Macroeconomics

Lecture 11: dynamic programming applications, part two

Chris Edmond

1st Semester 2019

This class

- Dynamic programming applications, part two
- Job search and matching, applications to labor markets

Search frictions

- In a standard labor supply/demand models
  - firm can hire as much labor as it wants at prevailing wage
  - workers can find employment at prevailing wage
- In search model, neither of these is immediately true
  - unemployed workers need to find jobs
  - firms with vacancies need to find workers

  and these activities take *time and resources*
- Of course, search frictions not the only reason for unemployment

Search models of the labor market

- Tractable alternative to labor supply/demand models

- Emphasizes *labor market flows*
(e.g., transitions in/out employment, in/out labor force etc)

- Natural connection to data on *job creation* and *job destruction*

- We will consider two distinct examples:

  (i) individual decision problem, sequential search in the spirit of McCall

  (ii) general equilibrium, random matching in the spirit of Diamond, Mortensen and Pissarides

McCall approach

Setup

- Time $t = 0, 1, 2, \ldots$
- Single agent with risk neutral preferences

$$\mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t c_t\right\}, \qquad 0 < \beta < 1$$

- Each period, unemployed worker draws IID wage offer $w \sim F(w)$
- Two actions
  - accept offer: become employed and have $c_t = w$ forever
  - reject offer: remain unemployed, receive benefits $c_t = b$ this period and draw new w' next period

Dynamic programming problem

- Bellman equation can be written

$$v(w) = \max_{\text{accept, reject}} \left[\frac{1}{1-\beta} w \,,\; b + \beta \int_0^\infty v(w')\, dF(w') \right]$$

- Accept wage offer if

$$\frac{1}{1-\beta} w > b + \beta \int_0^\infty v(w')\, dF(w')$$

- Reject wage offer if

$$\frac{1}{1-\beta} w < b + \beta \int_0^\infty v(w')\, dF(w')$$

- RHS of these inequalities is a constant, independent of current w

Reservation wage

- Let $\bar{w}$ be such that

$$\frac{1}{1-\beta}\bar{w} = b + \beta \int_0^\infty v(w')\, dF(w')$$

- Then value function has the piecewise linear form

$$v(w) = \begin{cases} \dfrac{1}{1-\beta}\bar{w} & w \leq \bar{w} \\ \\ \dfrac{1}{1-\beta}w & w \geq \bar{w} \end{cases}$$

- This $\bar{w}$ is known as the *reservation wage*, unemployed worker will not work for $w < \bar{w}$

- But still need to determine $\bar{w}$

Determining $\bar{w}$

- Reservation wage $\bar{w}$ solves indifference condition

$$\frac{1}{1-\beta}\bar{w} = b + \beta \int_0^\infty v(w')\, dF(w')$$

or

$$\frac{1}{1-\beta}\bar{w} = b + \frac{\beta}{1-\beta}\left(\int_0^{\bar{w}} \bar{w}\, dF(w') + \int_{\bar{w}}^\infty w' dF(w')\right)$$

- Collecting terms and rearranging gives

$$\bar{w} - b = \frac{\beta}{1-\beta}\int_{\bar{w}}^\infty (w' - \bar{w})\, dF(w')$$

- LHS is opportunity cost of searching again with $w = \bar{w}$ in hand, RHS is expected discounted benefit of searching again given that only $w' > \bar{w}$ will be accepted

Determining $\bar{w}$

- Single equation in unknown scalar $\bar{w}$

$$\bar{w} - b = \frac{\beta}{1-\beta} \int_{\bar{w}}^{\infty} (w' - \bar{w})\, dF(w')$$

- Let $R(x)$ denote the function on the RHS

$$R(x) \equiv \frac{\beta}{1-\beta} \int_{x}^{\infty} (w' - x)\, dF(w')$$

which has the properties

$$R(0) = \frac{\beta}{1-\beta}\mathbb{E}\{w\} > 0, \qquad \text{and} \qquad R(\infty) = 0$$

with

$$R'(x) = -\frac{\beta}{1-\beta}[1 - F(x)] < 0$$

and

$$R''(x) = +\frac{\beta}{1-\beta}F'(x) > 0$$

Comparative statics of $\bar{w}$

- Hence there is indeed a unique $\bar{w}$ ($> b$) such that

$$\bar{w} - b = \frac{\beta}{1-\beta} \int_{\bar{w}}^{\infty} (w' - \bar{w})\, dF(w') \qquad (*)$$

- Implicitly determines $\bar{w}$ in terms of parameters $b, \beta, F(\cdot)$

- Comparative statics of reservation wage $\bar{w}$

  – higher benefits b increase reservation wage

  – higher discount factor β increases reservation wage

  – *mean-preserving* spread in $F(w')$ increases reservation wage

Mean-preserving spread in $F(w')$

- To see the effect of a mean-preserving spread, use integration-by-parts to rewrite $(*)$ as

$$\bar{w} - b = \beta\big(\mathbb{E}\{w\} - b\big) + \beta \int_0^{\bar{w}} F(w')\, dw' \qquad (**)$$

- Now consider two distributions $F_1(w')$ and $F_2(w')$ where $F_2(w')$ is a mean-preserving spread of $F_1(w')$. Then $F_1(w')$ *second-order stochastically dominates* $F_2(w')$ in the sense that

$$\int_0^x F_1(w')\, dw' < \int_0^x F_2(w')\, dw', \qquad \text{for any } x$$

- Hence reservation wage $\bar{w}_1$ for $F_1(w')$ is less than $\bar{w}_2$ for $F_2(w')$

- Key intuition is that more dispersion in $F(w')$ creates *option value*

Job loss

- Suppose employed worker loses job with exogenous probability δ

- Bellman equation can be written

$$v(w) = \max \left\{ \begin{array}{c} w + \beta\Big((1-\delta)v(w) + \delta(b + \beta \displaystyle\int_0^\infty v(w')\, dF(w')\Big) \\ \\ b + \beta \displaystyle\int_0^\infty v(w')\, dF(w') \end{array} \right\}$$

- Solution again characterized by a reservation wage $\bar{w}$, can show $\bar{w}$ lower than previous case with $\delta = 0$

Unemployment flows

- Let u_t denote aggregate unemployment rate

- Employed workers become unemployed with probability δ

- Unemployed workers become employed with probability $1 - F(\bar{w})$

- Hence unemployment evolves according to

$$u_{t+1} = \delta(1 - u_t) + F(\bar{w})u_t$$

- Steady-state unemployment rate

$$u = \frac{\delta}{\delta + 1 - F(\bar{w})}$$

(increasing in δ and increasing in $\bar{w}$)

Diamond-Mortensen-Pissarides approach

Matching function

- Let $L > 0$ denote size of the labor force

- Let mL denote number of job *matches*, uL number of unemployed, and vL number of vacant jobs

- Assume number matches given by *matching function*

$$mL = M(\,uL\,,\,vL\,)$$

that is increasing, concave and has *constant returns to scale* so that

$$m = M(u\,,\,v)$$

Matching function

- Job finding rate f

$$fu = m = M(u,v) \qquad \Rightarrow \qquad f = \frac{M(u,v)}{u}$$

- Other side of this is vacancy filling rate q

$$qv = m = M(u,v), \qquad \Rightarrow \qquad q = \frac{M(u,v)}{v} = \frac{f\,u}{v}$$

- With constant returns to scale

$$f = M\big(1, \frac{v}{u}\big) \equiv f(\theta)$$

$$q = M\big(\frac{u}{v}, 1\big) \equiv q(\theta) = f(\theta)/\theta$$

where $\theta \equiv v/u$ is known as '*labor market tightness*'

Matching function

- Job finding rate $f(\theta)$, increasing in labor market tightness. Expected duration unemployment $1/f(\theta)$, decreasing in θ

- Vacancy filling rate $q(\theta)$, decreasing in labor market tightness. Expected duration vacancy $1/q(\theta)$, increasing in θ

- **Example**: if $M(u,v) = u^{\alpha}v^{1-\alpha}$ for $0 < \alpha < 1$ then

$$f(\theta) = \theta^{1-\alpha}, \qquad q(\theta) = \theta^{-\alpha}$$

Unemployment flows revisited

- Employed workers become unemployed with probability δ
- Unemployed workers become employed with probability $f(\theta)$
- Hence unemployment evolves according to

$$u_{t+1} = \delta(1 - u_t) + (1 - f(\theta))u_t$$

- Steady-state unemployment rate in this setting

$$u = \frac{\delta}{\delta + f(\theta)}$$

Beveridge curve

- Now write steady state unemployment condition

$$\boxed{u = \frac{\delta}{\delta + f(\theta)}, \qquad \theta = v/u} \tag{1}$$

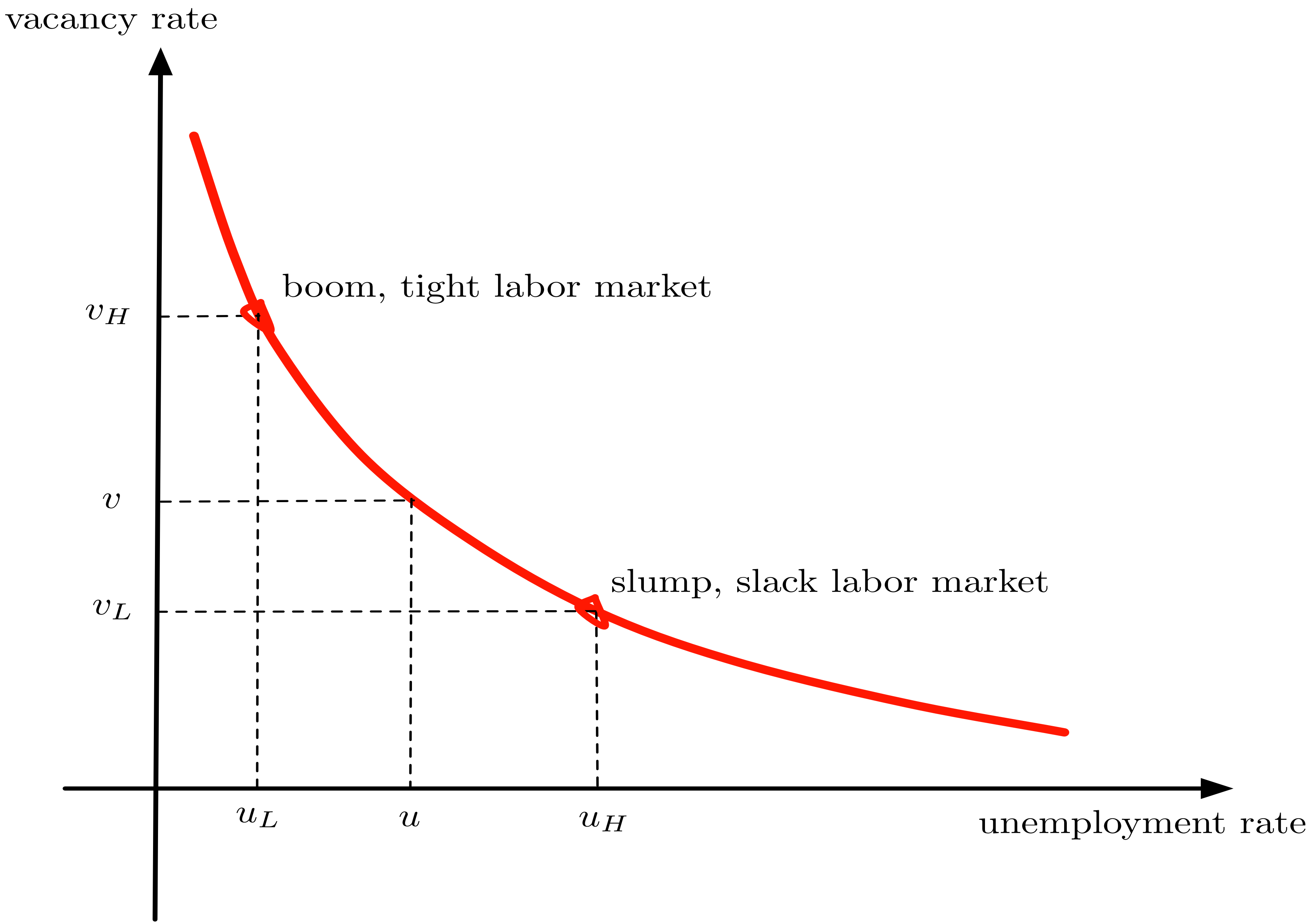
- Set of (v, u) satisfying (1) is known as the '*Beveridge curve*'

- An inverse relationship between v and u. Shifted by changes in the job destruction rate δ or the matching technology $f(\cdot)$

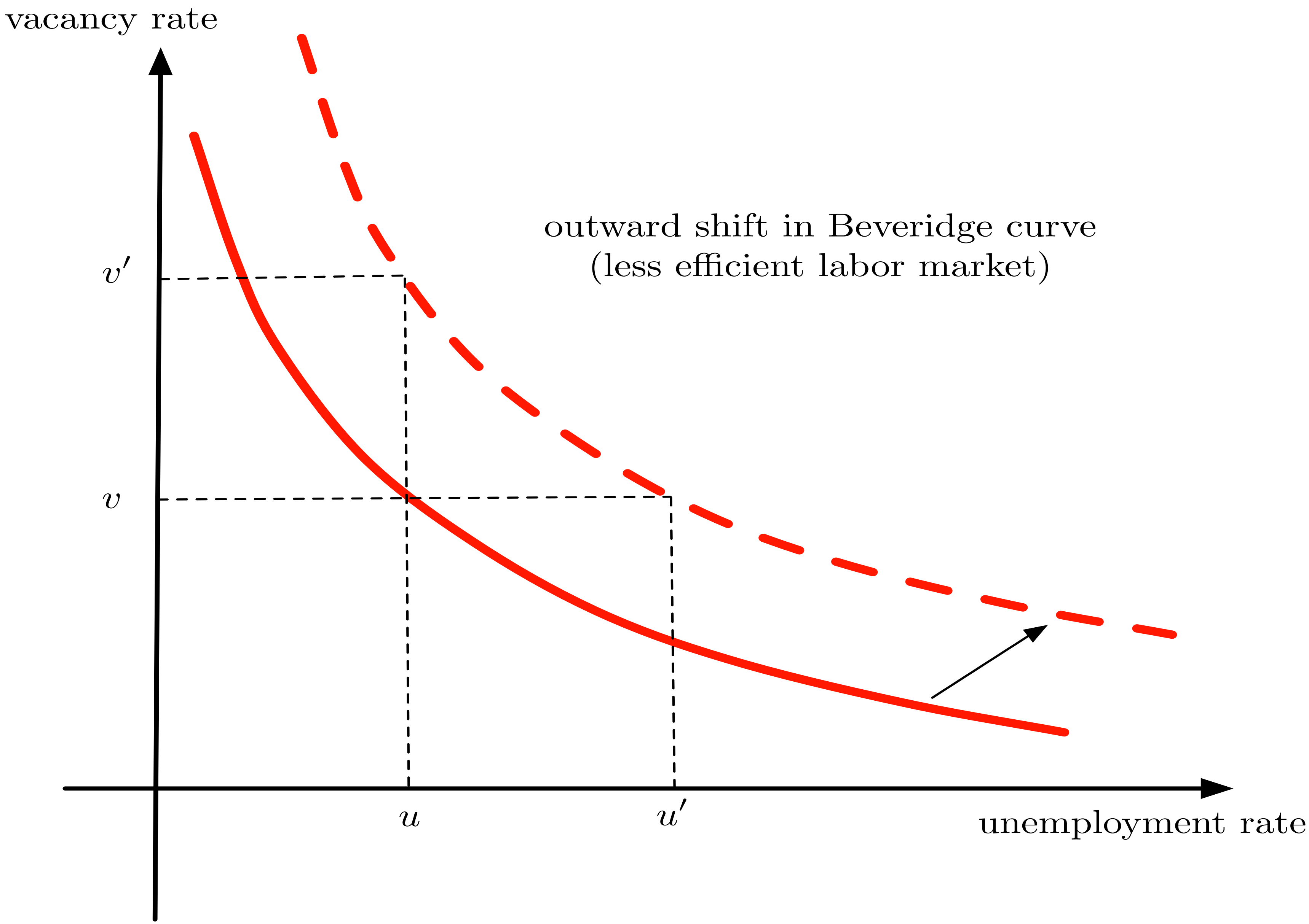
- **Example**: if $M(u, v) = A\, u^{\alpha} v^{1-\alpha}$ for $0 < \alpha < 1$ and $A > 0$ then

$$v = \left(\left(\frac{\delta}{A}\right)\left(\frac{1-u}{u^{\alpha}}\right)\right)^{1/(1-\alpha)}$$

Beveridge curve



Shifts in the Beveridge curve



Setup

- Risk neutral workers and firms, discount factor $\beta \in (0, 1)$
- Unemployed workers and firms with vacancies matched via $M(u, v)$
- Workers and firms *bargain* over wages w
- Free-entry into vacancy creation
- Focus on steady states

Job creation and destruction

- Firms can employ one worker
- Output from match $y = z > 0$
- Wage w paid to employed worker (no wage distribution)
- Jobs destroyed with probability $\delta \in (0, 1)$
- Jobs created by posting vacancies, cost $\kappa z > 0$
- Vacancy filled with probability $q(\theta)$

Value functions

- Let J denote the *value of a filled job* to a firm. Satisfies the steady-state Bellman equation

$$J = z - w + \beta\big(\,\delta V + (1-\delta)J\,\big)$$

hence

$$J = \frac{1}{1-\beta(1-\delta)}\big(\,z - w + \beta\delta V\,\big)$$

- Let V denote the *value of a vacancy* to a firm. Satisfies the steady-state Bellman equation

$$V = -\kappa z + \beta\big(\,q(\theta)J + (1-q(\theta))V\,\big)$$

Job creation

- Free-entry into job creation drives V to $V = 0$, so

$$0 = -\kappa z + \beta q(\theta) J \quad \Rightarrow \quad J = \frac{\kappa z}{\beta q(\theta)}$$

- Plugging this into first Bellman equation and collecting terms gives

$$\boxed{w = z - (1 - \beta(1 - \delta)) \frac{\kappa z}{\beta q(\theta)}} \tag{2}$$

- Wage equated to marginal product of labor less expected discounted search costs. Plays the role of a labor demand schedule

- For given wage w, this will determine labor market tightness θ.

Workers

- Let W denote the *value of a job* to a worker. Satisfies the steady-state Bellman equation

$$W = w + \beta\big(\,\delta U + (1-\delta)W\,\big)$$

  hence

$$W = \frac{1}{1-\beta(1-\delta)}\big(\,w + \beta\delta U\,\big)$$

- Let U denote the *value of being unemployed.* Satisfies the steady-state Bellman equation

$$U = b + \beta\big(\,f(\theta)W + (1-f(\theta))U\,\big)$$

  where $b \leq w$ denotes unemployment benefits etc

Wage determination

- Match between unemployed worker and firm with vacancy creates a mutual profit opportunity. How should these profits be split?

- Payments $z - w$ to firm, w to worker

- Wage w determined by *bargaining* between worker and firm

- Choice of w affects job value to individual firm $J(w)$ and to individual worker $W(w)$ taking as given aggregate market conditions U, V etc

- At a wage of w, the firm's surplus from a match is $J(w) - V$ and the worker's surplus is $W(w) - U$

Generalized Nash bargaining

- Wage w maximizes the *Nash product*

$$(W(w) - U)^{\phi}(J(w) - V)^{1-\phi}, \qquad 0 \leq \phi \leq 1$$

where the parameter ϕ denotes the workers' *bargaining power*

- First order condition for this problem can be written

$$\phi \frac{W'(w)}{W(w) - U} = -(1 - \phi)\frac{J'(w)}{J(w) - V}$$

Now note that, treating aggregate U, V as given,

$$W'(w) = \frac{1}{1 - \beta(1 - \delta)}, \qquad J'(w) = -\frac{1}{1 - \beta(1 - \delta)}$$

- So we can write

$$W = U + \phi S$$

where $S = W - U + J$ is the *total match surplus* (given $V = 0$)

Wages and the value of unemployment

- Recall that

$$W = \frac{1}{1-\beta(1-\delta)}\Big(\, w + \beta\delta U \,\Big), \qquad J = \frac{1}{1-\beta(1-\delta)}\Big(\, z - w \,\Big)$$

- Then given surplus splitting $W - U = \phi(W - U + J)$ we have

$$w - (1-\beta)U = \phi\Big(\, w - (1-\beta)U + z - w \,\Big)$$

- Collecting terms and simplifying

$$w = \phi z + (1-\phi)(1-\beta)U$$

- Wage is bargaining-weighted average of productivity z and flow value of unemployment $(1-\beta)U$

Wage curve

- From the Bellman equation for U

$$(1-\beta)U = b + \beta f(\theta)(W-U)$$

- But from the Nash bargain worker surplus proportional to firm surplus which is pinned down by free entry

$$W - U = \frac{\phi}{1-\phi} J = \frac{\phi}{1-\phi}\left(\frac{\kappa z}{\beta q(\theta)}\right)$$

Hence

$$(1-\beta)U = b + \beta f(\theta)\frac{\phi}{1-\phi}\left(\frac{\kappa z}{\beta q(\theta)}\right) = b + \frac{\phi}{1-\phi}\kappa z\theta$$

- Plugging this into our expression for wages and collecting terms

$$\boxed{w = (1-\phi)b + \phi(1+\kappa\theta)z} \tag{3}$$

This 'wage curve' plays the role of a labor supply schedule

Steady state equilibrium

- To summarize, in a steady state equilibrium we solve for w, θ simultaneously from (i) the wage curve

$$w = (1-\phi)b + \phi(1+\kappa\theta)z$$

and (ii) the marginal product condition

$$w = z - (1-\beta(1-\delta))\frac{z\kappa}{\beta q(\theta)}$$

- Given w, θ from these two equations we can back out the unemployment rate u from the Beveridge curve

$$u = \frac{\delta}{\delta + f(\theta)}$$

and then determine $v = \theta u$ and the present values W, U, J etc

Solving for w, θ