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ECON90003
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Macroeconomics
Tutorial #3

1. **Cake-eating.** Consider the problem of choosing consumption c_t for $t = 0, 1, \ldots$ to maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1 \tag{1}$$

subject to the constraints

$$c_t \geq 0, \qquad k_{t+1} \geq 0, \qquad c_t + k_{t+1} = k_t, \qquad t = 0, 1, \ldots$$

with given initial condition

$$k_0 > 0$$

Suppose that period utility has the isoelastic form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \qquad \sigma > 0$$

   (a) Consider a recursive formulation of this problem and let $v(k)$ denote the value function and let $c(k)$ denote the consumption policy function. Setup and explain the Bellman equation that determines these functions.

   (b) Solve for $c(k)$. Using this solution, explain the time paths of c_t and k_t starting from the given initial condition k_0. Explain how your answers depend on the parameters β and σ. Give intuition for your results.

2. **Borrowing and lending.** Consider a risk neutral consumer choosing consumption c_t for $t = 0, 1, \ldots$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \, c_t, \qquad 0 < \beta < 1 \tag{2}$$

The consumer can borrow or lend freely at gross interest rate $R = 1/\beta$ and so faces the sequence of constraints

$$0 \leq c_t \leq a_t - \beta a_{t+1}, \qquad t = 0, 1, \ldots$$

with given initial wealth a_0. The interpretation here is that $a_t > 0$ means the consumer is a net lender but $a_t < 0$ means the consumer is a net borrower. To begin with, suppose that there are no constraints on borrowing so that $a_t < 0$ is permitted.

   (a) Let $v^*(a_0)$ denote the solution of the sequence problem (maximizing (2) subject to the sequence of constraints $0 \leq c_t \leq a_t - \beta a_{t+1}$) and let $v(a)$ denote the value function that solves the associated Bellman equation. Do the solutions coincide, i.e., does $v(a_0) = v^*(a_0)$? Why or why not?

   (b) Now suppose that we prohibit borrowing, that is we add the constraint $a_{t+1} \geq 0$. How if at all does this change your answer to (a)?