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ECON90003
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Macroeconomics
Tutorial #9

1. **Income fluctuations with CARA utility**. Consider a single risk averse household that takes as given a constant interest rate $r > 0$ and that seeks to maximize

$$\mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^t\, u(c_t)\right\}, \qquad 0<\beta<1$$

subject to

$$c_t + a_{t+1} = (1+r)a_t + y_t$$

The household's income $y_t > 0$ fluctuates according to the autoregression

$$y_{t+1} = (1-\phi)\bar{y} + \phi y_t + \varepsilon_{t+1}, \qquad \bar{y}>0, \qquad 0<\phi<1$$

where the innovations ε_{t+1} are IID $N(0,\sigma_\varepsilon^2)$.

   (a) Let $v(a,y)$ denote the household's value function. Setup and explain the Bellman equation that determines $v(a,y)$.

Now suppose the utility function has the constant *absolute* risk aversion (CARA) form

$$u(c) = -\frac{\exp(-\alpha\, c)}{\alpha}, \qquad \alpha>0$$

   (b) Show that the value function that solves the Bellman equation is given by

$$v(a,y) = -\frac{\exp(-A\,(a+By+C))}{A}$$

   for some coefficients A, B, C. Solve for the coefficients A, B, C in terms of the parameters.

   (c) Let $c(a,y)$ denote the optimal consumption policy function. Solve for $c(a,y)$.

Now consider a Huggett-style incomplete markets model with many such households. Suppose the asset a is in zero net supply.

   (d) Define a stationary equilibrium for this economy. Give a computational procedure that would allow you to solve for a stationary equilibrium.