Macroeconomics
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Macroeconomics: Problem Set #1

Due Tuesday March 26 in class

1. **Simple difference equations.** Consider the linear difference equation

$$x_{t+1} = \bar{x} + a\,(x_t - \bar{x}), \qquad t = 0, 1, 2, \ldots, \qquad x_0 \in \mathbb{R} \text{ given}$$

   (a) Give a complete account of the possible dynamics of x_t implied by this linear difference equation. Explain how these dynamics depend on the value of the parameter a. Do these dynamics depend on the value of the initial condition x_0? Explain.

   Now consider the nonlinear difference equation

$$x_{t+1} = a\,x_t(1 - x_t), \qquad t = 0, 1, 2, \ldots, \qquad x_0 \in [0, 1] \text{ given}, \qquad a \in (0, 4]$$

   (b) Show that, for this difference equation, x_t lies in $[0, 1]$ for all t.

   (c) How many steady states does this difference equation have? How do these depend on the parameter a?

   (d) Give as complete an account as you can of the possible dynamics of x_t implied by this difference equation. Explain how these dynamics depend on the value of the parameter a. Do these dynamics depend on the value of the initial condition x_0? Explain.

   Hint: Consider the special cases

$$a \in \{\,0.5\,,\ 1.5\,,\ 2.5\,,\ 3.0\,,\ 3.5\,,\ 4.0\,\}$$

2. **Numerical dynamic programming by value function iteration.** Consider the infinite-horizon growth model. The planner chooses capital stocks k_{t+1} for $t = 0, 1, \ldots$ to maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1$$

   subject to the sequence of resource constraints

$$c_t + k_{t+1} \leq f(k_t) + (1 - \delta)k_t, \qquad 0 < \delta < 1$$

   with given initial condition

$$k_0 > 0$$

   (a) Let $v(k)$ denote the value function for this problem. Setup and explain the Bellman equation that determines $v(k)$.

Now suppose that the period utility function has the isoelastic form

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \qquad \sigma > 0$$

and that the production function is

$$f(k) = zk^{\alpha}, \qquad 0 < \alpha < 1$$

(b) Solve for the steady state values c^* and k^*. What is the steady state capital/output ratio? What is the steady state consumption/output ratio? What is the steady state savings rate? How does this compare to the 'golden rule' savings rate for this economy? Explain. How if at all do your answers depend on the value of σ? Explain.

(c) Now let $z = 1$, $\alpha = 0.3$, $\beta = 1/1.05$, $\delta = 0.05$ and $\sigma = 1$. Using these parameter values, discretize the state space on a grid of $n = 1001$ points calculate and plot the value function $v(k)$ on this grid of points. Let $c(k)$ be the associated policy function for consumption. Calculate and plot $c(k)$ for these parameter values. How does the savings behavior implied by this policy function compare to the steady-state savings rate from part (b)? Explain.

(d) Now suppose the economy is at steady state then suddenly at $t = 0$ the productivity level z permanently increases from $z = 1$ to $z' = 1.05$. Calculate and plot the new value function and consumption policy function associated with z'. Explain how these compare to the ones you found in part (c). Calculate and plot the transitional dynamics of the economy as it adjusts to its new long-run values. In particular, calculate and plot the time-paths of capital and consumption until they have converged to their new steady state levels. Use a phase diagram to explain these transitional dynamics.

(e) How if at all would your answers to parts (b) through (d) change if σ was lower, say $\sigma = 0.5$? Or higher, say $\sigma = 2$? Give intuition for your answers.