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Macroeconomics
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Macroeconomics: Problem Set #2

Due Tuesday April 16 in class

1. **Markov chains.** Consider a 2-state Markov chain on x_i, $i = 1, 2$ with transition probabilities $p_{ij} = \text{Prob}[x_{t+1} = x_j \,|\, x_t = x_i]$ for $i, j = 1, 2$ given by the matrix

$$\boldsymbol{P} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$$

with parameters $p \in [0, 1]$ and $q \in [0, 1]$.

   (a) Let λ_i for $i = 1, 2$ denote the eigenvalues of this transition matrix. Solve for λ_i in terms of the parameters p, q. Show that at least one $\lambda_i = 1$. Show that $\max_i |\lambda_i| = 1$. Can there be a zero eigenvalue? Can there be a negative eigenvalue? Explain.

   (b) Let $\boldsymbol{\psi}^*$ denote a stationary distribution of the Markov chain. Solve for $\boldsymbol{\psi}^*$ in terms of the parameters p, q. Can there be more than one stationary distribution? Does the sequence of distributions $\boldsymbol{\psi}_{t+1} = \boldsymbol{P}^\top \boldsymbol{\psi}_t$ always converge to such a stationary distribution? Explain.

2. **Stochastic growth with elastic labor supply.** Suppose the planner seeks to maximize

$$\mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t \, u(c_t, l_t)\right\}, \qquad 0 < \beta < 1$$

subject to the resource constraint

$$c_t + k_{t+1} = z_t f(k_t, l_t) + (1-\delta)k_t, \qquad 0 < \delta < 1$$

with initial conditions $k_0 > 0$ and $z_0 > 0$. Productivity z_t evolves according to a Markov process with transition probabilities $F(z' \,|\, z) = \text{Prob}[z_{t+1} \leq z' \,|\, z_t = z]$ and unconditional mean $\bar{z} > 0$.

In this problem the planner chooses how much labor l_t to supply. Assume that $u(c_t, l_t)$ is strictly increasing, strictly concave in c_t and strictly decreasing, strictly convex in l_t. The production function $f(k_t, l_t)$ is strictly increasing and strictly concave in both arguments and satisfies constant returns to scale.

   (a) Let $v(k, z)$ denote the planner's value function. Setup and explain the Bellman equation that determines $v(k, z)$.

   (b) Derive the planner's optimality conditions for consumption, capital and labor.

Now suppose that the utility function has the form

$$u(c, l) = \log c - \frac{l^{1+\varphi}}{1+\varphi}, \qquad \varphi > 0$$

and the production function has the form

$$f(k, l) = k^\alpha l^{1-\alpha}, \qquad 0 < \alpha < 1$$

(c) Solve for the non-stochastic steady state values of consumption, capital, and labor in terms of model parameters. Suppose there is a permanent increase in the level of productivity $\bar{z}$. Explain how this changes the steady state values of consumption, capital, and labor. Give economic intuition for your answers.

Now suppose that productivity is given by a stationary AR(1) process in logs

$$\log z_{t+1} = (1-\phi)\log \bar{z} + \phi \log z_t + \varepsilon_{t+1}, \qquad 0 < \phi < 1$$

where the innovations ε_t are IID $N(0, \sigma_\varepsilon^2)$.

(d) Let $\alpha = 0.3$, $\beta = 1/1.05$, $\delta = 0.05$, $\phi = 0.97$, $\varphi = 1$, $\bar{z} = 1$ and $\sigma_\varepsilon = 0.025$. Using these parameter values, use collocation methods to solve the model. In particular, use cubic splines with 99 breakpoints and discretize the shock process using 29 points for productivity z_t and 15 points for the innovations ε_t.

(e) Suppose the economy is at its non-stochastic steady state and that at $t = 0$ there is a 1 standard deviation innovation to productivity, i.e., $\varepsilon_0 = \sigma_\varepsilon = 0.025$. Use the functions you computed in part (d) to calculate and plot impulse responses for the log-deviations (from steady state) of consumption, capital, labor and output for $T = 250$ periods after the shock. Explain your findings.

(f) Simulate a sequence of productivity z_t of length $T = 1,000$ starting from $z_0 = \bar{z} = 1$. Use this simulated sequence of productivity and the functions you computed in part (c) to generate simulated sequences of the log-deviations (from steady-state) of consumption, capital, labor and output starting from $k_0 = \bar{k}$. Which of these variables move most closely together? Which of these variables is most volatile? Explain.

(g) How would your answers to (e) and (f) change if φ was much lower, say $\varphi = 0.1$? What about $\varphi = 10$? Give economic intuition for your answers.

(h) Suppose that capital was not needed for production, $\alpha \to 0$. Explain how this simplifies the determination of equilibrium consumption and employment. Explain the implications of this for fluctuations in consumption and employment. What does this suggest about the importance of capital in this model?