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ECON90003
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Macroeconomics
Tutorial #7

1. **Complete markets with CRRA preferences.** Suppose there are $i = 1, ..., I$ individuals with stochastic endowments $y_t^i(s^t)$ given by probabilities $\pi_t(s^t)$ and that these individuals all evaluate payoffs using the same expected utility function

$$U(c^i) = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c_t^i(s^t))\pi_t(s^t), \qquad 0 < \beta < 1$$

Moreover suppose that $u(c)$ has the constant coefficient of relative risk aversion (CRRA) form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \qquad \sigma > 0$$

(a) Consider a social planner that chooses $c^i = \{c_t^i(s^t)\}_{t=0}^{\infty}$ for each i to maximize the social welfare function

$$W = \sum_i \lambda_i U(c^i)$$

subject to the sequence of resource constraints

$$\sum_i c_t^i(s^t) \leq \sum_i y_t^i(s^t)$$

where the $\lambda_i \geq 0$ denote a set of given welfare weights. Solve for the planner's consumption allocation. What are the key *cross-sectional* properties of the consumption allocation? What are the key *time-series* properties of the consumption allocation? Explain how these depend on the coefficient of risk aversion σ and on the properties of the endowment processes $y_t^i(s^t)$.

(b) Now consider an Arrow-Debreu market economy where individuals can trade at time $t = 0$ in a complete set of contingent claims with prices $q_t^0(s^t)$ subject to the single intertemporal budget constraint

$$\sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) c_t^i(s^t) \leq \sum_{t=0}^{\infty} \sum_{s^t} q_t^0(s^t) y_t^i(s^t)$$

Let $\mu_i \geq 0$ denote the multiplier on an individual's intertemporal budget constraint. Solve for the equilibrium consumption allocation and the equilibrium prices. Explain how these compare to their counterparts in the planner's problem. Solve for the equilibrium multipliers μ_i. Explain how these depend on the coefficient of risk aversion σ and on the properties of the endowment processes $y_t^i(s^t)$.

2. **Existence of a representative consumer.** Consider a static economy with two individuals $i = 1, 2$ with utility functions $u_i(c_i)$ that are strictly increasing and strictly concave in consumption c_i. Consider the simple planning problem

$$W(y) = \max_{c_1, c_2} \left[\lambda_1 u_1(c_1) + \lambda_2 u_2(c_2) \right]$$

subject to the resource constraint

$$c_1 + c_2 \leq y$$

(a) Show that the solution of this problem is a strictly increasing strictly concave function $W(y)$ which depends on the weights λ_i. Derive a formula for $W'(y)$.

(b) Suppose that both individuals have utility functions that belong to the class of constant *absolute* risk aversion (CARA) utility functions

$$u_i(c_i) = -\frac{\exp(-\alpha_i c_i)}{\alpha_i}, \qquad \alpha_i > 0$$

with coefficients of absolute risk aversion α_i that potentially differs across individuals. Solve for $W(y)$. Explain how this function depends on the weights λ_i and the risk aversion coefficients α_i. Let $c = c_1 + c_2$ denote aggregate consumption and let $U(c)$ denote the utility of the '*representative consumer*' constructed in this way. In what sense is $U(c)$ representative? Does $U(c)$ belong to the class of CARA utility functions?