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**ECON90003**
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# Macroeconomics
## Tutorial #5

1. **Asset pricing with log utility.** Suppose there is a durable asset that pays dividends $y_t$ that follow a Markov chain with transition probabilities $\pi(y'\,|\,y) = \text{Prob}[y_{t+1} = y'\,|\,y_t = y]$. At the beginning of period $t$ the representative consumer's holdings of this asset are $k_t$ and they choose how many units of this asset to hold for the next period $k_{t+1}$. Their initial endowment of the asset is normalized to $k_0 = 1$. The price of this asset is $p_t$ and is taken as given. The representative consumer seeks to maximize

$$\mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t \log(c_t)\right\}, \qquad 0 < \beta < 1 \tag{1}$$

subject to the budget constraints

$$c_t + p_t k_{t+1} \leq (p_t + y_t) k_t$$

along with $c_t \geq 0$ and $k_{t+1} \geq 0$.

(a) Let $v(k, y)$ denote the representative consumer's value function. Setup the representative consumer's dynamic programming problem in terms of this value function and define a recursive competitive equilibrium for this economy.

(b) Let $p(y)$ denote the price of the asset when the current dividend state is $y$. Show that in equilibrium $p(y)$ solves the functional equation

$$p(y) = \beta \sum_{y'} \frac{y}{y'} \left(p(y') + y'\right) \pi(y'\,|\,y) \tag{2}$$

(c) Let $f(y) = p(y)/y$ so that $f(y)$ solves the functional equation

$$f(y) = \beta \sum_{y'} \left(f(y') + 1\right) \pi(y'\,|\,y)$$

Let $T$ denote the operator on the RHS of this equation

$$Tf(y) \equiv \beta \sum_{y'} \left(f(y') + 1\right) \pi(y'\,|\,y)$$

Show that $T$ satisfies Blackwell's sufficient conditions for a contraction mapping.

(d) Solve the functional equation (2) for $p(y)$.

2. **Asset pricing with CRRA utility and IID dividends.** Now suppose that the period utility function in (1) is the more general CRRA specification

$$u(c) = \frac{c^{1-\alpha} - 1}{1-\alpha}, \qquad \alpha > 0$$

and that dividends are IID over time so that $\pi(y' \,|\, y) = \pi(y')$ independent of the current $y$. Show that in equilibrium $p(y)$ now solves the functional equation

$$p(y) = \beta \sum_{y'} \left(\frac{y'}{y}\right)^{-\alpha} [p(y') + y']\, \pi(y') \tag{3}$$

Solve the functional equation (3) for $p(y)$. Other things equal, are asset prices higher or lower when the representative consumer is more risk averse? Give as much intuition as you can.