Faculty of Business and Economics
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THE UNIVERSITY OF MELBOURNE

ECON90003
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Macroeconomics
Tutorial #1

1. **Matlab**. If you have a laptop, bring it to class so that the tutors can assist you in getting Matlab up and running. I think it'd be advisable if at least one person from each group brings a laptop, but probably the more people who get sorted out the better. To obtain a copy, please follow the installation instructions here:

   https://github.com/resbaz/lessons/blob/master/matlab/unimelb_matlab_install.md

   If you need additional help, please let me know asap.

2. **Finite-horizon optimal growth model.** Consider a discrete-time finite-horizon optimal growth model where the planner chooses capital stocks k_{t+1} for $t = 0, 1, \ldots, T$ to maximize

$$\sum_{t=0}^{T} \beta^t u(c_t), \qquad 0 < \beta < 1$$

   subject to the sequence of resource constraints

$$c_t + k_{t+1} \leq f(k_t), \qquad t = 0, 1, \ldots, T$$

   with given initial condition

$$k_0 > 0$$

   To begin with, assume the utility and production functions satisfy $u'(c) > 0$, $u''(c) < 0$ and $f'(k) > 0$, $f''(k) < 0$ with $u'(0) = f'(0) = +\infty$ and $f(0) = 0$.

   (a) Show that the solution to this problem is characterized by the sequence of conditions

$$u'(f(k_t) - k_{t+1}) = \beta\, u'(f(k_{t+1}) - k_{t+2})\, f'(k_{t+1}), \qquad t = 0, 1, \ldots, T-1 \qquad (*)$$

   along with

$$k_{T+1} = 0$$

   Explain how these conditions pin down the sequence of capital stocks that solve the planning problem given the initial $k_0 > 0$.

   Now suppose in particular that $u(c) = \log c$ and $f(k) = k^\alpha$ for $0 < \alpha < 1$.

   (b) Show that the sequence of capital stocks

$$k_{t+1} = \alpha\beta \, \frac{1 - (\alpha\beta)^{T-t}}{1 - (\alpha\beta)^{T-t+1}} \, k_t^\alpha, \qquad t = 0, 1, \ldots, T$$

   satisfies the optimality conditions in part (a) above.

   (c) Consider the limit $T \to \infty$. Show that in this limit we have

$$k_{t+1} = \alpha\beta\, k_t^\alpha, \qquad c_t = (1 - \alpha\beta)\, k_t^\alpha$$

   for given $k_0 > 0$. Interpret these formulas in terms of the usual phase diagram for the discrete-time infinite-horizon optimal growth model.