Macroeconomics

Lecture 1: introduction and course overview

Chris Edmond

1st Semester 2019

This course

An advanced course in macroeconomic theory

First half: dynamic programming methods for deterministic and stochastic environments, applications to consumption-savings problems and job search, etc

Second half: more elaborate applications, including general equilibrium models with complete and incomplete asset markets, income and wealth inequality, firm dynamics, etc

Along the way, there will be a lot of programming in MATLAB (or your preferred language)

Course material

- No required text, but a useful supplement is
  - Ljungqvist and Sargent (2018): *Recursive Macroeconomic Theory.* 4th Edition.
- Also useful for the first half of the course
  - Miranda and Fackler (2002): *Applied Computational Economics and Finance.*
  - Stokey and Lucas with Prescott (1989): *Recursive Methods in Economic Dynamics.*
- Various journal articles and working papers, posted to the LMS
- Slides for each lecture, posted to the LMS

Assessment

Task	*Due date*	*Weight*
Problem set #1	Tuesday March 26	5%
Problem set #2	Tuesday April 16	5%
Problem set #3	Tuesday May 14	5%
Problem set #4	Tuesday May 28	5%
Mid-semester exam	Wednesday April 17	30%
Final exam	exam block	50%

Tutorials

Tutorial times

Fridays	15:15–16:15	Alan Gilbert G20

Tutor: Omid Mousavi.

Tutorials begin next week. An essential part of the course.

Lecture schedule

- First half: core material
  - *dynamic programming methods*, lectures 1–8

    including a fair amount of practical computational tools
  - *dynamic programming applications*, lectures 9–12

    consumption-savings problems, job search, asset pricing

- Mid-semester exam based on first half of the course

Lecture schedule

- Second half: more elaborate applications and extensions

  – *complete-markets general equilibrium*, lectures 15–16

    contingent claims, aggregate vs. idiosyncratic risk

  – *incomplete markets*, lectures 17–19

    implications for income and wealth inequality

  – *firm dynamics*, lectures 20–21

    entry and exit, job turnover, misallocation

  – *'behavioural' macro*, lectures 22–23

    dynamically inconsistent preferences, temptation and self-control

- Final exam based draws more on second half of course

Rest of this class

Introduction to intertemporal choice in discrete time

Endowments

- Discrete time $t = 0, 1, 2, \ldots$

- Single agent is *endowed* with a stream of goods

$$y_0, \quad y_1, \quad y_2, \quad \ldots$$

  representing the amount of resources they have at each date

- Write this endowment stream

$$\boldsymbol{y} = \{y_0, \quad y_1, \quad y_2, \quad \ldots \quad \}$$

Preferences

- Single agent has preferences over streams of consumption

$$c_0, \quad c_1, \quad c_2, \quad \ldots$$

- Write this consumption stream

$$\boldsymbol{c} = \{c_0, \quad c_1, \quad c_2, \quad \ldots \quad \}$$

- Preferences are represented by a *utility function*

$$U(\boldsymbol{c})$$

  which *ranks* consumption streams

- Suppose for simplicity $U(\boldsymbol{c})$ strictly increasing, concave

Autarky

- One feasible outcome is *autarky*, $c_t = y_t$ for each t, with payoff

$$U(\boldsymbol{y})$$

- Depending on market structure, may be possible to do better

Borrowing and lending

- Suppose agent can borrow or lend at real interest rate $r > 0$

- Agent with assets a_t at the beginning of t has interest payments

$$ra_t$$

  which may be positive or negative

- If $a_t > 0$, interest income $ra_t > 0$ adds to endowment y_t

- If $a_t < 0$, debt servicing $ra_t < 0$ subtracts from endowment y_t

Flow budget constraint

- Budget constraint for date t

$$c_t + a_{t+1} \leq (1+r)a_t + y_t$$

with the understanding that $a_0 = 0$

- Since utility function is strictly increasing in each c_t, budget constraint holds with equality

$$c_t + a_{t+1} = (1+r)a_t + y_t$$

- We will assume that it is not possible to rollover debt forever ('*no Ponzi schemes*')

Iterating forward

- At $t = 0$ we have

$$c_0 + a_1 = y_0$$

- At $t = 1$ we have

$$c_1 + a_2 = Ra_1 + y_1 = R(y_0 - c_0) + y_1$$

where $R := 1 + r$ denotes the gross real interest rate

- At $t = 2$ we have

$$c_2 + a_3 = Ra_2 + y_2 = R(R(y_0 - c_0) + (y_1 - c_1)) + y_2$$

Iterating forward

- Collecting terms and rearranging

$$R^2 c_0 + R c_1 + c_2 + a_3 = R^2 y_0 + R y_1 + y_2$$

- Iterating this out to some arbitrary date $T > 2$

$$R^T c_0 + R^{T-1} c_1 + R^{T-2} c_2 + \cdots + c_T + a_{T+1} = R^T y_0 + R^{T-1} y_1 + R^{T-2} y_2$$

- Dividing both sides by R^T gives

$$c_0 + R^{-1} c_1 + R^{-2} c_2 + \cdots + R^{-T} c_T + R^{-T} a_{T+1} = y_0 + R^{-1} y_1 + R^{-2} y_2$$

- Or more compactly

$$\sum_{t=0}^{T} R^{-t} c_t + R^{-T} a_{T+1} = \sum_{t=0}^{T} R^{-t} y_t$$

Intertemporal budget constraint

- Taking the limit as $T \to \infty$ and supposing that

$$\lim_{T \to \infty} R^{-T} a_{T+1} = 0$$

we get

$$\sum_{t=0}^{\infty} R^{-t} c_t = \sum_{t=0}^{\infty} R^{-t} y_t$$

- This is known as the agent's *intertemporal budget constraint* (as opposed to their single-period budget constraint)

Intertemporal prices

- Let $p_t := R^{-t}$ denote the *intertemporal price of consumption*. The price as of date zero of consumption to be delivered on date t

- Can then write the intertemporal budget constraint as

$$\sum_{t=0}^{\infty} p_t c_t = \sum_{t=0}^{\infty} p_t y_t$$

or in vector notation

$$\boldsymbol{p} \cdot \boldsymbol{c} = \boldsymbol{p} \cdot \boldsymbol{y}$$

- Notice that

$$\frac{p_{t+1}}{p_t} = \frac{R^{-(t+1)}}{R^{-t}} = R^{-1}$$

- The gross real interest rate is an intertemporal *relative price*

Intertemporal prices

- Aside: suppose we had time-varying real interest rates

$$R_t = 1 + r_t$$

- Then intertemporal price of consumption would be

$$p_t = \prod_{s=1}^{t} R_s^{-1}$$

with

$$\frac{p_{t+1}}{p_t} = R_{t+1}^{-1}$$

- But we will stick with constant R and hence $p_t = R^{-t}$ for now

Standard consumer problem

- Now looks like a standard consumer choice problem

- Choose bundle $\boldsymbol{c} \geq 0$ to maximize

$$U(\boldsymbol{c})$$

subject to the budget constraint

$$\boldsymbol{p} \cdot \boldsymbol{c} = \boldsymbol{p} \cdot \boldsymbol{y}$$

- Lagrangian with single multiplier $\lambda \geq 0$

$$U(\boldsymbol{c}) + \lambda \boldsymbol{p} \cdot (\boldsymbol{y} - \boldsymbol{c})$$

Standard consumer problem

- System of first order necessary conditions

$$\frac{\partial}{\partial c_t} U(\boldsymbol{c}) = \lambda p_t, \qquad t = 0, 1, 2, \ldots$$

- This system pins down optimal consumption choices given λ and $\boldsymbol{p}$

$$\boldsymbol{c}(\lambda, \boldsymbol{p})$$

- Budget constraint then pins down multiplier λ given $\boldsymbol{y}$ and $\boldsymbol{p}$

$$\boldsymbol{p} \cdot \boldsymbol{c}(\lambda, \boldsymbol{p}) = \boldsymbol{p} \cdot \boldsymbol{y} \quad \Rightarrow \quad \lambda(\boldsymbol{y}, \boldsymbol{p})$$

- Solution can then be written

$$\boldsymbol{c}^* = \boldsymbol{c}(\lambda(\boldsymbol{y}, \boldsymbol{p}), \boldsymbol{p})$$

- Prices $\boldsymbol{p}$ matter both directly (substitution effects) and indirectly via λ (income/wealth effects)

Marginal rates of substitution

- Let $U_{c,t}$ denote the marginal utility of date-t consumption

$$U_{c,t} := \frac{\partial}{\partial c_t} U(\boldsymbol{c})$$

- System of first order conditions can be written

$$U_{c,t} = \lambda p_t$$

- Take the ratio of first order conditions at $t+1$ and t

$$\frac{U_{c,t+1}}{U_{c,t}} = \frac{p_{t+1}}{p_t} = R^{-1}$$

- This is a standard 'marginal rate of substitution (MRS) equals relative price' tangency condition

Time-separable utility

- We will typically use the *time-separable* utility function

$$U(\boldsymbol{c}) = \sum_{t=0}^{\infty} \beta^t u(c_t), \qquad 0 < \beta < 1$$

with strictly concave period utility, $u'(c) > 0$, $u''(c) < 0$

- Future utility is discounted by constant factor β

$$1, \quad \beta, \quad \beta^2, \quad \beta^3, \quad \dots$$

- Marginal utility of date-t consumption

$$U_{c,t} := \frac{\partial}{\partial c_t} U(\boldsymbol{c}) = \beta^t u'(c_t)$$

depends only on t and c_t, not consumption on any other date

Marginal rates of substitution

- System of first order conditions is then

$$\beta^t u'(c_t) = \lambda p_t$$

- Marginal rate of substitution between $t+1$ and t is then

$$\frac{\beta^{t+1} u'(c_{t+1})}{\beta^t u'(c_t)} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$$

- So our tangency condition can be written

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} = \frac{p_{t+1}}{p_t} = R^{-1}$$

Consumption Euler equation

- This is often expressed as either

$$\beta \frac{u'(c_{t+1})}{u'(c_t)} R = 1$$

or

$$u'(c_t) = \beta R u'(c_{t+1})$$

- Known as the *consumption Euler equation*, the key optimality condition in many consumption-savings problems

- Main idea: consuming 1 unit less consumption today costs me $u'(c_t)$ utility today but earns me R units tomorrow which when converted to utility and discounted back to today is $\beta R u'(c_{t+1})$. At the optimum, these marginal costs and benefits are equated

Qualitative dynamics

- Recall that $u''(c) < 0$. This implies

$$c_{t+1} > c_t \qquad \Leftrightarrow \qquad u'(c_{t+1}) < u'(c_t) \qquad \Leftrightarrow \qquad \beta R > 1$$

- Recall $R = 1 + r$ and likewise define the *pure rate of time preference* $\rho > 0$ by $\beta = (1+\rho)^{-1}$

- Then we can write

$$c_{t+1} > c_t \qquad \Leftrightarrow \qquad r > \rho$$

- Consumption is growing when real interest rate is high relative to the rate of time preference

Example

- Suppose period utility function is

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \qquad \sigma > 0$$

- Consumption Euler equation

$$c_t^{-\sigma} = \beta R c_{t+1}^{-\sigma}$$

or

$$\frac{c_{t+1}}{c_t} = (\beta R)^{1/\sigma}$$

- Hence consumption growth is just

$$\log\left(\frac{c_{t+1}}{c_t}\right) = \frac{1}{\sigma}\log(\beta R) \approx \frac{r - \rho}{\sigma}$$

Elasticity of substitution

- Parameter $1/\sigma > 0$ measures the sensitivity of consumption growth to real interest rates

- Write consumption Euler equation in terms of intertemporal prices

$$\frac{c_{t+1}}{c_t} = \left(\beta \frac{p_t}{p_{t+1}}\right)^{1/\sigma}$$

- *Elasticity of substitution*, as we move along indifference curve in response to changing relative prices

$$\frac{d \log\left(\frac{c_{t+1}}{c_t}\right)}{d \log\left(\frac{p_{t+1}}{p_t}\right)} = -\frac{1}{\sigma}$$

Elasticity of substitution

- Three important special cases

  (i) *perfect substitutes*, $\sigma = 0$

  $\log\left(\frac{c_{t+1}}{c_t}\right)$ very sensitive to even small changes in r

  (ii) *log utility*, $\sigma = 1$ [use l'Hôpital's rule]

  $\log\left(\frac{c_{t+1}}{c_t}\right)$ responds 1-for-1 to changes in r

  (iii) *perfect complements*, $\sigma = \infty$

  $\log\left(\frac{c_{t+1}}{c_t}\right)$ insensitive to even large changes in r

Example

- Euler equation gives us consumption growth

$$\frac{c_{t+1}}{c_t} = (\beta R)^{1/\sigma}$$

- Hence iterating forward from date $t = 0$ we have

$$c_t = (\beta R)^{t/\sigma} c_0$$

- Pin down consumption *level* using intertemporal budget constraint

$$\sum_{t=0}^{\infty} R^{-t} (\beta R)^{t/\sigma} c_0 = \sum_{t=0}^{\infty} R^{-t} y_t$$

Example

- Hence initial consumption is

$$c_0 = (1 - [\beta R^{1-\sigma}]^{1/\sigma}) \sum_{t=0}^{\infty} R^{-t} y_t$$

from which consumption evolves according to Euler equation

- What matters for level of consumption is '*permanent income*'

$$\sum_{t=0}^{\infty} R^{-t} y_t$$

- Consumption should not respond much to transitory changes in y_t

Example

- What happened to borrowing and lending?

- Having solved for consumption c_t can back out the net asset positions a_t that implement this

$$a_{t+1} = Ra_t + y_t - c_t$$

- Can also directly use flow budget constraints to solve optimization problem (circumventing need to form intertemporal constraint)

Sequence of constraints approach

- Lagrangian with multiplier $\lambda_t \geq 0$ for each flow budget constraint

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \sum_{t=0}^{\infty} \lambda_t \big[R a_t + y_t - c_t - a_{t+1} \big]$$

- First order necessary conditions

$$c_t : \qquad \beta^t u'(c_t) - \lambda_t = 0$$

$$a_{t+1} : \qquad -\lambda_t + \lambda_{t+1} R = 0$$

$$\lambda_t : \qquad R a_t + y_t - c_t - a_{t+1} = 0$$

These need to hold at every date

Sequence of constraints approach

- Hence we get

$$\lambda_t = R\lambda_{t+1}$$

with

$$\lambda_t = \beta^t u'(c_t)$$

- This is just our consumption Euler equation again

$$u'(c_t) = \beta R u'(c_{t+1})$$

Next class

- Review of neoclassical growth model
  - production, not an endowment economy
  - endogenous return on capital
  - links capital accumulation with consumption/savings decisions