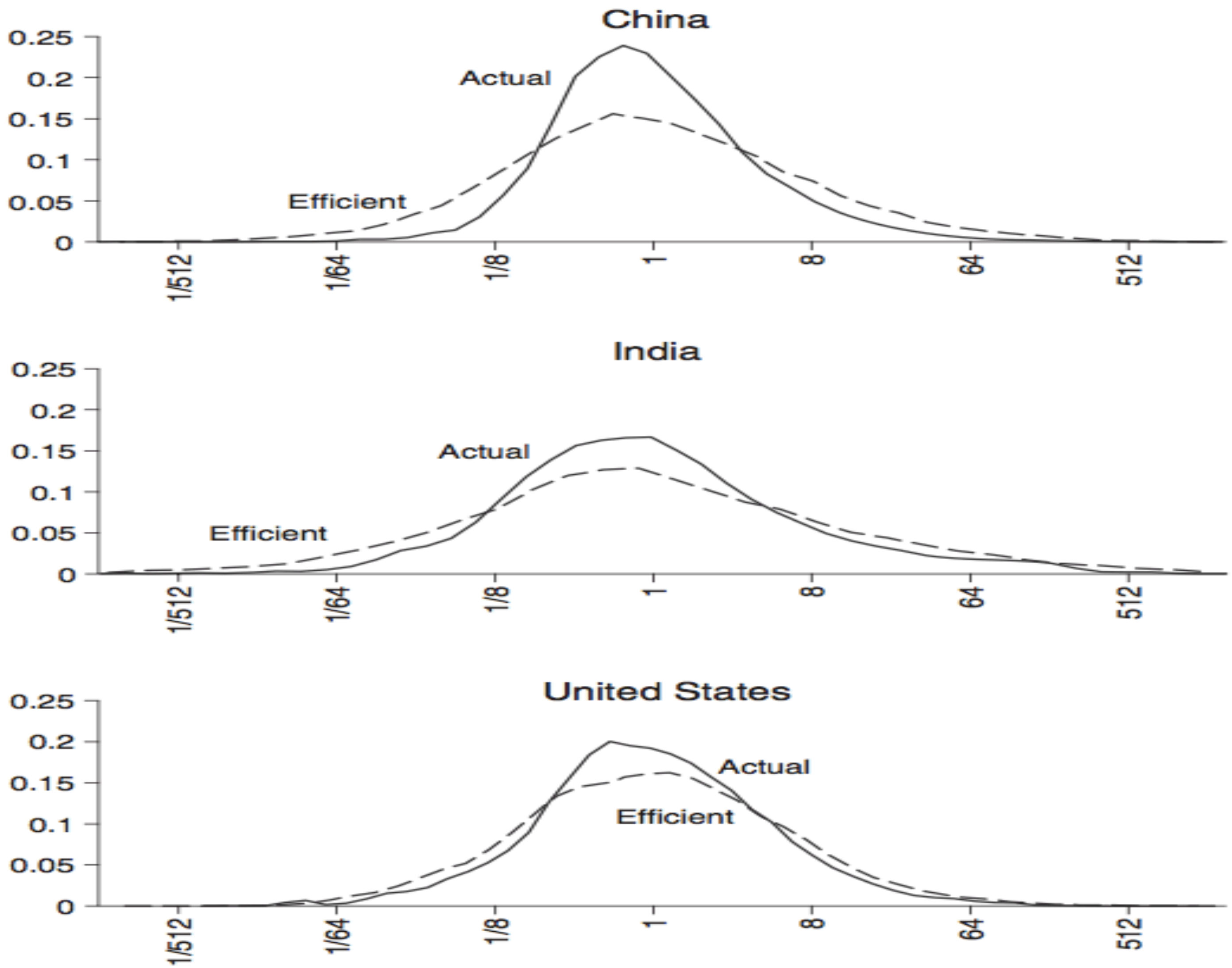
How large would the aggregate gains be if the cross-sectional allocation was more efficient?

# TFP Gains from Equal TFPR Within Industries

| China | 1998 | 2001 | 2005 |
|---|---|---|---|
| % | 115.1 | 95.8 | 86.6 |
| India | 1987 | 1991 | 1994 |
| % | 100.4 | 102.1 | 127.5 |
| United States | 1977 | 1987 | 1997 |
| % | 36.1 | 30.7 | 42.9 |

Gains from equalizing TFPR across all plants within each industry. Gains have been falling in China, suggesting actual distribution has been improving over time. Not so for India (and the US), at least in this sample.

# Distribution of Plant Size (= Value-Added)




Efficient distribution has *more dispersed plant size*, fewer middle but more large and more small plants.

# TFP gains from Equal TFPR, Relative to US

| China | 1998 | 2001 | 2005 |
|---|---|---|---|
| % | 50.5 | 37.0 | 30.5 |
| India | 1987 | 1991 | 1994 |
| % | 40.2 | 41.4 | 59.2 |

Gains from moving to "1997 US efficiency" (lowest US efficiency). Aggregate manufacturing TFP differences based on Penn World Tables suggest US TFP in 1998 was 2.3 times China and 2.6 times India. So reallocation could account for about $\log(1.5)/\log(2.3) \approx 0.49$ of the difference between China and the US.

Welfare gains would be magnified by endogenous capital accumulation.

The start of a huge literature, in macro, trade, and development.

# Outline

1. **Benchmark quality-ladder model**

2. **Firm dynamics in a quality-ladder model: Klette-Kortum (2004).**

3. **Static misallocation: Hsieh-Klenow (2009).**

4. **Dynamic misallocation in a quality-ladder model: Peters (2020).**

# Overview

- Motivation
  - Hsieh/Klenow (2009) takes marginal product gaps etc as exogenous
  - firms with higher TFPR are more 'constrained'
- Peters (2020) interpretation: misallocation through *endogenous markups*
  - quality ladder model with entry (simplified Klette/Kortum)
  - markups depend on productivity gap between incumbent and rivals
  - incumbent and entrant innovation determines productivity gaps
  - implied markup distribution is Pareto, thicker tails when low entry

# Model

- Continuous time $t \geq 0$.

- Quality ladder setup. Final output

$$\log Y(t) = \int_0^1 \log \Big[ \sum_{k=0}^{J(j,t)} y_k(j,t) \Big] \, dj$$

  horizontally differentiated intermediate goods $j \in [0,1]$, each of which comes in $k \in \{0, 1, \ldots, J(j,t)\}$ vertically differentiated vintages.

- Intermediate producers are heterogeneous in productivity.

- Intermediate producer with productivity $a$ has production function

$$y = a k^{\alpha} l^{1-\alpha}, \qquad 0 < \alpha < 1$$

  taking input prices $r$ and $w$ as given

# Costs, Pricing and Markups

- Marginal cost of intermediate producer with productivity $a$

$$\frac{c(r,w)}{a}$$

- Most efficient producer takes whole market and *limit prices*, sets price equal to marginal cost of *second-best* producer, the closest rival.

- Hence best producer has price

$$p^1 = \frac{c(r,w)}{a^2}$$

where $a^2$ is the productivity of the second-best producer.

- Then best producer has *markup* equal to its relative productivity

$$m \equiv \frac{p^1}{c(r,w)/a^1} = \frac{a^1}{a^2}$$

# Static Allocation

- For intermediate good $j$ with market taken by $a^1(j)$ producer

$$y(j) = \frac{1}{c(r,w)} \frac{a^1(j)}{m(j)} PY, \qquad m(j) = \frac{a^1(j)}{a^2(j)}$$

$$k(j) = \alpha \frac{c(r,w)}{a^1(j) r} y(j) = \frac{1}{m(j)} \frac{\alpha}{r} PY$$

$$l(j) = (1-\alpha) \frac{c(r,w)}{a^1(j) w} y(j) = \frac{1}{m(j)} \frac{(1-\alpha)}{w} PY$$

$$\pi(j) = \left( p(j) - \frac{c(r,w)}{a^1(j)} \right) y(j) = \left( \frac{m(j)-1}{m(j)} \right) PY$$

- But still need to determine distribution of relative productivities.

# TFPR

- Physical productivity of a producer is just $a^1(j)$.

- Revenue productivity is

$$p(j)a^1(j) = c(r, w)m(j)$$

- Since $c(r, w)$ is common, all cross-sectional variation in TFPR is coming from markup variation (i.e., relative productivity variation).

- Remark. In Hsieh/Klenow all cross-sectional variation in TFPR is coming from $(\tau_K, \tau_Y)$ variation and high TFPR indicates more distorted ('constrained') firms.

  But here, high TFPR indicates firms with high relative productivity.

# Aggregation

- Define aggregate productivity by

$$A \equiv \frac{Y}{K^{\alpha} L^{1-\alpha}}$$

where $K$ and $L$ are aggregate capital and labor used in production.

- Summing input demands over intermediate producers

$$K = \int_0^1 k(j)\, dj = \frac{\alpha}{r} PY \int_0^1 \frac{1}{m(j)}\, dj$$

$$L = \int_0^1 l(j)\, dj = \frac{1-\alpha}{w} PY \int_0^1 \frac{1}{m(j)}\, dj$$

# Aggregate TFPR

- Taking the geometric average of $K$ and $L$

$$K^\alpha L^{1-\alpha} = \left(\frac{\alpha}{r}\right)^\alpha \left(\frac{1-\alpha}{w}\right)^{1-\alpha} \left(\int_0^1 \frac{1}{m(j)}\, dj\right) PY$$

where the coefficient out the front is just $1/c(r,w)$.

- Hence aggregate TFPR is

$$PA = \frac{PY}{K^\alpha L^{1-\alpha}} = c(r,w) \left(\int_0^1 \frac{1}{m(j)}\, dj\right)^{-1}$$

- Need to decompose this into $P$ and $A$ using price index.

# Aggregate Price Index $P$

- With Cobb-Douglas preferences over the intermediates

$$\log P = \int_0^1 \log p(j)\, dj$$

(this is the limit of the usual CES index as $\sigma \to 1^+$)

- Plugging in for individual prices

$$\log P = \int_0^1 \log \left( m(j) \frac{c(r,w)}{a^1(j)} \right) dj$$

or

$$P = c(r,w) \exp \left( \int_0^1 \log \left( \frac{m(j)}{a^1(j)} \right) dj \right)$$

# Aggregate Productivity $A$

- Hence we can write aggregate productivity as

$$A = \frac{\exp\left(\int_0^1 \log\left(\frac{a^1(j)}{m(j)}\right) dj\right)}{\int_0^1 \frac{1}{m(j)}\, dj} = A^* D,$$

product of benchmark productivity $A^*$ and distortion index $D$.

- *Benchmark productivity* (first-best productivity)

$$A^* \equiv \exp\left(\int_0^1 \log a^1(j)\, dj\right)$$

- *Distortion index*

$$D \equiv \exp\left(-\int_0^1 \log m(j)\, dj\right) \Big/ \int_0^1 \frac{1}{m(j)}\, dj$$

# TFP Distortion Index

- Write the TFP distortion index as

$$D = \frac{\exp\Big(-\mathbb{E}[\,\log m\,]\Big)}{\mathbb{E}[\,1/m\,]}$$

- By Jensen's inequality $D \leq 1$ and $= 1$ only if $m$ degenerate.

- Is homogeneous degree zero in $m$: *A pure level shift in markups does not reduce aggregate productivity*.

- Is decreasing in a mean-preserving-spread of $\log m$: *More dispersed markups do reduce aggregate productivity*.

# Quality Ladder Dynamics

- Firm productivity follows ladder with constant step-size $q > 1$.

- If producer has had $n(j,t)$ innovations at $t$, their productivity is

$$a(j,t) = q^{n(j,t)}$$

- Markup is therefore

$$m(j,t) = \frac{a^1(j,t)}{a^2(j,t)} = \frac{q^{n^1(j,t)}}{q^{n^2(j,t)}} = q^{\Delta(j,t)}, \qquad \Delta \equiv n^1 - n^2 \geq 1$$

- Entry gives access to the current leading technology

$$qa^1(j,t)$$

# Innovation and Markups

Note contrast:

- Incumbent innovation: *increases* $a^1$ relative to $a^2$, increases markup by factor $q > 1$.
- Entrant innovation: *decreases* markup, by factor $q^{\Delta - 1}$.
- Innovating incumbent may be multiple steps ahead, innovating entrant is only one step ahead.

# Quality Gap Dynamics

- Let $\lambda$ denote incumbent innovation rate and let $\eta$ denote entry rate (both endogenous, will be constant along balanced growth path).

- Let $M(\Delta, t)$ denote measure of intermediates with quality gap $\Delta$.

- Law of motion

$$\dot{M}(\Delta, t) = -(\eta + \lambda) M(\Delta, t) + \lambda M(\Delta - 1, t), \qquad \text{for } \Delta \geq 2$$

and

$$\dot{M}(1, t) = -(\eta + \lambda) M(1, t) + \eta$$

# Stationary Quality Gap Distribution

- Setting $\dot{M}(\Delta, t) = 0$ for each $\Delta$ we have

$$M(1) = \frac{\eta}{\eta + \lambda}$$

and

$$M(\Delta) = \frac{\lambda}{\eta + \lambda} M(\Delta - 1), \qquad \text{for } \Delta \geq 2$$

- Iterating backwards we get

$$M(\Delta) = \left(\frac{\lambda}{\eta + \lambda}\right)^{\Delta - 1} \frac{\eta}{\eta + \lambda}$$

$$= \left(\frac{\lambda}{\eta + \lambda}\right)^{\Delta} \frac{\eta}{\lambda} = \left(\frac{1}{1 + \theta}\right)^{\Delta} \theta, \qquad \theta \equiv \frac{\eta}{\lambda}$$

# Quality Gaps and Markup Distribution

- Cumulative quality gap distribution

$$F_{\Delta}(n) \equiv \text{Prob}[\Delta \leq n] = \sum_{k=1}^{n} M(k) = 1 - \left(\frac{1}{1+\theta}\right)^{n}$$

- *Markup distribution is Pareto*

$$F(m) \equiv \text{Prob}[q^{\Delta} \leq m] = \text{Prob}[\Delta \leq \log m / \log q]$$

$$= 1 - m^{-\xi(\theta)}, \qquad \xi(\theta) \equiv \frac{\log(1+\theta)}{\log q}$$

with Pareto tail parameter $\xi(\theta)$ given by *entry intensity* $\theta \equiv \eta/\lambda$ (rate of entrant to incumbent innovation, as in Klette/Kortum above).

# Distortion Index $D \equiv \exp\Big(-\mathbb{E}[\log m]\Big)/\mathbb{E}[1/m]$

- Approximate moments, *treating markups as continuous*

$$\mathbb{E}[1/m] \approx \int_1^\infty (1/m)\, dF(m) = \int_1^\infty (1/m)\xi(\theta)(1/m)^{\xi(\theta)+1}\, dm = \frac{\xi(\theta)}{\xi(\theta)+1}$$

$$\mathbb{E}[\log m] \approx \int_1^\infty (\log m)\, dF(m) = \int_0^\infty z\, \exp(-\xi(\theta)z)\, dz = \frac{1}{\xi(\theta)}$$

- Gives distortion index

$$D(\theta) \approx \exp\Big(-\frac{1}{\xi(\theta)}\Big)\frac{\xi(\theta)+1}{\xi(\theta)}$$

- Remark. Can compute exact moments using discrete distribution of gaps $\Delta$, but messier.

# Effects of Higher Entry

- A higher entry intensity $\theta = \eta/\lambda$
  - reduces $F(m)$ in FOSD sense ($F(m)$ increasing in $\theta$ for all $m$)
  - increases $\xi(\theta)$ and hence reduces markup dispersion
  - reduces wedge between $A$ and first-best $A^*$, thereby increasing aggregate productivity
  - reduces wedge between factor prices and marginal products
- Now need to actually pin down innovation rates $\eta, \lambda$

# Innovation and Entry Costs

- Convex innovation cost function for incumbents

$$c(\lambda, \Delta) = q^{-\Delta} \lambda^{\gamma}, \qquad \gamma > 1$$

- This is the amount of labor required for an incumbent with advantage $\Delta$ to generate flow innovation rate $\lambda$.

- Workers generate ideas with Poisson intensity 1 (normalization), 'blueprint' operational after paying startup $l_S > 0$ units of labor

# Bellman Equation for Incumbents

- Focus on a balanced growth path with growth $g$, to be determined.

- Let $V(\Delta)$ denote the (detrended) value of a firm with quality gap $\Delta$. This can be written

$$(r+\eta)V(\Delta) = \pi(\Delta) + \max_{\lambda \geq 0} \left[ \lambda(V(\Delta+1) - V(\Delta)) - wq^{-\Delta}\lambda^{\gamma} \right] + gV(\Delta)$$

- Can show that, along BGP, value function $V(\Delta)$ has the form

$$V(\Delta) = v_0 - v_1 q^{-\Delta}$$

for some constants $v_0, v_1$ to be determined.

# General Equilibrium

- Representative consumer with preferences over final good

$$U = \int_0^\infty e^{-\rho t} \log C(t)\, dt$$

- Along BGP we then have

$$r = \rho + g$$

- With constant quality step $q$ and constant innovation rates

$$g = \frac{1}{1-\alpha}(\log q)(\lambda + \eta)$$

- To complete the solution of the model, need to solve for aggregate $\lambda, \eta$ etc.

# General Equilibrium

- Problem can be reduced to finding constants

$$(\eta^* \,,\, w^*)$$

consistent with firm optimization, i.e., $v_0(\eta, w), v_1(\eta, w), \lambda(\eta, w)$, and

**(i)** free entry condition

$$V(1) = (v_0 - v_1 q^{-1}) \leq w l_S$$

**(ii)** labor market clearing

$$L_X + L_R + L_S = L$$

- Compute equilibrium by solving fixed point problem in $\eta, w$.

- Then implies innovation intensity $\lambda^* = \lambda(\eta^*, w^*)$ etc.

# Calibrated Markup Distribution

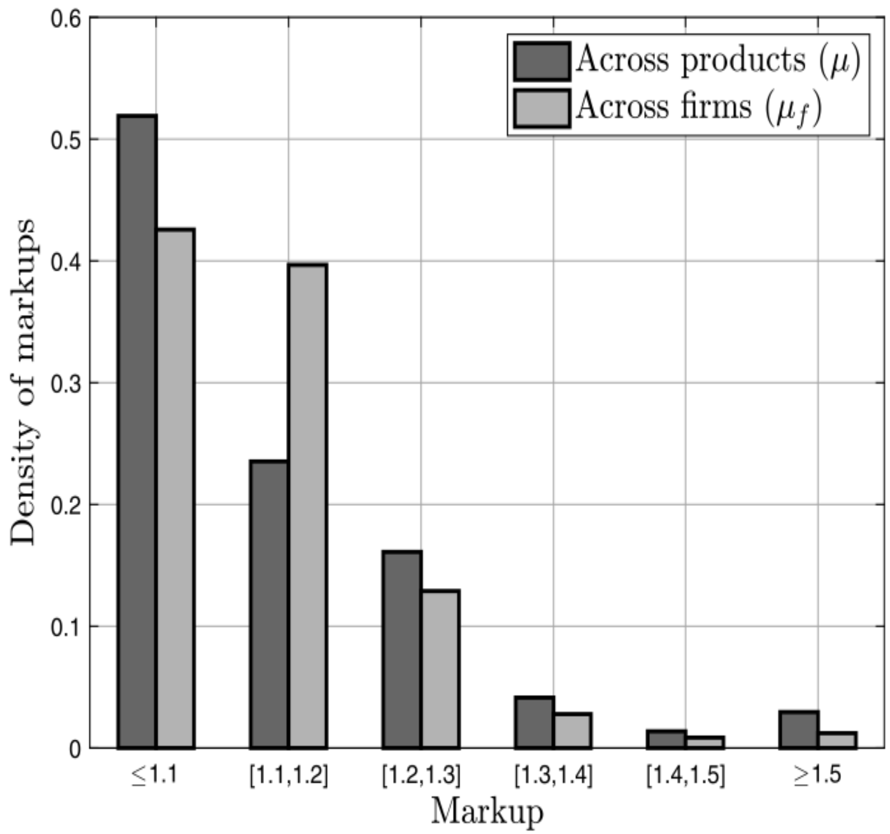



FIGURE 5.—THE STATIONARY DISTRIBUTION OF MARKUPS. *Notes*: The figure shows the distribution of markups at the product level (dark bars) and at the firm level (light bars). The results are based on the calibration reported in Table III.

# Application to Indonesian Manufacturing

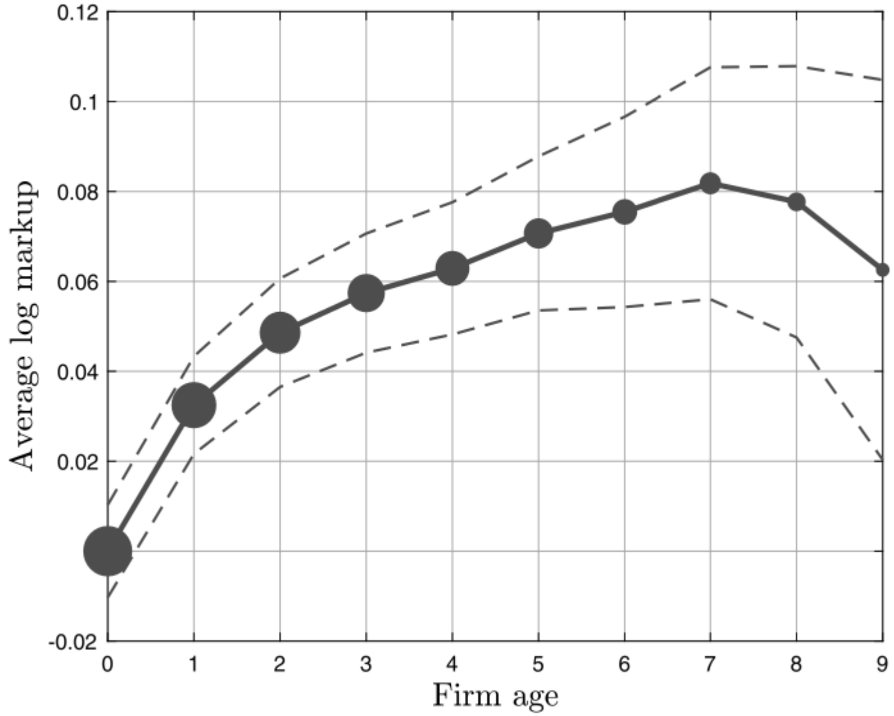
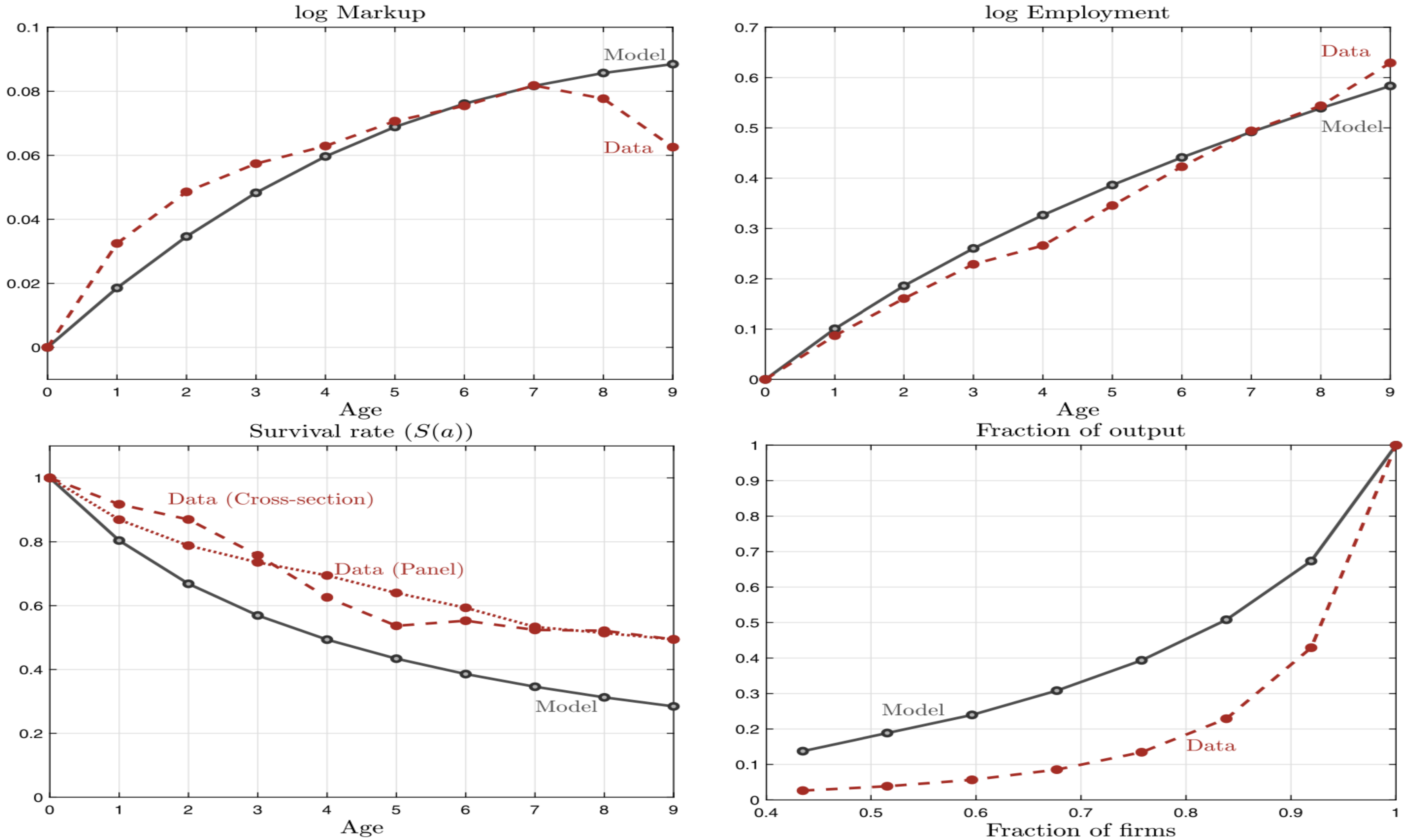



FIGURE 3.—THE LIFE-CYCLE OF MARKUPS IN INDONESIA. *Notes*: I focus on the unbalanced panel of firms entering the economy after 1990. I calculate log markups within 5-digit-industry-year cells, then calculate the average by the age of the cohort and normalize log markups of entering cohorts to zero. Because of attrition, the size of the cohort is declining in age. The dots reflect the size of the cohort. I also depict the 90% confidence intervals around the estimated average profile.

# Application to Indonesian Manufacturing



# Application to Indonesian Manufacturing

TABLE V

MARKUPS AND MISALLOCATION IN INDONESIA[a]

| Markups and Misallocation | | | | | |
|---|---|---|---|---|---|
| $\theta$ | $E[\mu]$ | $\sigma(\ln\mu)$ | $\sigma(\ln\mu_f)$ | $\mathcal{M}$ | $\Lambda$ |
| 9.5 | 11.8% | 0.103 | 0.079 | 0.995 | 0.9 |

[a]The table reports the endogenous tail parameter of the markup distribution $\theta$, the average markup ($E[\mu]$), the dispersion of log markups across products ($\sigma(\ln\mu)$) and firms ($\sigma(\ln\mu_f)$), and the two misallocation wedges $\mathcal{M}$ and $\Lambda$ (see (3) and (4)).

# Related Work

- If you find this work on endogenous markups interesting:

  - Edmond, Midrigan and Xu (2015) *Competition, Markups, and the Gains from International Trade* AER.

    (do reductions in trade barriers imply large 'pro-competitive' productivity gains from reduced misallocation? )

  - Edmond, Midrigan and Xu (2021) *How Costly Are Markups?* JPE r&r.

    (how large are the welfare costs of markup distortions? macro model calibrated to US census micro data)

# Thanks!