14.452 – Economic Growth

Final Exam, Fall 2021

This is an open-book exam. You may consult any materials you like, including materials on a tablet or laptop — but no use of the internet is permitted. Please answer all questions. Read the questions carefully before starting to answer. If you think you need to make additional assumptions to answer some questions, state them clearly in your answer. You have 180 minutes. Good luck!

Exercise 1. (35 points)

Capital accumulation and competition. Consider a neoclassical growth model in continuous time. Final output $Y(t)$ is the numeraire and is produced by *perfectly competitive* firms using a CES bundle of intermediate goods

$$Y(t) = \left(\int_0^1 y(v,t)^{\frac{\sigma-1}{\sigma}} \, dv \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1$$

The final good firms buy intermediate goods at prices $p(v,t)$ from intermediate producers $v \in [0,1]$. The intermediate producers are *monopolistically competitive* and choose prices $p(v,t)$ and output $y(v,t)$ to maximize their profits understanding their market power. The intermediate producers are symmetric. Each has the Cobb-Douglas production function

$$y(v,t) = k(v,t)^{\alpha} \, l(v,t)^{1-\alpha}, \qquad 0 < \alpha < 1$$

and take the wage $W(t)$ and rental rate of capital $R(t)$ as given.

The representative household inelastically supplies $L > 0$ units of labor and chooses consumption $C(t) \geq 0$ of the numeraire final good to maximize

$$\int_0^\infty e^{-\rho t} \, u(C(t)) \, dt$$

subject to the flow budget constraint

$$C(t) + \dot{K}(t) = W(t)L + (R(t) - \delta)K(t) + \Pi(t)$$

where $\Pi(t)$ denotes the aggregate profits of the intermediate firms distributed lump-sum.

The factor markets clear when $\int_0^1 l(v,t) \, dv = L$ and $\int_0^1 k(v,t) \, dv = K(t)$.

(a) [*7 points*] Set up a current-value Hamiltonian for the representative household and derive the key optimality conditions for $C(t)$ and $K(t)$. Explain how the time paths for $C(t)$ and $K(t)$ are determined.

(b) [*7 points*] Show that the final good producers' have residual demand curves

$$y(v,t) = p(v,t)^{-\sigma}Y(t)$$

Given these demand curves, explain why the equilibrium price of each intermediate is

$$p(v,t) = \frac{\sigma}{\sigma-1}\,c(R(t),W(t)), \qquad c(R,W) \equiv \left(\frac{R}{\alpha}\right)^{\alpha}\left(\frac{W}{1-\alpha}\right)^{1-\alpha}$$

where $c(R,W)$ is the intermediate producers' *marginal cost*.

(c) [*7 points*] Show that in equilibrium the factor shares in this economy are

$$\frac{R(t)K(t)}{Y(t)} = \frac{\sigma-1}{\sigma}\alpha, \qquad \frac{W(t)L}{Y(t)} = \frac{\sigma-1}{\sigma}(1-\alpha)$$

Give intuition for these expressions. Given these expressions, calculate monopoly profits as a share of final output, $\Pi(t)/Y(t)$. How do these profits depend on σ?

(d) [*7 points*] Solve for the steady-state values C^*, K^* and the capital/output ratio K^*/Y^* in terms of model parameters. How do these values compare to what they would be if the economy was perfectly competitive ($\sigma \to \infty$)? Does monopoly power lead to more or less capital accumulation in the long run? Explain.

(e) [*7 points*] Suppose the economy is in steady-state when suddenly σ decreases permanently to $\sigma' \in (1,\sigma)$. Explain the transitional dynamics of consumption $C(t)$ and capital $K(t)$ as the economy adjusts to this change.

Exercise 2. (30 points)

In his book *American and British Technology in the Nineteenth Century: The Search for Labor-Saving Inventions*, the economic historian John Habakkuk argues that labor scarcity and the search for labor-saving inventions were key to technological progress. In particular, Habakkuk argued that US technological progress in the 19th century was faster than in Britain because of the relative labor scarcity in the US (which pushed up wages and so encouraged firms to find adopt labor-saving technologies).

(a) [*10 points*] Suppose numeraire final output is produced with two factors, technology A

and labor L, according to

$$Y = F(A, L) = A^{\alpha} L^{1-\alpha}, \qquad 0 < \alpha < 1$$

The representative firm chooses L taking as given a competitive wage w and chooses A subject to a strictly convex technology adoption cost function

$$c(A) = \frac{A^{1+\varphi}}{1+\varphi}, \qquad \varphi > 0$$

Consider two countries $i = 1, 2$ identical except that country i has labor supply L_i with $L_1 > L_2$. Compare the equilibrium wages w_i and technology choices A_i for countries $i = 1, 2$. Are these findings consistent with the Habakkuk thesis? Why or why not? Give intuition for your findings.

(b) [*10 points*] Consider a lab-equipment model of *directed technological change.* Suppose there are two factors, labor L and land Z both in inelastic supply. Following the usual arguments, along a balanced growth path the equilibrium mix of technologies is

$$\frac{N_L}{N_Z} = \left(\frac{\eta_L}{\eta_Z}\right)^{\sigma} \left(\frac{\gamma_L}{\gamma_Z}\right)^{\varepsilon} \left(\frac{L}{Z}\right)^{\sigma-1}$$

where $\sigma > 0$ denotes the elasticity of substitution between L and Z, where η_L, η_Z denote the arrival rates of innovations in each sector, and where γ_L, γ_Z denote the 'distribution parameters' and $\varepsilon > 0$ denotes the elasticity of substitution in the final goods aggregator. The equilibrium factor price ratio is

$$\frac{w_L}{w_Z} = \left(\frac{\gamma_L}{\gamma_Z}\right)^{\frac{\varepsilon}{\sigma}} \left(\frac{N_L}{N_Z}\right)^{\frac{\sigma-1}{\sigma}} \left(\frac{L}{Z}\right)^{-\frac{1}{\sigma}}$$

For what, if any, values of σ does this model provide results consistent with the Habakkuk thesis? Explain.

(c) [*10 points*] Along a balanced growth path, the real interest rate in this economy is

$$r = \left(\gamma_L^{\varepsilon}(\eta_L L)^{\sigma-1} + \gamma_Z^{\varepsilon}(\eta_Z Z)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \bar{\pi}$$

Now consider two countries $i = 1, 2$ identical except that country i has relative factor supply L_i/Z_i with $L_1/Z_1 > L_2/Z_2$. Can we determine which of these countries *grows* faster? Is this consistent with the Habakkuk thesis? Explain.

Exercise 3. (35 points)

Automation in an OLG economy. Consider a two-period OLG model where there are a constant $L > 0$ young workers at each date $t = 0, 1, 2, \ldots$ each of whom is endowed with 1 unit of labor supplied inelastically and with preferences

$$U_t = \log c_t^1 + \beta \log c_{t+1}^2, \qquad 0 < \beta < 1$$

where c_t^a denotes their consumption at age $a = 1, 2$. Each worker maximizes utility subject to the budget constraints

$$0 \leq c_t^1 \leq W_t - s_t, \qquad \text{and} \qquad 0 \leq c_{t+1}^2 \leq R_{t+1} s_t$$

Production is as in a *task-based* model. The numeraire final good Y_t is produced by perfectly competitive firms using a Cobb-Douglas bundle of tasks

$$\log Y_t = \int_{N-1}^{N} \log y_t(z)\, dz$$

for some given interval $z \in [N-1, N]$. All tasks can be done by labor, but some tasks can be done by labor or capital. In particular, there is a threshold task I such that the production technology for tasks $z > I$ is

$$y_t(z) = A_L l_t(z), \qquad z > I$$

while the production technology for tasks $z \leq I$ is

$$y_t(z) = A_K k_t(z) + A_L l_t(z), \qquad z \leq I$$

for some constant coefficients $A_L, A_K > 0$.

Each task is produced under competitive conditions taking as given the wage rate W_t and the rental rate R_t. Capital fully depreciates each period. The factor markets clear when $\int_{N-1}^{N} l_t(z)\, dz = L$ and $\int_{N-1}^{N} k_t(z)\, dz = K_t$.

To simplify the analysis, we tentatively suppose that W_t, R_t are such that

$$\frac{R_t}{A_K} < \frac{W_t}{A_L} \tag{$*$}$$

(a) [*10 points*] Let $Y_t = F(K_t, L)$ denote the aggregate production function for this economy. Show that this has the form

$$Y_t = F(K_t, L) = Z K_t^{s_K} L^{s_L}$$

for some $Z > 0$ and $s_K, s_L > 0$. Solve for Z, s_K, s_L in terms of the underlying parameters of the model.

(b) [*8 points*] Show that in order for condition $(*)$ to hold the aggregate capital stock K_t must exceed a certain threshold

$$K_t > \hat{K}$$

Provide a formula for this threshold $\hat{K}$ in terms of the underlying parameters of the model and give economic intuition for this condition.

(c) [*5 points*] Solve for the steady state capital stock. Is condition $(*)$ always satisfied in steady state? Explain.

To simplify the following algebra, suppose $A_K = A_L = A = (1+\beta)/\beta$ and $N = L = 1$.

(d) [*4 points*] Show that condition $(*)$ is satisfied in steady state if and only if the automation threshold is

$$I < I^*$$

for some I^* to be determined.

(e) [*8 points*] Now suppose $I < I^*$ and the automation threshold increases to some $I' \in (I, I^*)$. Explain the effect of the increase from I to I' on steady-state wages. Compare your results to the static automation model with exogenous K discussed in class. Does capital accumulation amplify either the 'displacement effect' or the 'labor productivity effect'? Explain.