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ECON90003
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Macroeconomics
Tutorial #8

1. **Tauchen-Hussey by hand.** Consider the continuous-state AR(1) process

$$x_{t+1} = (1-\rho)\bar{x} + \rho x_t + \varepsilon_{t+1}, \qquad 0 < \rho < 1$$

where the innovations ε_t are

$$\varepsilon_t \sim \text{IID } N(0, \sigma^2)$$

This process is completely characterized by three parameters: $\bar{x}, \rho$ and σ. Now consider a 2-state Markov chain on x_i, $i = 1, 2$ with transition probabilities $p_{ij} = \text{Prob}[x_{t+1} = x_j \,|\, x_t = x_i]$ for $i, j = 1, 2$ given by a symmetric matrix of the form

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}, \qquad 0 < p < 1$$

This 2-state Markov chain is completely characterized by three parameters, x_1, x_2 and p. We can make this 2-state Markov chain *mimic* the continuous-state AR(1) by choosing the three parameters x_1, x_2 and p to match three moments of the AR(1). The natural moments to target are the unconditional mean $\mathbb{E}[x]$, unconditional variance $\text{Var}[x]$, and AR(1) coefficient.

Using this approach, derive expressions for the three parameters of the Markov chain x_1, x_2 and p that will allow you to mimic a continuous-state AR(1) with parameters $\bar{x}, \rho$ and σ.

2. **Discount factor shocks.** Consider a Huggett-style incomplete markets model where individual households face *discount factor* shocks. In particular, their time discount factor $\beta_t \in (0, 1)$ evolves according to a Markov chain with transition probabilities $\phi(\beta' \,|\, \beta) = \text{Prob}[\beta_{t+1} = \beta' \,|\, \beta_t = \beta]$. Their preferences are represented recursively by

$$v_t = u(c_t) + \beta_t \mathbb{E}_t\big[v_{t+1}\big]$$

which they maximize subject to the sequence of budget constraints

$$c_t + q a_{t+1} \leq a_t + y_t$$

and a borrowing constraint of the form $a_{t+1} \geq -\phi$ for some ϕ. Their income $y_t > 0$ evolves exogenously according to a Markov chain with transition probabilities $\pi(y' \,|\, y) = \text{Prob}[y_{t+1} = y' \,|\, y_t = y]$.

   (a) Let $v(a, y, \beta)$ denote the value function of a household of type a, y when their discount factor is β. Setup the household dynamic programming problem in terms of this value function and define a stationary equilibrium for this economy.

   (b) Give a computational procedure that would allow you to solve for a stationary equilibrium.