Macroeconomics

Lecture 15: incomplete markets, part one

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This class

- Introduction to incomplete markets
- Huggett (1993)
  - endowment economy
  - idiosyncratic risk but no aggregate risk
  - implications for borrowing, saving, and interest rates

Setup

- Time $t = 0, 1, 2, \ldots$
- Continuum $i \in [0, 1]$ of heterogeneous agents
- Idiosyncratic endowment risk

$$\pi(y' \,|\, y) = \text{Prob}[y_{it+1} = y' \,|\, y_{it} = y]$$

- Aggregate endowment constant, Y

Riskless bond

- Extreme form of market incompleteness, just a single riskless bond

- Let q_t denote the price at which bond can be bought or sold

- Let a_{it+1} denote agent i's end-of-period asset holdings

  – $a_{it+1} < 0$ is borrowing (selling bond),
  get q_t at t, pay 1 unit of consumption at $t + 1$

  – $a_{it+1} > 0$ is saving (buying bond),
  pay q_t at t, get 1 unit of consumption at $t + 1$

- Agents will acquire a *buffer-stock* of savings in an attempt to *self-insure* against their idiosyncratic risk

Borrowing constraint

- If current y_{it} is particularly low, may borrow to keep c_{it} smooth

- Such borrowing is limited by a constraint

$$a_{it} \geq \underline{a}, \qquad \underline{a} \leq 0$$

- For this model we take $\underline{a}$ to be an exogenous parameter

Aggregate state

- There are two key endogenous variables

  (i) the price q_t of the riskless bond

  (ii) the cross-sectional (joint) *distribution* of types

$$\mu_t(a, y) = \text{Prob}[a_{it} = a\, ,\; y_{it} = y]$$

- The *aggregate state* of the economy is the distribution (function) $\mu_t(\cdot)$ and in principle this distribution changes over time

- We will study a simpler problem where distribution is not changing

Stationary equilibrium

- In particular, we focus on a stationary (steady-state) equilibrium

- In such an equilibrium

  - aggregate variables are constant

    $$q \,,\ \mu(\cdot)$$

  - but individual-level variables are not constant

    $$c_{it} \,,\ y_{it} \,,\ a_{it}$$

- An *initial distribution* $\mu_0(a, y) \neq \mu(a, y)$ would induce transitional dynamics, but we focus on the 'long run' where such transitional dynamics of the distribution have played out

Dynamic programming problem

- Bellman equation for an agent of type a, y given q

$$v(a, y\,;\,q) = \max_{a' \geq \underline{a}} \left[u(c) + \beta \sum_{y'} v(a', y'\,;\,q)\,\pi(y' \,|\, y) \right]$$

subject to

$$c + qa' \leq a + y$$

- Let $a' = g(a, y\,;\,q)$ denote the policy function implied by the maximization on the RHS of the Bellman equation

- Note that q is constant and that individuals do not need to know $\mu(\cdot)$ to solve their problem

Stationary equilibrium

- A *stationary equilibrium* is a value function $v(a, y)$, policy function $g(a, y)$, distribution $\mu(a, y)$ and price q such that:

  (i) taking q as given, $v(a, y)$ and $g(a, y)$ solve the dynamic programming problem for an agent of type a, y

  (ii) the asset market clears

$$\sum_a \sum_y g(a, y)\mu(a, y) = 0$$

  (iii) the distribution $\mu(a, y)$ is stationary

$$\mu(a', y') = \sum_a \sum_y \text{Prob}[a', y' \,|\, a, y]\, \mu(a, y)$$

  where the conditional distribution $\text{Prob}[a', y' \,|\, a, y]$ is given by $a' = g(a, y)$ and $\pi(y' \,|\, y)$

Market clearing

- At the equilibrium price, demand equals supply

$$\sum_a \sum_y g(a,y)\mu(a,y) = 0$$

- Assets are in *zero net supply*

  – those with $a' > 0$ are on the demand side, buying the asset at price q
  – those with $a' < 0$ are on the supply side, selling the asset at price q

- If the asset market clears, we also have goods market clearing

$$\sum_a \sum_y c(a,y)\mu(a,y) = \sum_a \sum_y y\mu(a,y) \equiv Y$$

where $c(a,y)$ denotes the consumption policy implied by $g(a,y)$

Borrowing constraint

- Let $\lambda \geq 0$ denote the Lagrange multiplier on the borrowing constraint $a' \geq \underline{a}$

- The Lagrangian for the RHS of the Bellman equation can be written

$$L = u(a + y - qa') + \beta \sum_{y'} v(a', y'\,;\, q)\, \pi(y' \,|\, y) + \lambda(a' - \underline{a})$$

- The first order condition with respect to a' is then

$$-qu_1(c) + \beta \sum_{y'} v_1(a', y'\,;\, q)\, \pi(y' \,|\, y) + \lambda \leq 0, \qquad \text{with} = \text{if } a' > \underline{a}$$

while the envelope condition is, as usual,

$$v_1(a, y) = u_1(c)$$

Complementary slackness

- Equivalently

$$qu_1(c) - \beta \sum_{y'} u_1(c')\,\pi(y' \,|\, y) \geq \lambda \geq 0$$

where λ and c, c' are evaluated at the optimum

- Either (i) the borrowing constraint is *slack* ($a' > \underline{a}$ and $\lambda = 0$) and we have the usual kind of consumption Euler equation

$$qu_1(c) = \beta \sum_{y'} u_1(c')\,\pi(y' \,|\, y)$$

or (ii) the borrowing constraint *binds* ($a' = \underline{a}$ and $\lambda > 0$) so that we have the Euler inequality

$$qu_1(c) > \beta \sum_{y'} u_1(c')\,\pi(y' \,|\, y)$$

Solution algorithm

- Start with an initial guess q^0
- Solve individual's problem for $v(a, y\,;\,q^0)$ and $g(a, y\,;\,q^0)$ given q^0
- Solve for the stationary distribution $\mu(a, y\,;\,q^0)$ implied by $g(a, y\,;\,q^0)$ and the exogenous $\pi(y' \mid y)$
- Compute the error on the market-clearing condition

$$\left\| \sum_a \sum_y g(a, y\,;\,q^0)\mu(a, y\,;\,q^0) \right\|$$

  If this error is less than some pre-specified *tolerance* $\varepsilon > 0$, stop. Otherwise update to q^1 and try again

Updating the price

- We are trying to find q such that the asset market clears

- If for any q^n (for $n = 0, 1, 2, \ldots$) we have

$$\sum_a \sum_y g(a, y\,;\, q^n)\mu(a, y\,;\, q^n) > 0$$

then there is *excess demand* and so we should increase the price, updating to some $q^{n+1} > q^n$

- Likewise if for any q^n we have

$$\sum_a \sum_y g(a, y\,;\, q^n)\mu(a, y\,;\, q^n) < 0$$

then there is *excess supply* and so we should decrease the price, updating to some $q^{n+1} < q^n$

Inner and outer problems

- Think of this whole procedure as a mapping of the form

$$q \to F(q) = 0$$

with

$$v(\cdot\,;\,q) = Tv(\cdot\,;\,q)$$

- We have an *inner problem*, namely the individual agent's dynamic programming problem $v(\cdot\,;\,q) = Tv(\cdot\,;\,q)$

- And an *outer problem*, namely finding the market clearing price q given individual optimality

Stationary equilibrium

- In this stationary equilibrium, individual outcomes c_{it}, a_{it} are stochastic processes induced by Markov process for y_{it}, namely

$$a_{it+1} = g(a_{it}, y_{it})$$

and

$$c_{it} = c(a_{it}, y_{it}) \equiv a_{it} + y_{it} - qg(a_{it}, y_{it})$$

- While individual outcomes fluctuate over time, the cross-sectional distribution of them does not

- Cross-sectional distribution equivalent to the time-series distribution of individual outcomes

Huggett's example

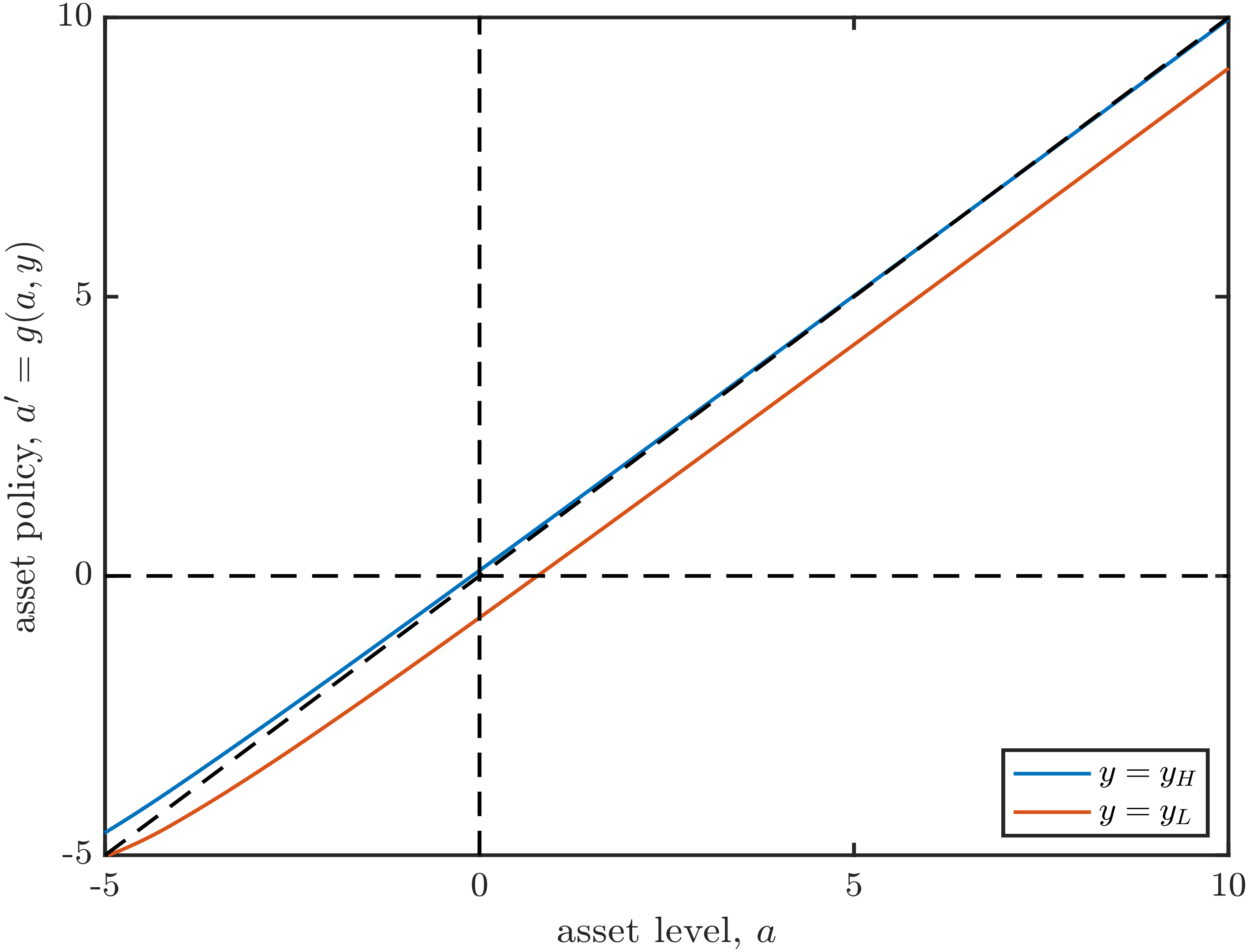
- Six periods per year
- Time discount factor $\beta = 0.96^{1/6} = 0.993$ per period
- CRRA $u(c)$ with various α
- Two-state Markov chain with $y_H = 1, y_L = 0.1$ and transition probabilities

  $$\pi_{HH} \equiv \pi(y_H \,|\, y_H) = 0.925$$
  $$\pi_{LL} \equiv \pi(y_L \,|\, y_L) = 0.500$$

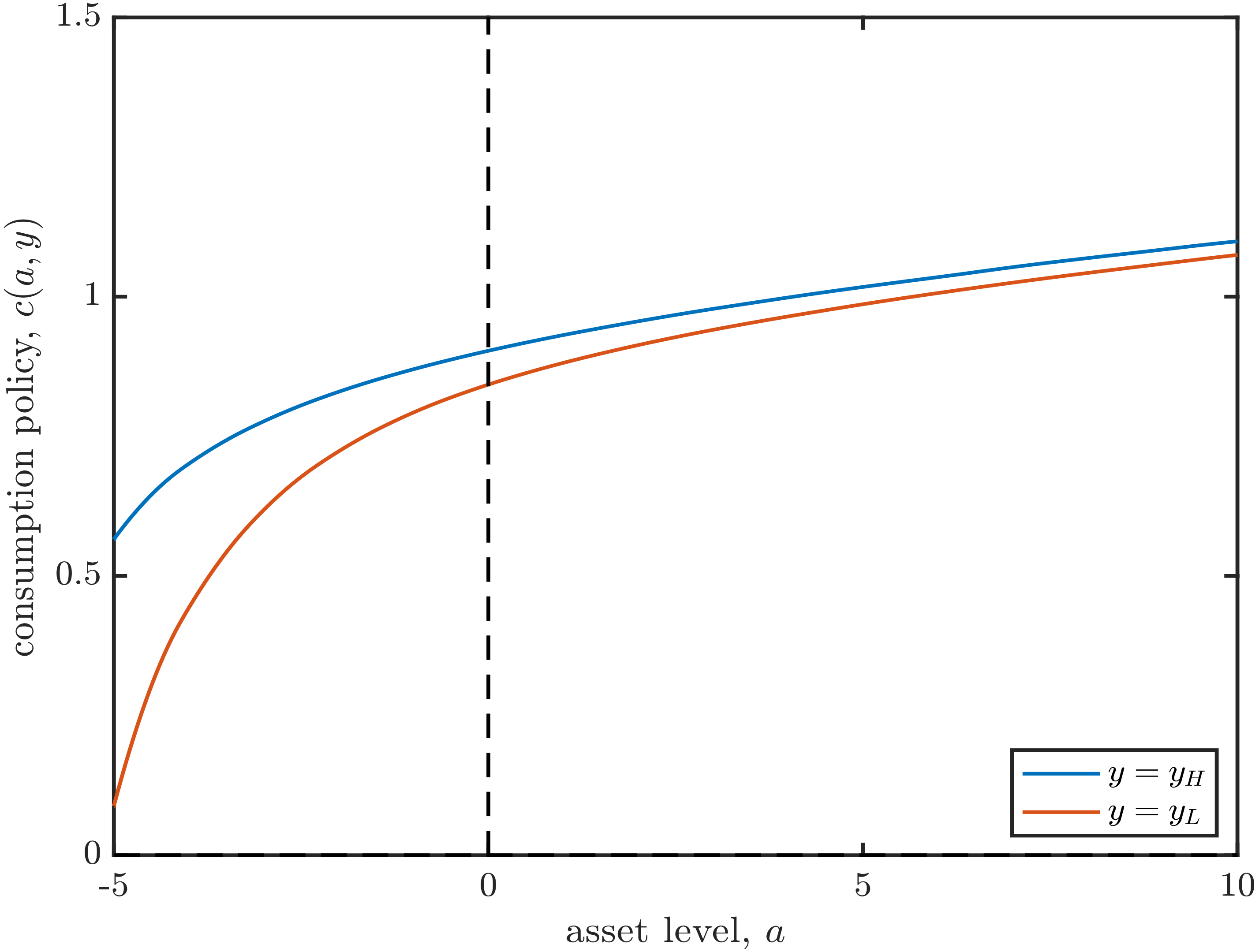
  Implies average duration L state is two periods (≈ 17 weeks)

- Solve on a grid of $a \in \{\underline{a}, \ldots\}$ for various $\underline{a}$. Benchmark $\underline{a} = -5.3$ (approximately 1 year's average endowment)

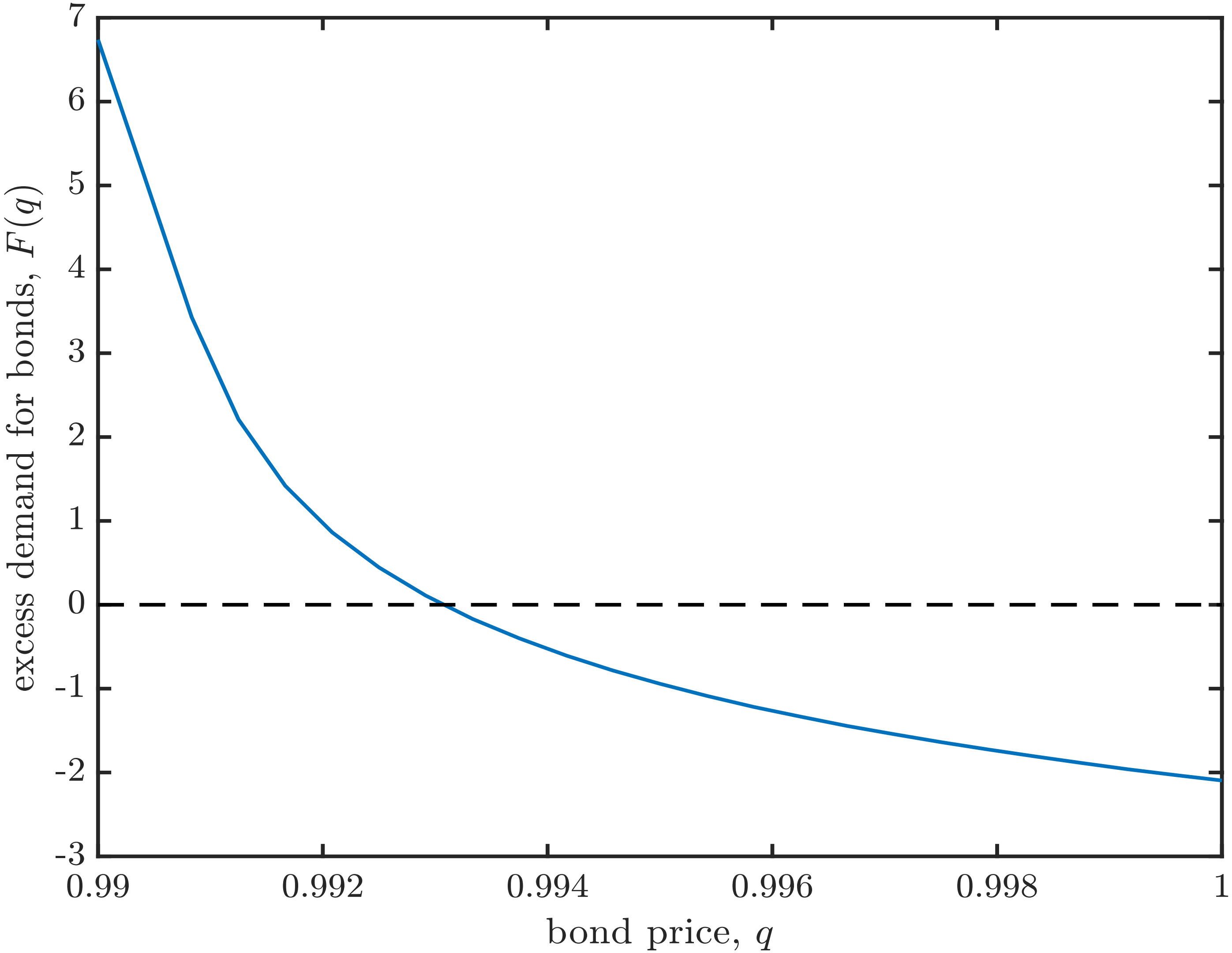
Asset policy $a' = g(a, y)$



Consumption policy $c(a, y)$



Excess demand $F(q) = \sum_a \sum_y g(a, y\,;\,q)\mu(a, y\,;\,q)$



Complete markets benchmark

- Complete risk-sharing

$$c_{it} = Y$$

- Implies bond price

$$q = \beta$$

- Assets then follow

$$a_{it+1} = (1+r)(a_{it} + y_{it} - Y)$$

where $r = 1/q - 1$ denotes the risk-free rate

Low risk aversion, $\alpha = 1.5$

- Risk-free rate r in annual percent, for various $\underline{a}$

$\underline{a}$	r	q
-2	-7.1%	1.0124
-4	2.3%	0.9962
-6	3.4%	0.9944
-8	4.0%	0.9935

- As borrowing constraint becomes tight (higher $\underline{a}$) there is high demand for saving, pushes up equilibrium q and pushes down r

- As borrowing constraint becomes slack (lower $\underline{a}$), there is more borrowing, pushes down equilibrium q and pushes up r

- Approach complete market case $q \approx \beta$ if $\underline{a}$ low enough

Higher risk aversion, $\alpha = 3$

- Risk-free rate r in annual percent, for various $\underline{a}$

$\underline{a}$	r	q
-2	-23.0%	1.0448
-4	-2.6%	1.0045
-6	1.8%	0.9970
-8	3.7%	0.9940

- Higher risk aversion α reduces r for all $\underline{a}$

- Higher risk aversion plus tight borrowing constraints leads to massive demand for saving and hence very low interest rates

Next class

- Solving the Huggett model
  - further computational details
  - solving consumption/savings problems with borrowing constraints
  - solving the general equilibrium problem