PhD Topics in Macro: Problem Set #3

Due Monday September 15th in class

Misallocation. Consider the Restuccia/Rogerson (2008) model. There is a large number of competitive firms that produce a homogeneous numeraire good. Each firm has a *time-invariant* productivity level a and produces output y using capital k and labor n according to the production function $y = ak^{\alpha}n^{\gamma}$ with $0 < \alpha + \gamma < 1$. Each firm faces a distortion $\tau \in (-1, +1)$ to its revenue so that from selling output y it takes home $(1-\tau)y$, i.e., $\tau > 0$ is a tax and $\tau < 0$ is a subsidy. Let $H(a)$ denote the cross-sectional distribution of productivity and let $P(\tau \,|\, a)$ denote the cross-sectional distribution of distortions conditional on productivity.

There are no fixed costs of operating, but to enter a firm pays a one-time cost $f_e > 0$. On entering a firm draws a pair (a, τ) from the joint distribution $G(a, \tau) = P(\tau \,|\, a)H(a)$. Each firm has time discount factor $0 < \beta < 1$ and also faces a constant probability $1 - \phi$ of being forced to exit. The representative consumer accumulates capital (which has a depreciation rate $0 < \delta < 1$) and inelastically supplies one unit of labor.

(a) Let w, r denote the wage rate and rental rate for capital. Let $\pi(a, \tau)$ denote the per-period profits of a firm with productivity a and distortion τ and let $k(a, \tau)$ and $n(a, \tau)$ denote the associated factor demands (given w, r). Solve for $\pi(a, \tau), k(a, \tau)$ and $n(a, \tau)$.

(b) Let $\mu(a, \tau)$ denote the measure of firms of type (a, τ) and let m denote the measure of entering firms in a stationary equilibrium. Provide a formula for $\mu(a, \tau)/m$ in terms of model primitives. Sketch a procedure for computing a stationary equilibrium. Explain how the wage rate w and capital rental rate r are determined in such an equilibrium.

Now consider the following annual parameterization. Normalize $f_e = 1$ and let $\beta = .96$, $\phi = .9$, $\alpha = .28$, $\gamma = .57$, and $\delta = .08$. Let $H(a)$ be Pareto on $a \geq 1$, that is

$$H(a) := \text{Prob}[a' \leq a] = 1 - a^{-\xi}$$

with shape parameter be $\xi = 3$. For computational purposes, discretize the productivity levels a to a grid with 1000 points. To do this, let u_i for $i = 1, ..., 1000$ be a uniform grid on $(.001, .999)$ and let $a_i = H^{-1}(u_i)$.

(c) Suppose $\tau = 0$ for all firms. Compute the stationary equilibrium of this no-distortion benchmark economy. Report aggregate output Y and aggregate capital K.

(d) Now consider *size-independent* distortions, $P(\tau \,|\, a)$ independent of a. In particular, let τ have a binary distribution $\tau \in \{\tau_-, \tau_+\}$ with $\tau_- \leq 0 \leq \tau_+$ and $\text{Prob}[\tau = \tau_+] = 0.5$. Consider the following distortions

$$\tau_+ \in \{0, 0.1, 0.2, 0.3, 0.4\}$$

For each of the distortions τ_+, compute the 'counterpart' subsidy τ_- such that the level of aggregate capital in this distorted economy is the same as in the benchmark non-distorted economy from part (c) above. To do this, start by guessing $\tau_- = -\tau_+$, solve the model, calculate aggregate capital, and then update τ_-, iterating on this until τ_- is such that aggregate capital is the same as in part (c) within a numerical tolerance of 10^{-6}. Report these counterpart subsidies. Once the appropriate counterpart subsidy has been found, also report the level of aggregate output in this distorted economy expressed relative to the aggregate output in the benchmark economy from part (c). Explain your findings.

(e) Now consider *size-dependent* distortions. Suppose the bottom 50% of firms are subsidized while the top 50% of firms are taxed with tax rates $\tau_+ \in \{0, 0.1, 0.2, 0.3, 0.4\}$. Again compute the counterpart subsidies that leave aggregate capital the same as in part (c) and report the implied levels of aggregate output expressed relative to the aggregate output in the benchmark economy from part (c). Compare these results to those you obtained in part (d). Explain any differences you find.

(f) Redo your calculations from parts (c) and (d) but now with alternate Pareto shape parameters $\xi = 1.5$ and $\xi = 5$. Report aggregate output for the no-distortion benchmark and for the economy with the size-independent distortions in (d) for both these alternate shape parameters (you will also have to recompute the counterpart subsidies). Explain any differences you find. Give economic intuition for your results.