PhD Topics in Macro: Problem Set #2

Due Monday September 1st in class

1. **Endogenous step-size in a quality-ladder model.** Consider a basic quality-ladder model where all firms have a single product and so all incumbent firms are identical. All incumbents receive flow profits $\pi = 1 - 1/q$ where $q > 1$ is the quality step-size. Now let the the amount of labor required to innovate to generate step size q at Poisson arrival rate $\lambda \geq 0$ be

$$l(q, \lambda) = c(q)\lambda$$

where $c(q)$ is strictly increasing and strictly convex. Firms choose both the innovation intensity λ *and* the step-size q.

   (a) Let $v(q)$ denote the steady-state value of an incumbent firm that innovates with step-size q and let w and r denote the wage and interest rate. What Bellman equation does $v(q)$ solve? Using this Bellman equation, derive a first order condition for the optimal q.

   (b) Suppose there is free entry. What restriction does this place on $v(q)$ and the economy-wide innovation intensity λ?

   (c) Now consider a steady-state with $r = \rho$ (the representative consumer's rate of time preference) and inelastically supplied labor L. Explain how the key endogenous variables q, λ, w and $v(q)$ are determined in such an equilibrium. Now suppose $c(q)$ has the particular form

$$c(q) = \frac{q^{1+\varphi}}{1+\varphi}, \qquad \varphi > 0$$

   Solve for the steady state values of q, λ, w and $v(q)$. Under what conditions on the parameters ρ, L and φ will this economy exhibit growth? Supposing there is growth, does a higher cost elasticity φ increase or decrease the growth rate? Explain.

2. **Solving a Klette/Kortum model.** Consider the Klette/Kortum model of innovation and firm dynamics. Let the representative consumer's discount rate be ρ per year with inelastic labor supply L. Suppose the flow profits of an incumbent who produces at the current state of the art are $\pi = 1 - 1/q$ per product where q is an exogenous step-size. The amount of labor required for an incumbent to innovate at rate $\lambda \geq 0$ per product is $l_R(\lambda) = c(\lambda)$ where

$$c(\lambda) = c_0 \frac{\lambda^{1+\varphi}}{1+\varphi}, \qquad c_0, \varphi > 0$$

Incumbents face a product destruction rate $\mu \geq 0$ per product that they take as given (but that is determined in equilibrium). Potential entrants need to pay a fixed cost $l_S > 0$ units of labor to innovate at rate 1. Let $\eta \geq 0$ denote the entry rate.

(a) Let $\rho = 0.04$ per year, $L = 1$, $\pi = 0.05$, $c_0 = 8$, $\varphi = 1$, and $l_S = 0.4$. Calculate the equilibrium allocations of labor between goods production L_X, research at incumbents L_R and research at 'startups' L_S. Is more labor employed at incumbents or startups? What are the innovation intensities at incumbents and startups? What is the aggregate rate of product destruction? What is the annual aggregate growth rate of the economy?

(b) Consider firms with $n = 1, 2, \ldots$ products. Let M_n denote the mass of firms with n products, M the total mass of firms and $P_n = M_n/M$ the size distribution of firms (in terms of number of products). For the parameter values in part (a), plot the size distribution P_n as a function of n. Also calculate the mean, median and standard deviation of the size distribution. (consider values of n until P_n drops below 10^{-8}).

Now let $a \geq 0$ denote the *age* of a firm. Calculate the expected number of products of a firm of age

$$a \in \{1\,,\,5\,,\,10\,,\,15\,,\,20\,,\,25\,,\,50\,,\,100\}$$

and plot the expected number of products as a function of a. Also calculate the expected life of a firm (time to exit). Explain all your findings.

(c) Suppose instead the profit rate was $\pi = 0.1$. Redo your calculations from parts (a) and (b) above and comment on the differences you find. What happens to employment in research at incumbents? At startups? What happens to the aggregate growth rate? How does the size distribution of firms change? What about the expected number of products by age? Give as much intuition as you can.

(d) Now keep $\pi = 0.05$ as in part (a) but suppose instead that $l_S = 0.6$. Again redo your calculations from parts (a) and (b) above and comment on the differences you find. What happens to employment in research at incumbents? At startups? What happens to the aggregate growth rate? How does the size distribution of firms change? What about the expected number of products by age? Again, give as much intuition as you can.