PhD Topics in Macroeconomics

Lecture 7: innovation and firm dynamics, part three

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This lecture

- Lentz/Mortensen applications of Klette/Kortum-style model
  - 2005 paper: extension to ex ante heterogeneous firms; implications for growth through reallocation/selection
  - 2008 paper: structural estimation using Danish panel data

Lentz/Mortensen (2005): overview

- Fundamental heterogeneity in Klette/Kortum model

- At birth, firm draws profitability type π ($\Leftrightarrow$ step size q). Applies to all of a firm's portfolio of n products

- Growth through two kinds of *reallocation*

  (i) from obsolete vintages to newest vintages, as in Klette/Kortum

  (ii) from low-π to high-π firms within set of continuing firms

  This second kind of reallocation is a *selection effect*

Lentz/Mortensen (2005): model

- Continuous time $t \geq 0$

- Representative household

$$U = \int_0^\infty e^{-\rho t} \log C_t \, dt, \qquad \rho > 0$$

- Expenditure $E_t = P_t C_t$ satisfies intertemporal Euler equation

$$\frac{\dot{E}_t}{E_t} = r_t - \rho$$

- Choose $E_t = 1$ as numeraire, then $r_t = \rho$

$$g_t = \frac{\dot{C}_t}{C_t} = -\frac{\dot{P}}{P_t}$$

Quality ladder

- Instantaneous utility

$$\log C_t = \int_0^1 \log \Big[\sum_{k=0}^{J_t(j)} z_t(j,k) x_t(j,k) \Big] \, dj$$

from $j \in [0,1]$ horizontal varieties, each of which comes in $k \in \{0, 1, \dots, J_t(j)\}$ vintages of increasing quality

- Random quality increments

$$z_t(j,k) = \prod_{k'=0}^{k} q_t(j,k')$$

(determined by π-type of successfully innovating firm)

Pricing

- In equilibrium, only highest quality supplier of product j operates, limit prices to deter entry by follower

$$p = qw, \qquad x = 1/qw$$

with profits

$$\pi = (p - w)x = \frac{q-1}{q} \in (0,1)$$

Firms

- Portfolio of $n \in \{0, 1, \dots\}$ products
- Profitability $\pi \sim \phi(\pi)$ drawn at firm *entry*, applies to *all products*
- Choose innovation intensity λ per product, total intensity λn
  - cost $c(\lambda)$ labor per product, strictly increasing and convex in λ
  - in general, innovation intensity will vary with π, i.e., $\lambda(\pi)$
- Firm takes as given destruction intensity μ per product

Value of a firm

- Let $V_n(\pi)$ denote value of type π firm with n products

- Bellman equation for firm with $n > 0$ products

$$rV_n(\pi) = \max_\lambda \Big[\pi n - wc(\lambda)n + \lambda n(V_{n+1}(\pi) - V_n(\pi)) - \mu n(V_n(\pi) - V_{n-1}(\pi))\Big]$$

- Guess-and-verify that $V_n(\pi)$ is linear in n

$$V_n(\pi) = v(\pi)n$$

for some $v(\pi) > 0$ to be determined

Value of a firm

- So value $v(\pi)$ per product satisfies

$$(r+\mu)v(\pi) = \max_{\lambda} \left[\pi - wc(\lambda) + \lambda v(\pi)\right]$$

- Equivalently

$$v(\pi) = \max_{\lambda} \left[\frac{\pi - wc(\lambda)}{r+\mu-\lambda}\right], \qquad \lambda(\pi) = \operatorname*{argmax}_{\lambda} \left[\frac{\pi - wc(\lambda)}{r+\mu-\lambda}\right]$$

with first order condition for innovation intensity

$$wc'(\lambda) = v(\pi)$$

- From envelope condition, $v'(\pi) > 0$ so $\lambda'(\pi) > 0$ too

Size distribution

- Let $M_n(\pi)$ denote steady-state measure of firms of size n

- For $n > 1$ we have the steady-state balance condition

$$(\lambda(\pi) + \mu)nM_n(\pi) = \lambda(\pi)(n-1)M_{n-1}(\pi) + \mu(n+1)M_{n+1}(\pi)$$

- For $n = 1$ we have

$$(\lambda(\pi) + \mu)M_1(\pi) = \eta\phi(\pi) + \mu 2M_2(\pi)$$

where η is the equilibrium entry rate (to be determined) and $\phi(\pi)$ is the measure of entering firms that draw π (exogenous)

- Finally, since only $n = 1$ firms are at risk of exit

$$\eta\phi(\pi) = \mu M_1(\pi)$$

Size distribution

- Rearranging, we have for $n = 1$ firms

$$M_1(\pi) = \frac{\eta}{\mu}\phi(\pi)$$

- So for size $n = 2$ firms

$$M_2(\pi) = \frac{1}{2\mu}[(\lambda(\pi) + \mu)M_1(\pi) - \eta\phi(\pi)] = \frac{\eta\lambda(\pi)}{2\mu^2}\phi(\pi)$$

- And so on, by induction, for any n

$$M_n(\pi) = \frac{\eta\lambda(\pi)^{n-1}}{n\mu^n}\phi(\pi), \qquad n = 1, 2, \ldots$$

Size distribution

- Total measure of type π firms

$$
\begin{aligned}
M(\pi) &:= \sum_{n=1}^{\infty} M_n(\pi) \\
&= \sum_{n=1}^{\infty} \frac{\eta\lambda(\pi)^{n-1}}{n\mu^n}\phi(\pi) = \eta\frac{\phi(\pi)}{\lambda(\pi)}\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{\lambda(\pi)}{\mu}\right)^n \\
&= \eta\frac{\phi(\pi)}{\lambda(\pi)}\log\left(\frac{\mu}{\mu-\lambda(\pi)}\right)
\end{aligned}
$$

- So conditional distribution is given by

$$
\frac{M_n(\pi)}{M(\pi)} = \frac{\frac{1}{n}\left(\frac{\lambda(\pi)}{\mu}\right)^n}{\log\left(\frac{\mu}{\mu-\lambda(\pi)}\right)}
$$

Again, the *log-series* distribution, now with type-specific parameter $\lambda(\pi)/\mu \in (0,1)$

Size distribution

- Conditional mean

$$\mathbb{E}[n \,|\, \pi] = \sum_{n=1}^{\infty} n \frac{M_n(\pi)}{M(\pi)} = \frac{\frac{\lambda(\pi)}{\lambda(\pi)-\mu}}{\log\left(\frac{\mu}{\mu-\lambda(\pi)}\right)}$$

which is increasing in $\lambda(\pi)$ and hence increasing in π

- Total mass of products produced by type π firms

$$K(\pi) := \sum_{n=1}^{\infty} n M_n(\pi) = \frac{\eta}{\mu - \lambda(\pi)} \phi(\pi)$$

with $K(\pi)/\phi(\pi)$ increasing in π and

$$\sum_{\pi} K(\pi) = 1$$

Selection and reallocation

- More profitable firms $\pi' > \pi$ are *over-represented* relative to intrinsic frequency

$$\frac{M_n(\pi')}{M_n(\pi)} - \frac{\phi(\pi')}{\phi(\pi)} = \frac{\phi(\pi')}{\phi(\pi)} \left[\left(\frac{\lambda(\pi')}{\lambda(\pi)} \right)^{n-1} - 1 \right]$$

is positive and increasing in n since $\lambda(\pi') > \lambda(\pi)$ for $\pi' > \pi$

- This 'selection bias' reflects *reallocation* from less to more profitable surviving firms and from exiting to entering

(Of course *all* growth in the model is reallocation of one kind or another, but this term reflects reallocation *across types*)

Entry

- Cost wl_S for Poisson intensity 1 of entering with $n = 1$ product

- Entry rate η adjusts to satisfy free-entry condition

$$\mathbb{E}[v] = \sum_{\pi} v(\pi)\phi(\pi) \leq wl_S$$

  with equality whenever $\eta > 0$

- Aggregate product destruction rate is then

$$\mu = \eta + \sum_{\pi} \sum_{n=1}^{\infty} n\lambda(\pi)M_n(\pi) = \eta + \sum_{\pi} \lambda(\pi)K(\pi)$$

$$= \eta \sum_{\pi} \frac{\mu}{\mu - \lambda(\pi)} \phi(\pi)$$

Equilibrium

- Constants

$$(w^* \, , \, \eta^* \, , \, \mu^*)$$

consistent with firm optimization, i.e., $v(\pi), \lambda(\pi)$, and

(i) free entry condition

$$\sum_{\pi} v(\pi)\phi(\pi) \leq w l_S \tag{*}$$

(ii) product destruction rate

$$\mu = \eta \sum_{\pi} \frac{\mu}{\mu - \lambda(\pi)} \phi(\pi) \tag{**}$$

(iii) labor market clearing

$$L_X + L_R + L_S = L$$

- Note: $v(\pi)$ and $\lambda(\pi)$ themselves depend on (w, μ)
  Compute equilibrium by solving fixed point problem in w, η, μ

$$L_X + L_R + L_S = L$$

- Labor employed in goods production

$$L_X = \sum_{\pi} \sum_{n=1}^{\infty} l_X(\pi, n) M_n(\pi), \qquad l_X(\pi, n) = \frac{1-\pi}{w} n$$

- Labor employed in innovation at incumbents

$$L_R = \sum_{\pi} \sum_{n=1}^{\infty} l_R(\pi, n) M_n(\pi), \qquad l_R(\pi, n) = c(\lambda(\pi)) n$$

- Labor employed in attempt to enter ('startups')

$$L_S = \eta l_S$$

$$L_X + L_R + L_S = L$$

- So labor employed in production is

$$L_X = \sum_{\pi} \sum_{n=1}^{\infty} l_X(\pi, n) M_n(\pi) = \sum_{\pi} \frac{1-\pi}{w} K(\pi)$$

- And labor employed in innovation at incumbents is

$$L_R = \sum_{\pi} \sum_{n=1}^{\infty} l_R(\pi, n) M_n(\pi) = \sum_{\pi} c(\lambda(\pi)) K(\pi)$$

- Which means labor market clearing can be written

$$\eta \left(\sum_{\pi} \left(\frac{1-\pi}{w} + c(\lambda(\pi)) \right) \frac{\phi(\pi)}{\mu - \lambda(\pi)} + l_S \right) = L \qquad (***)$$

Growth decomposition

- Aggregate growth is

$$g = \mu \mathbb{E}[\log q]$$

the rate of innovation times the average quality improvement

- And in equilibrium, the aggregate innovation rate is

$$\mu = \eta + \sum_{\pi} \lambda(\pi) K(\pi)$$

- Gives growth decomposition

$$g = \eta \mathbb{E}[\log q] + \sum_{\pi} \lambda(\pi) \mathbb{E}[\log q][K(\pi) - \phi(\pi)] + \sum_{\pi} \lambda(\pi) \mathbb{E}[\log q] \phi(\pi)$$

that is, growth = net entry + selection + residual

selection $= \sum_\pi \lambda(\pi)\mathbb{E}[\log q][K(\pi) - \phi(\pi)]$

- Recall that mass of products accounted for by type π firms is

$$K(\pi) = \frac{\eta}{\mu - \lambda(\pi)}\phi(\pi), \qquad \mu = \eta + \sum_\pi \lambda(\pi)K(\pi)$$

- So

$$K(\pi) > \phi(\pi) \qquad \Leftrightarrow \qquad \lambda(\pi) > \sum_\pi \lambda(\pi)K(\pi)$$

- Collecting terms and rearranging

$$\text{selection} = \frac{1}{\eta}\left(\sum_\pi \lambda(\pi)^2 K(\pi) - \left(\sum_\pi \lambda(\pi)K(\pi)\right)^2\right)\mathbb{E}[\log q] > 0$$

Lentz and Mortensen (2008): overview

- Structural estimation of Klette/Kortum model
  - Danish firm-level panel data on productivity and employment
  - selection effects account for about *half* of aggregate growth

Additional features

- General CES preferences over horizontal varieties $j \in [0,1]$

$$C_t = \left(\int_0^1 \alpha(j) \Big[\sum_k z_t(j,k) x_t(j,k) \Big]^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 0$$

with qualities

$$z_t(j,k) := \prod_{k'=0}^{k} q_t(j,k')$$

- *Random quality steps*: on entry, firm learns type $\pi \sim \phi(\pi)$. Firm of type π draws innovation steps $q \sim F(q \,|\, \pi)$ with higher π's giving a more favorable distribution (FOSD)

- *Random initial demand*: each product variety has an initial demand realization $\xi \sim G(\xi)$, independent of q and π, that persistently affects profitability

Overview of steady-state equilibrium

- Let $q^n := (q_1, \dots, q_n)$ denote vector of quality steps, $\xi^n = (\xi_1, \dots, \xi_n)$ vector of initial demand conditions

- Lentz/Mortensen derive closed form solution for value function $V_n(\pi, q^n, \xi^n)$ of type π firm

- Again, each type π chooses a different innovation rate $\lambda(\pi)$

- Aggregate innovation rate given by

$$\mu = \eta + \sum_{\pi} \lambda(\pi) K(\pi)$$

  where again $K(\pi) = \eta\phi(\pi)/(\mu - \lambda(\pi))$ is mass of products produced by type π firms, η is entry rate (both endogenous)

- Aggregate innovation rate then pins down aggregate growth rate g

Data moments used for structural estimation

Data Moments (Standard Errors in Parentheses)[a]

	1992	1997		1992	1996
Survivors	4872.000	3628.000	$Cor[\frac{Y}{N^*}, \frac{Y_{+1}}{N^*_{+1}}]$	0.476	0.550
	—	(32.132)		(0.088)	(0.091)
$E[Y]$	26,277.262	31,860.850	$Cor[\frac{Y}{N^*}, \Delta\frac{Y}{N^*}]$	−0.227	−0.193
	(747.001)	(1031.252)		(0.103)	(0.057)
$Med[Y]$	13,472.812	16,448.965	$Cor[\frac{Y}{N^*}, \frac{\Delta Y}{Y}]$	−0.120	
	(211.851)	(329.417)		(0.016)	
$Std[Y]$	52,793.105	64,120.233	$Cor[\frac{Y}{N^*}, \frac{\Delta N^*}{N^*}]$	0.119	
	(5663.047)	(7741.448)		(0.032)	
$E[W]$	13,294.479	15,705.087	$E[\frac{\Delta Y}{Y}]$	−0.029	
	(457.466)	(609.595)		(0.008)	
$Med[W]$	7231.812	8671.939	$Std[\frac{\Delta Y}{Y}]$	0.550	
	(92.720)	(154.767)		(0.067)	
$Std[W]$	30,613.801	35,555.701	$Cor[\frac{\Delta Y}{Y}, Y]$	−0.061	
	(6750.399)	(8137.541)		(0.012)	
$E[\frac{Y}{N^*}]$	384.401	432.118	Within	1.015	
	(2.907)	(5.103)		(0.146)	
$Med[\frac{Y}{N^*}]$	348.148	375.739	Between	0.453	
	(1.829)	(2.139)		(0.112)	
$Std[\frac{Y}{N^*}]$	205.074	305.306	Cross	−0.551	
	(19.633)	(42.491)		(0.196)	
$Cor[Y, W]$	0.852	0.857	Exit	0.084	
	(0.035)	(0.045)		(0.066)	
$Cor[\frac{Y}{N^*}, N^*]$	−0.018	−0.026			
	(0.013)	(0.011)			
$Cor[\frac{Y}{N^*}, Y]$	0.198	0.143			
	(0.036)	(0.038)			

[a] Unit of measurement is 1000 DKK. The BHC growth decomposition moments are stated as fractions of total growth.

Model fit

Model Fit (Data in Top Row, Estimated Model in Bottom Row)

	1992	1997		1992	1996
Survivors	4872.000	3628.000	$\mathrm{Cor}[\frac{Y}{N^*}, \frac{Y_{+1}}{N^*_{+1}}]$	0.476	0.550
	4872.000	3604.315		0.716	0.718
$E[Y]$	26,277.262	31,860.850	$\mathrm{Cor}[\frac{Y}{N^*}, \Delta\frac{Y}{N^*}]$	−0.227	−0.193
	23,023.834	27,252.981		−0.342	−0.352
$\mathrm{Med}[Y]$	13,472.812	16,448.965	$\mathrm{Cor}[\frac{Y}{N^*}, \frac{\Delta Y}{Y}]$	−0.120	
	13,352.797	15,382.112		−0.094	
$\mathrm{Std}[Y]$	52,793.105	64,120.233	$\mathrm{Cor}[\frac{Y}{N^*}, \frac{\Delta N^*}{N^*}]$	0.119	
	31,013.660	37,224.359		0.123	
$E[W]$	13,294.479	15,705.087	$E[\frac{\Delta Y}{Y}]$	−0.029	
	11,772.341	13,735.062		0.024	
$\mathrm{Med}[W]$	7231.812	8671.939	$\mathrm{Std}[\frac{\Delta Y}{Y}]$	0.550	
	7122.721	8154.757		0.771	
$\mathrm{Std}[W]$	30,613.801	35,555.701	$\mathrm{Cor}[\frac{\Delta Y}{Y}, Y]$	−0.061	
	14,716.584	17,431.300		−0.042	
$E[\frac{Y}{N^*}]$	384.401	432.118	Within	1.015	
	379.930	417.041		0.969	
$\mathrm{Med}[\frac{Y}{N^*}]$	348.148	375.739	Between	0.453	
	346.456	378.623		0.364	
$\mathrm{Std}[\frac{Y}{N^*}]$	205.074	305.306	Cross	−0.551	
	202.134	223.173		−0.446	
$\mathrm{Cor}[Y, W]$	0.852	0.857	Exit	0.084	
	0.927	0.928		0.113	
$\mathrm{Cor}[\frac{Y}{N^*}, N^*]$	−0.018	−0.026			
	−0.031	−0.024			
$\mathrm{Cor}[\frac{Y}{N^*}, Y]$	0.198	0.143			
	0.170	0.176			

Estimated parameters (for $\sigma = 1$)

- Cost function

$$c(\lambda) = c_0 \lambda^{c_1+1}, \qquad c_1 = 3.7$$

- Three-point type distribution

$$\phi(\pi) = (0.85\,,\, 0.10\,,\, 0.05)$$

(and quality $F(q\,|\,\pi)$ is Weibull with type-specific scale parameter)

- Equilibrium mass of products by type

$$K(\pi) = (0.54\,,\, 0.28\,,\, 0.18)$$

- Innovation intensities

$$\lambda(\pi) = (0.000\,,\, 0.055\,,\, 0.057), \quad \eta = 0.045 \quad \Rightarrow \quad \mu = 0.071$$

Theoretical growth decomposition

- In $\sigma = 1$ case, can derive similar growth decomposition

$$g = \eta \sum_{\pi} \mathbb{E}[\log q \mid \pi]\phi(\pi) + \sum_{\pi} \lambda(\pi)\mathbb{E}[\log q \mid \pi][K(\pi) - \phi(\pi)] + \sum_{\pi} \lambda(\pi)\mathbb{E}[\log q \mid \pi]\phi(\pi)$$

again, growth = net entry + selection + residual

Selection contributes about half of growth

THE PRODUCTIVITY GROWTH RATE AND ITS COMPONENTS (STD ERROR IN PARENTHESES)[a]

	$\sigma = 1.000$	$\sigma = 0.500$	$\sigma = 0.750$	$\sigma = 1.250$
g	0.0139	0.0123	0.0131	0.0144
	(0.0006)	—	—	—
Decomposition (fraction of g)				
Entry/exit	0.2110	0.1876	0.1959	0.2498
	(0.0149)	—	—	—
Residual	0.2608	0.2316	0.2425	0.2788
	(0.0128)	—	—	—
Selection	0.5282	0.5809	0.5615	0.4715
	(0.0270)	—	—	—

Next class

- Innovation and firm dynamics, part four

- Embodied technical change and diffusion of new technologies

  ◇ Atkeson and Kehoe (2007): Modeling the transition to a new economy: Lessons from two technological revolutions, *American Economic Review*.