Macroeconomics

Lecture 20: firm dynamics, part two

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This lecture

1- Hopenhyan (1992) in general equilibrium

2- Hopenhayn/Rogerson (1993)

- quantitative application of Hopenhayn model
- nonconvex adjustment costs; a firm's lagged employment is an endogenous state variable
- adjustment costs induce *misallocation* of resources across heterogeneous producers
- how much does this misallocation matter?

General equilibrium version of Hopenhyan

- Representative consumer

$$U(C, N) = \theta \log C - N, \qquad \theta > 0$$

- Steady state with discount factor $\beta = 1/(1 + r)$

- Problem reduces to maximizing period utility subject to static budget constraint

$$pC \leq N + \Pi, \qquad (w = 1 \text{ is numeraire})$$

where Π denotes aggregate profits, distributed lump-sum

General equilibrium version of Hopenhyan

- First order conditions imply demand curve

$$C(p) = \frac{\theta}{p}$$

- Perfectly elastic labor supply then

$$N = \theta - \Pi$$

Aggregate profits

- Profits of incumbent with productivity z

$$\pi(z) = py(z) - n(z) - k$$

- Aggregate profits

$$\Pi = \int \pi(z)\,\mu(z)\,dz$$

$$= p\int y(z)\,\mu(z)\,dz - \int (n(z) + k)\,\mu(z)\,dz$$

Market clearing

- Goods market clearing

$$Y = \int y(z)\,\mu(z)\,dz = C(p) = \frac{\theta}{p}$$

- Labor market clearing

$$N = \int (n(z) + k)\,\mu(z)\,dz = \theta - \Pi$$

- So indeed if goods market clears at price p, labor market also clears

Hopenhayn/Rogerson (1993)

- Background: large labor market flows at individual firm level (job creation and job destruction)
- What are the consequences of policies that make it costly for firms to adjust employment levels? (e.g., taxes on job destruction)
- Nonconvex adjustment costs implies a firm's lagged employment is an endogenous state variable

Model

- Time $t = 0, 1, 2, \ldots$

- Output and input prices p_t and $w_t = 1$ (numeraire) taken as given

- Output $y_t = z_t F(n_t)$ produced with labor n_t given productivity z_t

- Static profits

$$p_t z_t F(n_t) - n_t - H(n_t, n_{t-1}) - k$$

where k is per-period fixed cost of operating and $H(n_t, n_{t-1})$ captures *labor adjustment costs*, both in units of labor

- A tax τ on job destruction implies adjustment cost function

$$H(n_t, n_{t-1}) = \tau \times \max[0\,,\, n_{t-1} - n_t]$$

(but other specifications straightforward too)

Timing within period

- Incumbent begins period with (z_{-1}, n_{-1})
- Decides to exit or not
- If exit, pay $H(0, n_{-1})$ this period and zero in future
- If stay, draw new productivity $z \sim f(z \,|\, z_{-1})$ and choose n to max

  $$pzF(n) - n - H(n, n_{-1}) - k$$

  and receive profits, then start next period

Incumbent's problem

- Consider stationary equilibrium with constant price p

- Let $v(z, n\,;\,p)$ denote value function for firm that *had* employment n last period, that has decided to operate and has just drawn z

- Bellman equation

$$v(z,n\,;\,p) = \max_{n'\geq 0}\Big\{pzF(n') - n' - H(n',n) - k$$
$$+\,\beta \max\Big[-H(0,n')\,,\,\int v(z',n'\,;\,p)\,f(z'\,|\,z)\,dz'\Big]\Big\}$$

- Let $n' = \eta(z,n\,;\,p)$ denote optimal employment policy and $\chi(z,n\,;\,p) \in \{0,1\}$ denote optimal exit policy ($\chi = 1$ is exit)

- Let $\mu(z,n)$ denote the distribution of firms across states z, n

Entrant's problem

- Potential entrants ex ante identical
- Begin with employment size $n = 0$
- Pay $k_e > 0$ to enter, initial draw from $g(z)$ if they do
- Start producing next period
- Let $m > 0$ denote the mass of entrants, free entry condition

$$\beta \int v(z, 0\,;\, p)\, g(z)\, dz \leq k_e$$

with strict equality whenever $m > 0$

Aggregation

- Aggregate output

$$Y = \iint zF(\eta(z,n\,;\,p))\,\mu(z,n)\,dzdn$$

- Aggregate employment

$$N = \iint (\eta(z,n\,;\,p) + k)\,\mu(z,n)\,dzdn$$

- Representative consumer's budget constraint

$$pC \leq N + \Pi + T$$

where T denotes revenues from adjustment costs rebated lump-sum

Computing an equilibrium (sketch)

- **Step 1.** Guess price p^0 and solve incumbent's Bellman equation for the value function $v(z, n\,; p^0)$

- **Step 2.** Check that price p^0 satisfies the free entry condition

$$\beta \int v(z, 0\,; p^0)\, g(z)\, dz = k_e$$

  If yes, proceed to Step 3. If no, return to Step 1 with new guess p^1

- **Step 3.** Given a p^* that satisfies the free-entry condition and the associated value and optimal policy functions of incumbent firms, solve for the stationary distribution $\mu(z, n)$ associated with measure $m = 1$ of entrants

- **Step 4.** Find the scale factor m^* for the distribution $\mu(z, n)$ that ensures the goods market clears

Stationary distribution

- Let $\phi(z', n' \mid z, n)$ denote transition from (z, n) to (z', n')

$$\phi(z', n' \mid z, n) \equiv f(z' \mid z)\, \mathbb{1}[n' = \eta(z, n\,;\, p)]\, \mathbb{1}[\chi(z, n\,;\, p) = 0]$$

- Stationary distribution $\mu(z, n)$ then solves linear system of the form

$$\mu(z', n') = \iint \phi(z', n' \mid z, n)\, \mu(z, n)\, dz dn + m\, g(z') \mathbb{1}[n' = 0]$$

Given p^* from Steps 1–2, solve this once for $m = 1$ then find the scale factor m^* that ensures the goods market clears

Numerical example

- Suppose production function and adjustment cost function

$$y = zn^{\alpha}, \qquad \text{and} \qquad H(n', n) = \tau \times \max[0\,,\, n - n']$$

- And that firm productivity follows AR(1) in logs

$$\log z' = (1-\rho)\log \bar{z} + \rho \log z + \sigma\varepsilon'$$

- Parameter values (period 5 years $\Rightarrow \tau = 0.1$ is 6 months pay)

$$\alpha = 2/3, \quad \beta = 0.80, \quad k = 20, \quad k_e = 40$$
$$\log \bar{z} = 1.40, \quad \sigma = 0.20, \quad \rho = 0.9, \quad \theta = 100$$

- Approximate AR(1) with Markov chain on 33 nodes

Size Distribution of Firms and Employment

firms		<20	<50	<100	<500	rest
tau =	0.000	0.24	0.32	0.15	0.28	0.02
tau =	0.100	0.17	0.29	0.24	0.28	0.03
tau =	0.200	0.20	0.22	0.17	0.37	0.03
tau =	0.500	0.26	0.13	0.26	0.31	0.04

employment		<20	<50	<100	<500	rest
tau =	0.000	0.02	0.12	0.11	0.54	0.21
tau =	0.100	0.00	0.10	0.16	0.50	0.23
tau =	0.200	0.01	0.08	0.10	0.59	0.24
tau =	0.500	0.00	0.04	0.16	0.52	0.27

As τ increases, employment even more concentrated in large and very large firms.

Optimal employment policy

- If no adjustment costs ($\tau = 0$), then employment given by

$$n' = \eta(z, n\,;\, p) = (\alpha z p)^{\frac{1}{1-\alpha}}, \qquad \text{independent of } n$$

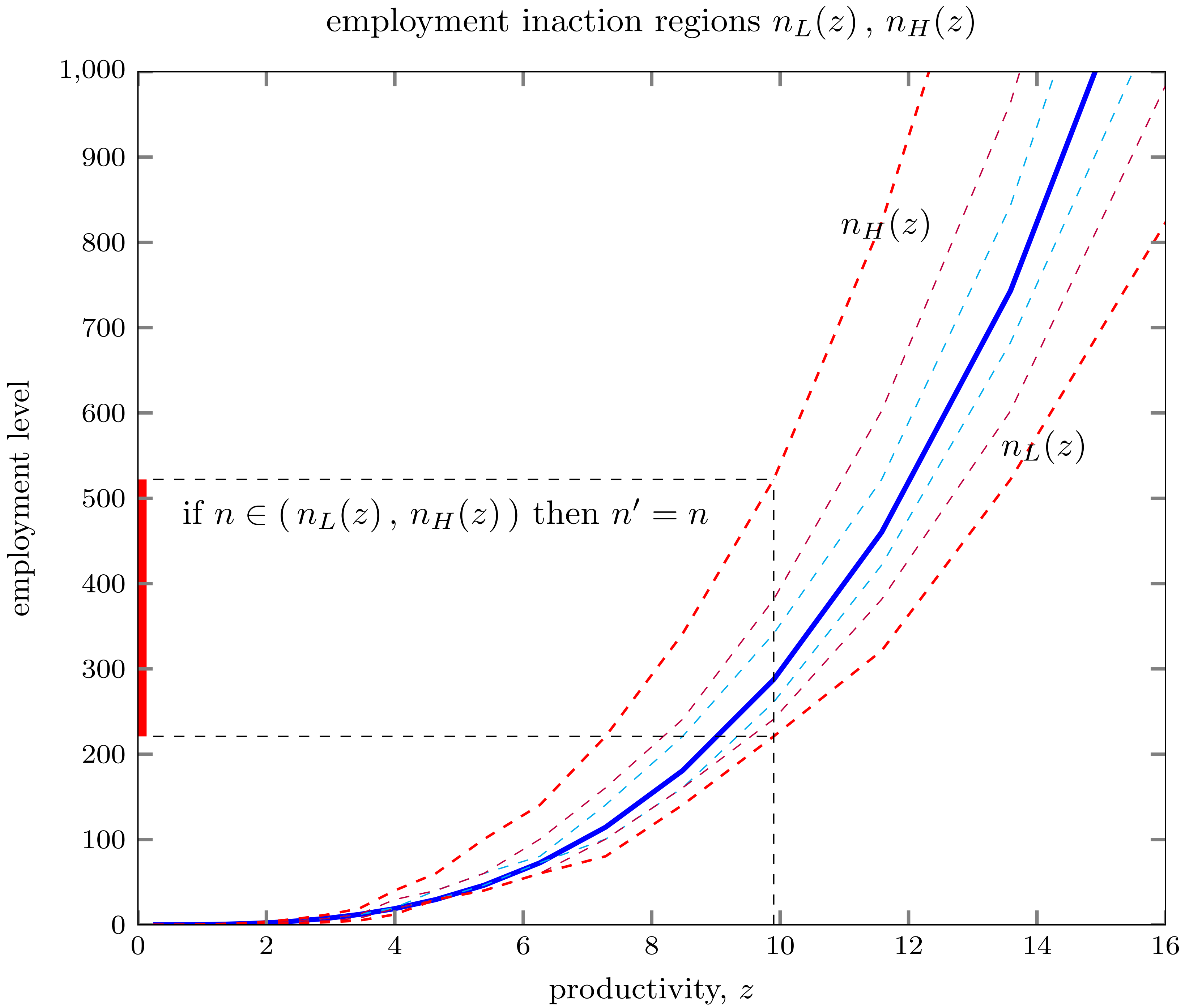
(log employment proportional to log productivity)

- If adjustment costs ($\tau > 0$), then employment

$$n' = \eta(z, n\,;\, p) = n, \qquad \text{whenever} \qquad n \in (\, n_L(z)\,,\, n_H(z)\,)$$

and otherwise resets to value independent of n

- Higher τ widens the *inaction region* for each z



employment inaction regions $n_L(z)$, $n_H(z)$
1,000
900
800
700
600
500
400
300
200
100
0
employment level
0
2
4
6
8
10
12
14
16
productivity, z
$n_H(z)$
$n_L(z)$
if $n \in (\, n_L(z)\, ,\, n_H(z)\,)$ then $n' = n$

Misallocation

- If no adjustment costs ($\tau = 0$), marginal product of labor is

$$\alpha \eta(z, n\,;\, p)^{\alpha-1} = \frac{1}{p}, \qquad \text{for all } z, n$$

- Implies aggregate productivity

$$A = \frac{1}{\alpha p}$$

- If adjustment costs ($\tau > 0$), many firms have marginal product of labor $\neq 1/p$, *inefficient scale*

- Higher τ increases the size of marginal product deviations from $1/p$, reduces aggregate productivity and aggregate output

Misallocation

mpl deviation, pct	<1	<5	<10	<20	rest
tau = 0.000	1.00	0.00	0.00	0.00	0.00
tau = 0.100	0.08	0.52	0.40	0.00	0.00
tau = 0.200	0.00	0.12	0.74	0.14	0.00
tau = 0.500	0.05	0.15	0.10	0.53	0.17

Distribution of marginal product deviations from $1/p$. With high τ many firms not adjusting employment and so have inefficient scale.

Aggregate Statistics				
adjustment cost, tau	0.000	0.100	0.200	0.500
price	1.000	1.013	1.023	1.044
aggregate output	100.000	98.715	97.778	95.801
aggregate productivity	1.500	1.490	1.483	1.450
aggregate employment, production	66.667	66.251	65.948	66.078
aggregate employment, overhead	13.111	12.466	11.708	11.230
aggregate profit	20.223	22.618	24.487	25.945
aggregate firing costs/wage bill	0.000	0.017	0.028	0.042

Misallocation reduces aggregate productivity and aggregate output.

Misallocation

- The misallocation here is induced by an *aggregate* friction that applies to all firms

- Recent literature (Restuccia/Rogerson 2008, Hsieh/Klenow 2009) focuses on *idiosyncratic* frictions

Role of persistence ρ

- When shocks very persistent, efficient scale does not change often

  $\Rightarrow$ adjustment costs less important

- But when shocks less persistent, efficient scale changes often

  $\Rightarrow$ adjustment costs more important

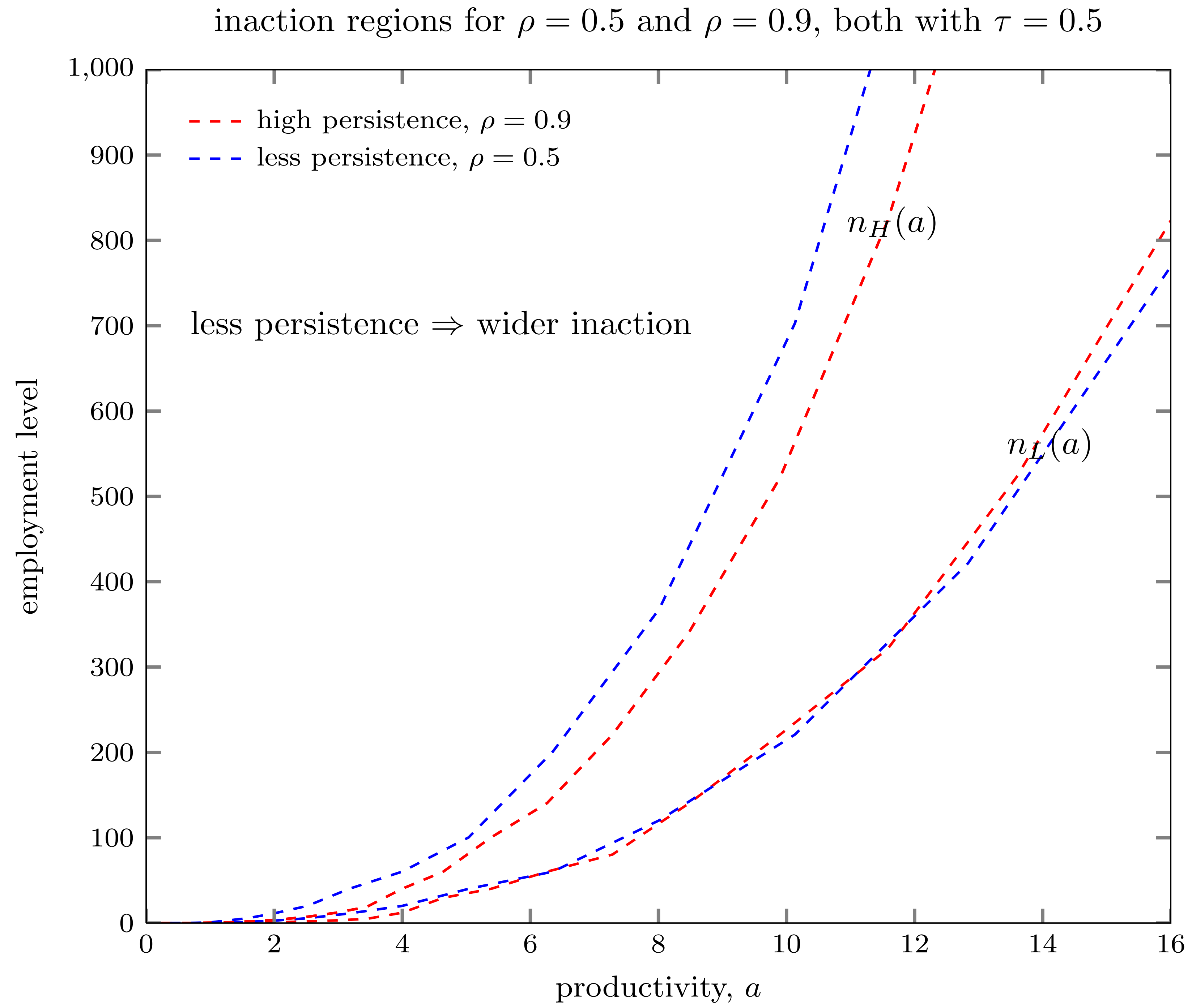
- Lower ρ increases employment share of small firms, widens inaction region, increases misallocation

Size Distribution of Firms and Employment (tau=0.5)

firms		<20	<50	<100	<500	rest
rho =	0.900	0.26	0.13	0.26	0.31	0.04
rho =	0.500	0.22	0.42	0.19	0.16	0.00

employment		<20	<50	<100	<500	rest
rho =	0.900	0.00	0.04	0.16	0.52	0.27
rho =	0.500	0.03	0.24	0.21	0.47	0.05

For lower ρ, employment relatively more concentrated in small-medium firms rather than large firms



inaction regions for $\rho = 0.5$ and $\rho = 0.9$, both with $\tau = 0.5$
high persistence, $\rho = 0.9$
less persistence, $\rho = 0.5$
$n_H(a)$
less persistence ⇒ wider inaction
$n_L(a)$
employment level
productivity, a
0
100
200
300
400
500
600
700
800
900
1,000
0
2
4
6
8
10
12
14
16

Misallocation (tau=0.5)

mpl deviation, pct		<1	<5	<10	<20	rest
rho =	0.900	0.05	0.15	0.10	0.53	0.17
rho =	0.500	0.06	0.01	0.08	0.36	0.50

For lower ρ, wider inaction region at each level of productivity and more frequently the case that deviations from $1/p$ are very large.

Aggregate Statistics (tau=0.5)		
persistence, rho	0.900	0.500
price	1.044	1.161
aggregate output	95.801	86.129
aggregate productivity	1.450	1.385
aggregate employment, production	66.078	62.207
aggregate employment, overhead	11.230	20.938
aggregate profit	25.945	22.970
aggregate firing costs/wage bill	0.042	0.074
entry/exit rate	0.219	0.068

Hence for lower ρ, aggregate productivity and aggregate output are lower, firing costs are higher, and there is less entry and exit.