Faculty of Business and Economics
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ECON90003
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Macroeconomics
Tutorial #2

1. **An exactly-solved Bellman equation.** Consider a discrete-time infinite-horizon optimal growth model where the planner chooses capital stocks k_{t+1} for $t = 0, 1, \ldots, T$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \log c_t, \qquad 0 < \beta < 1 \tag{1}$$

subject to the sequence of resource constraints

$$c_t + k_{t+1} \leq k_t^\alpha, \qquad t = 0, 1, \ldots$$

with given initial condition

$$k_0 > 0$$

Let $v(k)$ denote the value function for this problem. The value function $v(k)$ solves the Bellman equation

$$v(k) = \max_x \Big[\log(k^\alpha - x) + \beta v(x) \Big]$$

Verify that the solution for $v(k)$ is

$$v(k) = A + B \log k$$

where

$$A = \frac{1}{1-\beta} \left(\log(1-\alpha\beta) + \frac{\alpha\beta}{1-\alpha\beta} \log(\alpha\beta) \right)$$

and

$$B = \frac{\alpha}{1-\alpha\beta} > 0$$

2. **Properties of the policy function.** Consider a strictly increasing strictly concave production function $f(k)$ that satisfies $f(0) = 0$ and $f'(0) = +\infty$ and $f'(\infty) < 1$. Suppose the policy function $k_{t+1} = g(k_t)$ has the form $g(k) = sf(k)$ for some $s \in (0, 1)$.

   (a) Show that there is exactly one steady state $k^* > 0$.

   (b) Show that for any $k_0 > 0$ the sequence $\{k_{t+1}\}_{t=0}^{\infty}$ induced by $g(k)$ converges monotonically to k^*.