ECON90003
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Macroeconomics
Tutorial #6

1. **McCall model with wage growth.** Consider an unemployed worker with risk neutral preferences

$$\mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^t c_t\right\}, \qquad 0<\beta<1$$

Each period the worker draws an IID initial wage offer w from a distribution $F(w)$. If they accept the wage offer they become employed and have wage $w_t = wg^t$ growing at $g \geq 1$ thereafter. If they reject the wage offer they remain unemployed and draw a new initial wage offer w' next period. Assume $\beta g < 1$.

   (a) Let $v(w)$ denote the unemployed worker's value function. Setup the unemployed worker's dynamic programming problem in terms of this value function.

   (b) Show that the unemployed worker's problem is characterized by a reservation wage $\bar{w}$ such that the worker rejects the offer if $w < \bar{w}$ and accepts the offer if $w > \bar{w}$. Provide an expression for $v(w)$ in terms of $\bar{w}$.

   (c) Consider two economies exactly the same except for different rates of wage growth, $g_1 > g_2$ say. Which economy has the higher reservation wage? Explain.

2. **Diamond-Mortensen-Pissarides model.** Consider a search model of the labor market in discrete time $t = 0, 1, 2....$ Risk neutral workers and firms have common discount factor $\beta \in (0,1)$. Workers and firms are matched via a standard constant-returns-to-scale matching function $M(u, v)$ where u_t denotes the unemployment rate and v_t the vacancy rate at time t. When a match forms, a firm is able to produce a constant amount of output $z > 0$. The worker receives a wage of w_t and the firm makes a flow profit of $z - w_t$. Job matches between workers and firms are destroyed at an exogenous rate $\delta \in (0,1)$. Firms can create jobs by posting vacancies with a per period cost κz proportional to z. There is free-entry into job creation. When unemployed, workers receive constant flow utility $b \leq w_t$ from unemployment benefits.

   (a) Let V_t, J_t denote the time t value to a firm of a vacancy and a filled job respectively and let U_t, W_t denote the value to a worker of unemployment and employment respectively. Setup and explain the Bellman equations that determine the evolution of these four values over time.

Now suppose that wages are determined by Nash-Bargaining between a worker and firm such that in equilibrium the worker's surplus is a constant fraction $\phi \in (0,1)$ of the total match surplus

$$W_t - U_t = \phi S_t, \qquad S_t \equiv W_t - U_t + J_t - V_t$$

Suppose also that free entry drives the value of a vacancy to $V_t = 0$ for all t.

(b) Let the matching function be $M(u, v) = u^{\alpha}v^{1-\alpha}$. Explain how the steady state wage, labor market tightness $\theta = v/u$ and unemployment rate are determined.

(c) Now suppose productivity increases from z to $z' > z$. Explain what happens to the steady-state wage, labor market tightness, unemployment rate, vacancy rate, and vacancy filling rate. What about if β decreases to $\beta' < \beta$? Explain.

(d) Now consider what happens if the wage is fixed at $\bar{w}$ corresponding to the initial z and β. How if at all do your answers to (c) change if the wage is kept fixed at at $\bar{w}$? Explain.