

PhD Topics in Macro: Problem Set #4
Due Monday October 6th

Monopolistic competition and trade. Consider the following version of the Melitz (2003) model. There are two identical countries each populated by a representative consumer that inelastically supplies labor L and that has symmetric CES preferences over differentiated varieties

$$C = \left(\int_{\Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1$$

which they maximise subject to the budget constraint

$$PC = \int_{\Omega} p(\omega)c(\omega) d\omega \leq wL$$

Each variety $\omega \in \Omega$ is produced by single firm that operates with an increasing returns technology. A firm with productivity a can produce y units of output using

$$l(y, a) = f + \frac{y}{a}$$

units of labor. That is, there is a fixed operating cost $f > 0$ and a constant marginal cost of $1/a$ (both in units of labor). Prior to entry firms do not know their productivity level a . On paying a sunk entry cost $f_e > 0$, each firm draws a productivity level $a \sim G(a)$ from a Pareto distribution on $a \geq 1$, that is

$$G(a) := \text{Prob}[a' \leq a] = 1 - a^{-\xi}, \quad \xi > \sigma - 1$$

On learning a , each firm makes a once-and-for-all decision to operate (which involves paying the operating cost f) or not. A firm that exits obtains an outside option normalised to zero.

Firms in both countries may export to the other. To export, a firm must pay a fixed export cost $f_x > 0$ (in addition to its fixed operating cost f) and exports also incur a variable trade cost $\tau \geq 1$ of the iceberg form, for every τ units shipped only 1 unit arrives. By symmetry the wage rate in both countries will be the same. Normalise this to $w = 1$.

To begin with, consider a closed economy ($f_x = +\infty$).

- (a) Let $\pi(a)$ denote the profits of a firm with productivity a (given the aggregate price index P and aggregate consumption C). Derive a formula for $\pi(a)$ in terms of model parameters.
- (b) Let a^* denote the lowest productivity such that a firm will be willing to operate and let $\mu(a)$ denote the probability density of operating firms. Solve for a^* and $\mu(a)$ in terms of model parameters. Also let $P(a^*)$ denote the aggregate price index, $C(a^*)$ aggregate consumption, $A(a^*)$ aggregate productivity and $n(a^*) = \int_{\Omega} d\omega$ the number of producers. Derive formulas for these aggregate statistics in terms of a^* .

- (c) Suppose the parameter values are $\sigma = 4$, $\xi = 4.5$, $f = 0.5$, $f_e = 1$ and $L = 100$. Use these parameter values and your answers from (b) above to compute a^* and thereby compute the aggregate statistics $P(a^*)$, $C(a^*)$, $A(a^*)$ and $n(a^*)$. Also compute the real wage. Using these answers, plot the productivity density $\mu(a)$ of operating firms against the latent density $g(a) = G'(a)$ of productivity draws. Explain the relationship between these distributions. Also plot the the productivity density of operating firms against the densities of firm revenue and firm production. Are these more or less dispersed than the productivity distribution? Explain.

Now consider an open economy.

- (d) Let $\pi_d(a)$ denote the profits of a firm in its domestic market, $\pi_x(a)$ the profits of a firm in its export market, and let $\pi(a)$ denote overall firm profits. Derive formulas for these functions in terms of model parameters. Let a^* denote the lowest productivity such that a firm will be willing to operate and let a_x^* denote the lowest productivity such that a firm will export. Solve for a^* and a_x^* in terms of model parameters.
- (e) Now suppose the parameter values $\tau = 1.05$ and $f_x = 2$ with all other parameters as in (c) above. Using these specific parameter values, calculate a^* and a_x^* . How does the value you find for a^* compare to the closed economy case from (c) above? Explain. What fraction of firms export? Using your solutions, plot the functions $\pi_d(a)$, $\pi_x(a)$ and $\pi(a)$ for this open economy. Explain which firms benefit and which firms lose from opening to trade.
- (f) Using your solutions from (e), compute the aggregate statistics $P(a^*)$, $C(a^*)$, $A(a^*)$ and $n(a^*)$ and the densities of firm revenue and firm production. Explain how these compare to their closed economy counterparts. Explain the pattern of reallocation across firms induced by opening the economy to trade. Is the representative consumer better off in this open economy? Explain.