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ECON90003
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Macroeconomics
Tutorial #4

1. **An exactly solved Bellman equation, revisited.** Consider the problem of maximizing

$$\mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^t \log(c_t)\right\}, \qquad 0<\beta<1 \tag{1}$$

subject to the constraints

$$c_t \geq 0, \qquad k_{t+1}\geq 0, \qquad c_t + k_{t+1} \leq z_t k_t^{\alpha}, \qquad 0<\alpha<1, \qquad t=0,1,\ldots$$

for given initial condition $k_0 > 0$ and some given stochastic process $\{z_t\}$.

Let $v(k,z)$ denote the value function for this problem. The value function $v(k,z)$ solves the Bellman equation

$$v(k,z) = \max_x \Big\{ \log(zk^{\alpha} - x) + \beta\mathbb{E}[v(x,z')\,|\,z] \Big\}$$

Verify that the solution for $v(k,z)$ is

$$v(k,z) = A + B\log k + C\log z$$

where the coefficients are given by

$$A \equiv \frac{1}{1-\beta}\left(\log(1-\alpha\beta) + \frac{\alpha\beta}{1-\alpha\beta}\log(\alpha\beta) + \frac{\beta}{1-\alpha\beta}\mathbb{E}\{\log z'|z\}\right)$$

and

$$B \equiv \frac{\alpha}{1-\alpha\beta} > \alpha > 0$$

and

$$C \equiv \frac{1}{1-\alpha\beta} > 1$$

2. **Stationary distribution with lognormal shocks.** Suppose that the shocks z_t are IID lognormal

$$\log(z_t) \sim N(\mu, \sigma^2)$$

Let $\mu_{k,t}$ and $\sigma^2_{k,t}$ denote the mean and variance of the time-t distribution of the log capital stock $\log(k_t)$. Using (2), solve for the sequence of means $\mu_{k,t}$ and variances $\sigma^2_{k,t}$ and calculate their limiting values as $t\to\infty$. What is the stationary distribution of the log capital stock?