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Macroeconomics
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Macroeconomics: Problem Set #4

Due Tuesday May 28 in class

1. **Income fluctuations.** Consider a single risk averse household that takes as given a constant bond price $q > 0$ and that seeks to maximize

$$\mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^t u(c_t)\right\}, \qquad 0<\beta<1$$

subject to the sequence of budget constraints

$$c_t + q a_{t+1} \leq a_t + y_t$$

along with $c_t \geq 0$ and $a_{t+1} \geq -\phi$ for some limit ϕ. The household's income $y_t > 0$ fluctuates according to a Markov chain with transition probabilities $\pi(y' \,|\, y) = \text{Prob}[y_{t+1} = y' \,|\, y_t = y]$.

   (a) Let $v(a, y)$ denote the household's value function. Setup and explain the Bellman equation that determines $v(a, y)$.

Now suppose the utility function is CRRA

$$u(c) = \frac{c^{1-\alpha}-1}{1-\alpha}, \qquad \alpha > 0$$

and that the income process is a 2-state Markov chain with support y_i and transition probabilities $\pi_{ij} = \text{Prob}[y_{t+1} = y_j \,|\, y_t = y_i]$ for $i, j = 1, 2$.

Let the parameters be $\alpha = 1$, $\beta = 0.97$, $y_1 = 1$, $y_2 = 0.5$, $p_{11} = 0.95$, $p_{22} = 0.5$ with borrowing limit $\phi = 5$ (i.e., the borrowing constraint is $a_{t+1} \geq -5$). Let the bond price be $q = 0.98$.

   (b) Solve the household's dynamic programming problem using your preferred numerical methods. Let $a' = g(a, y)$ and $c(a, y)$ denote the asset accumulation and consumption policy functions for this problem. Plot $g(a, y)$ and $c(a, y)$ for each of $y = y_1$ and $y = y_2$ and explain your findings.
   (c) How do your answers to (b) change for $\alpha = 2$ and $\alpha = 5$? Explain.
   (d) How do your answers to (b) change for $y_2 = 0.9$ and $y_2 = 0.1$? Explain.
   (e) How do your answers to (b) change for $\phi = 8$ and $\phi = 1$? Explain.
   (f) How do your answers to (b) change for $q = 0.96$ and $q = 0.99$? Explain.

2. **Aiyagari model.** Consider an Aiyagari model where the typical household seeks to maximize

$$\mathbb{E}\left\{\sum_{t=0}^{\infty}\beta^t u(c_{it})\right\}, \qquad 0<\beta<1$$

subject to the budget constraints

$$c_{it}+a_{it+1}\leq(1+r_t)a_{it}+w_t n_{it}$$

along with $c_{it}\geq 0$ and $a_{it+1}\geq -\phi$ for some limit ϕ. The household's labor endowment evolves exogenously according to the AR(1) process

$$\log n_{it+1}=(1-\varphi)\log\bar{n}+\varphi\log n_{it}+\varepsilon_{it+1}, \qquad 0\leq\varphi<1$$

where the innovations ε_{it+1} are IID $N(0,\sigma_\varepsilon^2)$ over time and in the cross section of households. Suppose that the representative firm uses the aggregate production function

$$Y_t=K_t^\theta N_t^{1-\theta}, \qquad 0<\theta<1$$

and that physical capital evolves according to

$$K_{t+1}=(1-\delta)K_t+I_t, \qquad 0<\delta<1$$

(a) Let $v(a,n)$ denote the value function of a household of type a,n. Setup and explain the Bellman equation that determines $v(a,n)$ and define a stationary equilibrium for this economy.

(b) Let $\sigma_\varepsilon=0.4$ and $\varphi=0.9$. Choose $\bar{n}$ so that $N=1$. Then use the method of Tauchen and Hussey to calibrate a 7-state Markov chain that approximates the AR(1) process. In particular, calculate the 7 nodes for n and the 7×7 probability transition matrix.

(c) Let $u(c)$ be CRRA with coefficient $\alpha=3$ and suppose the time discount factor is $\beta=0.96$, and that the borrowing constraint is $\phi=0$. Suppose also that capital's share is $\theta=0.36$ and the depreciation rate is $\delta=0.08$. Using these parameter values and your preferred numerical methods, solve for the stationary equilibrium. In particular, calculate the risk-free rate r, aggregate capital stock K, aggregate savings rate, and stationary distribution $\mu(a,n)$. Report the cross-sectional coefficient of variation for consumption, wealth, and labor earnings in this stationary equilibrium.

(d) How do your answers to (b)-(c) change for $\varphi=0,0.3$ and 0.6. Explain your findings.

(e) How do your answers to (c) change for $\phi=3$ and $\phi=6$. Explain your findings.