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ECON90003
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Macroeconomics
Tutorial #10

1. **Static Hopenhayn.** Firms discount flow profits according to a constant discount factor $0 < \beta < 1$. There is an unlimited number of potential entrants. On paying an entry cost $k_e > 0$ an entrant receives a once-and-for-all productivity draw $z \sim g(z)$ and then makes a once-and-for-all decision to operate or exit. On paying a fixed operating cost $k > 0$, a firm that hires n workers can produce output $y = zn^\alpha$ for $0 < \alpha < 1$. Let w denote the wage and p the price of of their output. Let $w = 1$ be the numeraire.

   (a) Let $n(z;p)$, $y(z;p)$ and $\pi(z;p)$ denote the optimal employment policy, output, and profits of a firm with productivity z when the price is p. Solve for these functions. Let $z^*(p)$ denote the lowest level of productivity such that a firm does not exit. Solve for $z^*(p)$. How does z^* depend on p? Explain.

   (b) Let $v(z;p)$ denote the value function of a firm. Solve for $v(z;p)$.

   (c) Use the free-entry condition and the cutoff productivity condition to derive the comparative statics of z^* and p^* with respect to k, k_e and α. Give intuition for your results.

   Now suppose that productivity is drawn from the *Pareto distribution* with density

   $$g(z) = \xi z^{-\xi-1}, \qquad z \geq 1, \qquad \xi > 1$$

   (d) Solve explicitly for z^* and p^*. How do z^* and p^* depend on the shape parameter ξ? What is the productivity distribution of actively producing firms? Explain.

2. **Hopenhayn with aggregate risk.** Firms discount flow profits according to a constant discount factor $0 < \beta < 1$. There is an unlimited number of potential entrants. On paying a sunk entry cost $k_e > 0$, an entrant receives an initial productivity draw $z_0 \sim g(z_0)$ and then starts operating the next period as an incumbent firm. On paying a fixed operating cost $k > 0$, an incumbent firm that hires n workers produces flow output $y = zn^\alpha$ with $0 < \alpha < 1$. The firm's productivity z evolves according to a Markov process with transition density $f(z' \,|\, z)$.

   Unlike the basic Hopenhayn model, the demand curve facing the firms fluctuates. Let $D_t(p_t) = d_t/p_t$ denote the demand facing firms if the price is p_t and the state of demand is d_t. The state of demand d_t evolves according to a 2-state Markov chain $d_t \in \{d_l, d_h\}$ with transition probabilities $h(d' \,|\, d)$. Let $w_t = 1$ be the numeraire.

   (a) What are the aggregate state variables in this economy? Setup the dynamic programming problem for incumbent firms and define a recursive competitive equilibrium for this economy. Be clear as to how all of the endogenous variables are determined in this equilibrium.

   (b) Does the cross-sectional distribution of productivity fluctuate in this economy? Why or why not? What about the price p_t and the mass of entrants m_t? Explain.

   (c) Outline an algorithm by which approximate solutions to this model can be computed.