PhD Topics in Macroeconomics

Lecture 6: innovation and firm dynamics, part two

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This lecture

- Klette/Kortum (2004) model of innovation and firm dynamics
  - integrated treatment of quality-ladder model of endogenous growth with firm dynamics
  - seeks to reconcile micro-data on R&D, patenting, and productivity with firm growth, entry, exit

Klette/Kortum: outline

1- Motivating facts on R&D and firm-size

2- Individual firm innovation decisions, firm life-cycle

3- Industry equilibrium, firm size distribution

4- General equilibrium and aggregate growth

Klette/Kortum: ten stylized facts

1. Productivity and R&D positively correlated across firms, but productivity growth not strongly correlated with firm R&D
2. Patents and R&D positively correlated, both in the cross-section of firms and over-time for a given firm
3. R&D intensity uncorrelated with firm size
4. R&D intensity is highly skewed across firms; many firms do zero R&D
5. Differences in R&D intensity across firms very persistent
6. Firm-level R&D investment follows geometric random walk
7. Size distribution also highly skewed
8. Smaller firms have low survival probability, but those that do survive grow faster than large firms. Among large firms, growth independent of firm size
9. Variance of growth rates higher for smaller firms
10. Younger firms are small, have low survival probability, but those that survive grow faster than older firms. Market share of a cohort declines with age

Klette/Kortum: model overview

- Each firm is small relative to industry
- Focus on stationary equilibrium with entry and exit
- Firm characterized by portfolio of n products
- Firms engage in R&D to produce innovations, drives firm growth
- An innovation allows firm to take over production of a good, old producer priced out of market

Klette/Kortum: model overview

- Continuous time $t \geq 0$
- Firm size n follows discrete stochastic *birth/death process*
  - births: new products added when innovation is successful
  - deaths: products lost when competing firms innovate
- No natural size of a firm (unlike Lucas span-of-control)
- Firms can grow unboundedly large, but takes time and luck
- Firms that hit a string of bad luck exit

Innovation technology

- Innovation production function

$$I = G(R, n)$$

where I is innovation rate, R is R&D effort, n current size

- Innovation technology $G(\cdot)$ is

  – strictly increasing in R and n
  (existing knowledge capital facilitates innovation)

  – strictly concave in R

  – homogenous degree one in R and n
  (neutralizes effect of firm size on innovation)

- Use homogeneity to write as

$$R = nc(\lambda)$$

where $\lambda := I/n$ is *innovation intensity* (cf., quality ladders)

Value of a firm

- Product line gives constant profit flow $\pi \in (0,1)$
- Let V_n denote value of firm with n products, $V_0 = 0$ (exit)
- Bellman equation for firm with $n > 0$ products

$$rV_n = \max_{\lambda} \Big[\pi n - c(\lambda)n + \lambda n(V_{n+1} - V_n) - \mu n(V_n - V_{n-1}) \Big]$$

with interest rate $r > 0$ and product destruction rate $\mu > 0$

- Value is linear in n, $V_n = vn$, for some $v > 0$ to be determined

$$(r + \mu)v = \max_{\lambda} \Big[\pi - c(\lambda) + \lambda v \Big]$$

with $c'(\lambda) = v$ for $\lambda > 0$ [or $c'(0) > v$ and $\lambda = 0$]. Innovation intensity *independent of firm size*, increasing in π, decreasing in r, μ

Firm dynamics and life-cycle

- Let $p_n(t\,;\,n_0)$ denote prob. firm is size n at t given size n_0 at 0

- Law of motion for $n \geq 1$ products

$$\dot{p}_n(t\,;\,n_0) = (n-1)\lambda p_{n-1}(t\,;\,n_0) + (n+1)\mu p_{n+1}(t\,;\,n_0)$$
$$- n(\lambda+\mu)p_n(t\,;\,n_0)$$

- Firms with no products exit, $n = 0$ is an absorbing state

$$\dot{p}_0(t\,;\,n_0) = \mu p_1(t\,;\,n_0)$$

Firm dynamics and life-cycle

- Consider a firm of size $n = 1$. Let $p_n(t) := p_n(t\,;\,1)$

- Solving the system of differential equations gives

$$p_0(t) = \frac{\mu}{\lambda}\gamma(t), \qquad \gamma(t) := \frac{\lambda - \lambda e^{-(\mu-\lambda)t}}{\mu - \lambda e^{-(\mu-\lambda)t}}$$

and

$$p_1(t) = [1 - p_0(t)][1 - \gamma(t)], \quad p_n(t) = p_{n-1}(t)\gamma(t) \quad \text{for } n = 2, 3, \ldots$$

- *Geometric distribution* conditional on survival

$$\frac{p_n(t)}{1 - p_0(t)} = [1 - \gamma(t)]\gamma(t)^{n-1}, \qquad n = 1, 2, \ldots$$

- Firms eventually exit, $\lim_{t\to\infty} p_0(t) = 1$

- Geometric distribution with parameter $\gamma(t)$ increasing in t

  - distribution grows stochastically over time
  - conditional on survival, mean and variance of size increase with t

- Firm with n_0 products at $t = 0$ behaves as if n_0 independent firms each of size 1

  $$p_0(t\,;\,n_0) = p_0(t)^{n_0}$$

  Larger firms have smaller exit hazard

Firm age

- Let A denote random age of exiting firm

$$\text{Prob}[A \le a] = p_0(a)$$

- Expected life of a firm

$$\mathbb{E}[A] = \int_0^\infty [1 - p_0(a)]\, da = \frac{1}{\lambda} \log\left(\frac{\mu}{\mu - \lambda}\right)$$

increasing in λ, decreasing in μ

- Exit hazard

$$\frac{\dot{p}_0(a)}{1 - p_0(a)} = \mu(1 - \gamma(a))$$

declines with age a, approaches $\mu - \lambda$ as $a \to \infty$

Firm age

- Expected size of firm, conditional on survival

$$\sum_{n=1}^{\infty} n \frac{p_n(a)}{1 - p_0(a)} = \frac{1}{1 - \gamma(a)}$$

  increasing with age

- Expected number of products produced by a cohort of size m

$$m \sum_{n=1}^{\infty} n p_n(a) = m \frac{1 - p_0(a)}{1 - (\lambda/\mu) p_0(a)}$$

  declines with age a if $\mu > \lambda$

Firm growth

- Let N_t denote random size of firm at date t, $G_t := (N_t - N_0)/N_0$

- Expected growth conditional on initial size

$$\mathbb{E}\left[G_t \,\middle|\, N_0 = n\right] = e^{-(\mu-\lambda)t} - 1$$

independent of initial size (Gibrat's law)

- Variance conditional on initial size

$$\mathrm{Var}\left[G_t \,\middle|\, N_0 = n\right] = \frac{\lambda + \mu}{n(\mu - \lambda)}\, e^{-(\mu-\lambda)t}\, [1 - e^{-(\mu-\lambda)t}]$$

which declines in initial size

Firm growth

- Expected growth conditional on initial size *and survival*

$$\mathbb{E}\left[G_t \,\middle|\, N_t > 0,\ N_0 = n\right] = \frac{e^{-(\mu-\lambda)t}}{1 - p_0(t)^n} - 1$$

which also declines in initial size

- But for firms that are initially large (or have grown very fast) probability of survival to t is high, so Gibrat's law will be good approximation

Aggregation

- Let $M_n(t)$ denote measure of size n firms at date t and let

$$M(t) := \sum_{n=1}^{\infty} M_n(t)$$

- Unit mass of products, each product produced by exactly one firm

$$1 = \sum_{n=1}^{\infty} nM_n(t)$$

- Total innovation rate by incumbents

$$\sum_{n=1}^{\infty} I(n)M_n(t) = \sum_{n=1}^{\infty} \lambda nM_n(t) = \lambda$$

independent of size distribution of firms

Industry equilibrium

- Unlimited potential entrants. If entrants have innovation rate η, total product destruction rate is

$$\mu = \lambda + \eta$$

- Pay sunk cost $k_e > 0$ to enter, gives Poisson intensity 1 of entering with $n = 1$ products. Free entry condition

$$v = k_e, \qquad \text{whenever } \eta > 0$$

- Recall incumbents' first order condition $c'(\lambda) = v$, so this pins down R&D intensity, λ^* that solves

$$c'(\lambda^*) = v = k_e$$

- Then from the incumbent's Bellman equation

$$(r + \mu)v = (\pi - c(\lambda) + v\lambda) \qquad \Rightarrow \qquad \eta^* = \frac{\pi - c(\lambda^*)}{k_e} - r$$

(or $\eta^* = 0$ if the last is negative, in which case $v < k_e$)

Size distribution

- Law of motion is then, for $n = 1$

$$\dot{M}_1(t) = \eta + 2\mu M_2(t) - (\lambda + \mu) M_1(t)$$

- Similarly for $n = 2, 3, ...$

$$\dot{M}_n(t) = (n-1)\lambda M_{n-1}(t) + (n+1)\mu M_{n+1}(t) - n(\lambda + \mu) M_n(t)$$

- And, by our adding up condition, the total measure $M(t)$ follows

$$\dot{M}(t) = \eta - \mu M_1(t)$$

Size distribution

- For stationary distribution, set time derivatives to zero and solve

- From the adding up condition

$$M_1 = \eta/\mu$$

- Plugging into the law of motion for $n = 1$ and solving for M_2

$$M_2 = ((\lambda + \mu)M_1 - \eta)/2\mu = \lambda\eta/(2\mu^2)$$

- And so on, by induction

$$M_n = \frac{\lambda^{n-1}\eta}{n\mu^n} = \frac{\theta}{n}\left(\frac{1}{1+\theta}\right)^n, \qquad \theta := \eta/\lambda$$

(for $\lambda > 0, \eta > 0$)

Size distribution

- Total mass of firms

$$M = \sum_{n=1}^{\infty} M_n = \sum_{n=1}^{\infty} \frac{\theta}{n}\left(\frac{1}{1+\theta}\right)^n = \theta \log\left(\frac{1+\theta}{\theta}\right)$$

- So finally, size distribution $P_n := M_n/M$ is given by

$$P_n = \frac{(1/(1+\theta))^n}{n\log((1+\theta)/\theta)}$$

the *logarithmic* or *log-series* distribution with parameter $1/(1+\theta)$

- Endogenously skewed size distribution. Mean given by

$$\sum_{n=1}^{\infty} nP_n = \frac{1/\theta}{\log((1+\theta)/\theta)}$$

which is decreasing in θ

  – when θ small, some firms have time to get very large
  – when θ large, entry dominates and there are many $n = 1$ firms

General equilibrium

- Horizontal varieties $j \in [0,1]$

- Inelastic supply of aggregate labor

$$L = L_X + L_S + L_R$$

  L_X producing goods, L_S in research at 'startups' trying to enter, L_R in research at incumbent firms

- Labor requirements for research

  l_S researchers for size 0 firm (entrant) to innovate at rate 1 (i.e., sunk entry cost is $k_e = wl_S$ for w to be determined)

  $l_R(\lambda)$ researchers for size 1 firm (incumbent) to innovate at rate λ (i.e., innovation cost function is $c(\lambda) = wl_R(\lambda)$ for each n)

  assumed strictly increasing, strictly convex in λ

Stochastic quality ladders

- Each innovation (by new or incumbent) is *quality improvement* to randomly drawn variety $j \in [0, 1]$

- Improvements arrive with endogenous Poisson intensity μ

- Let $J_t(j)$ denote *number of improvements* that have hit j at time t, this is Poisson with intensity μt

- Let $z(j, k)$ denote the *quality* of the k'th vintage of variety j

$$1 =: z(j, 0) < z(j, 1) < \cdots < z(j, k) < \cdots < z(j, J_t(j))$$

- Quality step is *random* (not constant)

$$q(j, k) := \frac{z(j, k)}{z(j, k-1)} > 1, \qquad q \sim \text{IID}\, \Psi(q)$$

Preferences and expenditure

- Representative household

$$U = \int_0^\infty e^{-\rho t} \log C_t \, dt$$

- As before, varieties $j \in [0,1]$ are imperfect (Cobb-Douglas) substitutes while vintages $k \in \{0, \ldots, J_t(j)\}$ are perfect substitutes

$$\log C_t = \int_0^1 \log \left[\sum_{k=0}^{J_t(j)} z(j,k) x_t(j,k) \right] dj$$

- In equilibrium only highest quality vintage is sold, limit price

$$p_t(j) = w q_t(j), \qquad q_t(j) := q(j, J_t(j))$$

- Take aggregate expenditure as numeraire, $P_t C_t = E_t = 1$. Then expenditure per j

$$1 = p_t(j) x_t(j) \quad \Rightarrow x_t(j) := x_t(j, J_t(j)) = \frac{1}{w q_t(j)}$$

Profits and income accounting

- Flow profit per variety j

$$\pi_t(j) = (p_t(j) - w)x_t(j) = \frac{q_t(j) - 1}{q_t(j)}$$

- Quality step distribution $\Psi(q)$ implies profit distribution $\Phi(\pi)$ [previous analysis goes through replacing π with mean $\bar{\pi}$]

- Average profits

$$\bar{\pi} = \int_0^1 \left[1 - q_t(j)^{-1}\right] dj = 1 - \int_1^\infty q^{-1}\, d\Psi(q)$$

- Aggregate household income, in steady-state

$$PC = Y = wL + rv$$

Stationary equilibrium

- Constants

$$(r^*, w^*, v^*, \lambda^*, \eta^*)$$

such that (i) no further incentive to enter, (ii) incumbents maximize firm value, (iii) household maximizes utility subject to their budget constraint, and (iv) labor market clears

- Entry condition and incumbent's innovation decision

$$v = wl_S = wl_R'(\lambda) \qquad \text{determines } \lambda^* \text{ in terms of } l_S$$

- Since $PC = 1$ is numeraire, household maximization implies

$$r^* = \rho$$

and then from household budget constraint

$$1 = wL + \rho v = wL + \rho w l_S \qquad \Rightarrow \qquad w^* = \frac{1}{L + \rho l_S}$$

Stationary equilibrium

- Then since $v = wl_S$ by free entry, firm value is

$$v^* = w^* l_S = \frac{l_S}{L + \rho l_S}$$

- Demand for researchers at incumbent firms

$$L_R = l_R(\lambda^*)$$

- Demand for production workers

$$L_X = \frac{1 - \bar{\pi}}{w^*} = (1 - \bar{\pi})(L + \rho l_S)$$

- Demand for startup workers

$$L_S = \eta l_S \qquad \Rightarrow \qquad \eta^* = \frac{L - L_X - L_R}{l_S} = \frac{\bar{\pi} L - (1 - \bar{\pi})\rho l_S - l_R(\lambda^*))}{l_S}$$

Can now calculate all other objects of interest

Aggregate growth

- Aggregate rate of innovation

$$\mu^* = \lambda^* + \eta^*$$

- Aggregate growth rate of consumption (and real wage etc)

$$g^* = \mu^* \log \bar{q}, \qquad \log \bar{q} := \int_1^\infty \log q \, d\Psi(q)$$

- Simple comparative statics

  – increasing in labor force L and in average profits $\bar{\pi}$
  – decreasing in impatience ρ and in entry labor requirement l_S

All these calculations presume an 'interior' steady state with entry, i.e., $L_S > 0$.

Next

- Innovation and firm dynamics, part three

- Lentz/Mortensen estimation of Klette/Kortum-style model

  ◇ Lentz and Mortensen (2005): Productivity growth and worker reallocation, *International Economic Review*.

  ◇ Lentz and Mortensen (2008): An empirical model of growth through product innovation, *Econometrica*.