# PhD Topics in Macroeconomics

## Lecture 2: firm dynamics, part two

Chris Edmond

2nd Semester 2014

# This lecture

**1-** Hopenhayn's (1992) model of entry, exit and long-run equilibrium

- model exposition
- sketch of computational procedure
- simplified static version for intuition

**2-** Application to Taiwanese and Korean firm-dynamics micro data

# Hopenhayn: overview

- Workhorse model of industry dynamics
- Steady-state model: firms enter, grow and decline, and exit, but overall distribution of firms is unchanging
- Endogenous stationary distribution of firm-size etc, straightforward comparative statics
- Competitive firms, no strategic interactions

# Key elements

- Continuum of firms, each measure zero, produce with DRS
- No aggregate risk: deterministic paths for producer price and factor price(s) taken as given
- But idiosyncratic risk: individual firm productivities follow a first-order Markov process
- Fixed cost to enter, fixed cost to operate each period

# Model

- Time $t = 0, 1, 2, ...$

- Output and input prices $p$ and $w$ taken as given

- Output $y$ produced with labor $n$ given productivity $a$

$$y = af(n)$$

- Static profits

$$\pi(a, p, w) := \max_n \left[ paf(n) - wn - k \right]$$

where $k > 0$ is per-period fixed cost of operating

- Let $n(a, p, w)$ denote optimal employment and let $y(a, p, w)$ denote associated output

# Model

- Assumptions
  - $n(\cdot), y(\cdot), \pi(\cdot)$ are all strictly increasing in productivity $a$
  - productivity draws follow a first-order Markov process with distribution function $F(a' \,|\, a)$ and $F(\cdot \,|\, a)$ is strictly decreasing in $a$

    i.e., if $a_1 > a_2$ then $F(\cdot \,|\, a_1)$ FOSD's $F(\cdot \,|\, a_2)$
  - entrants draw initial productivity $a_0$ from separate distribution $G(a)$, pay sunk cost $k_e > 0$ to do so
- Timing within a period
  - incumbents decide to stay or exit, entrants decide to enter or not
  - incumbents that stay pay $k$, entrants pay $k_e$
  - *after* paying $k$ or $k_e$, operating firms learn their productivity draws

# Incumbent's problem

- Let $z = \{p_t, w_t\}_{t=0}^{\infty}$ denote sequence of prices a firm takes as given

- Let $v_t(a, z)$ denote the value of incumbency to a firm with current productivity draw $a$

- Bellman equation for an incumbent firm

$$v_t(a, z) = \pi(a, p_t, w_t) + \beta \max \left[ 0 \,, \int v_{t+1}(a', z) \, dF(a' \,|\, a) \right]$$

- An *exit threshold* $a_t^*(z)$ such that firm exits if $a_t < a_t^*(z)$, solves

$$\int v_{t+1}(a', z) \, dF(a' \,|\, a^*) = 0$$

(for interior cases)

# Entrant's problem

- Potential entrants are ex ante identical
- Pay $k_e > 0$ to enter, initial draw from $G(a)$ if they do
- Start producing next period
- Let $m_t \geq 0$ denote the mass of entrants, *free entry* condition

$$\beta \int v_{t+1}(a, z)\, dG(a) \leq k_e$$

with strict equality whenever $m_t > 0$

# Aggregate state $\mu_t(\mathcal{A})$

- Let $\mu_t(\mathcal{A})$ be the measure of incumbents with productivity $a \in \mathcal{A}$
- $\mu_t(\mathcal{A})$ is the state variable for the aggregate economy
- $\mu_t(\mathcal{A})$ is endogenous and, in general, evolves over time

# Law of motion for the state

- The measure of incumbents with productivity $a \in [0, a')$ at $t+1$ is

$$\mu_{t+1}([0,a')) = \int F(a' \,|\, a)\mathbb{1}[a \geq a_t^*]\mu_t(da) + m_{t+1}G(a'), \quad \text{all } a'$$

(suppressing the dependence on $z$)

- Suppose we discretize to a grid with $N$ elements. Then this is a linear system of the form

$$\boldsymbol{\mu}_{t+1} = \boldsymbol{\Psi}_t \boldsymbol{\mu}_t + m_{t+1}\mathbf{g}$$

where $\boldsymbol{\Psi}$ is a $N \times N$ matrix that depends on the productivity process and exit threshold $a_t^*$, where $\boldsymbol{\mu}$ and $\mathbf{g}$ are $N \times 1$ vectors, and where $m$ is a scalar

# Industry demand and supply

- Industry demand curve $D(p)$, exogenous
- Industry supply curve, endogenous

$$Y = \int y(a, p_t, w_t)\mu_t(da)$$

- Market clears when

$$Y = D(p_t)$$

- Choose either $p_t$ or $w_t$ as numeraire. We will choose $w_t = 1$

# Equilibrium: overview

- Given an initial distribution $\mu_0$, a *perfect foresight equilibrium* consists of sequences

$$\{p_t, m_t, a_t^*, \mu_t\}_{t=0}^{\infty}$$

such that (i) goods market clears, (ii) incumbents make optimal exit decisions, (iii) no further incentives to enter, and (iv) distribution $\mu_t$ defined recursively by law of motion above

- We will focus on a *stationary equilibrium*, constants

$$(p^*, m^*, a^*, \mu^*)$$

that corresponds to a steady-state of the dynamical system implied by the perfect foresight equilibrium

# Computing an equilibrium (sketch)

- **Step 1.** Guess output price $p_0$. For this price, solve the incumbent's dynamic programming problem

$$v(a, p_0) = \pi(a, p_0) + \beta \max \left[ 0\,, \int v(a', p_0) dF(a' \,|\, a) \right]$$

The solution of this problem also implies the optimal exit rule, i.e., the $a^*(p_0)$ that solves

$$\int v(a', p_0) dF(a' \,|\, a^*) = 0$$

- **Step 2.** Check that this price $p_0$ satisfies the free-entry condition

$$\beta \int v(a', p_0) dG(a') = k_e$$

For example, if the LHS is too high, then go back to Step 2 and guess a new price $p_1 < p_0$. Continue until a price $p^*$ is found that solves the free-entry condition

# Computing an equilibrium (sketch)

- **Step 3.** Guess a measure of entrants, $m_0$. Given this, calculate the stationary distribution $\mu_0$. This solves the linear system

$$\mu_0([0, a')) = \int_{a \geq a^*(p^*)} F(a' \mid a)\mu_0(da) + m_0 G(a'), \quad \text{for all } a'$$

  Observe that the RHS depends on the price found at Step 2 via the exit threshold $a^*(p^*)$

- **Step 4.** Given this $\mu_0$, calculate the total industry supply and check the market clearing condition

$$Y = \int y(a, p^*)\mu_0(da) = D(p^*)$$

  For example, if the LHS is too low, then go back to Step 3 and guess new entrants $m_1 > m_0$. Continue until a $m^*$ is found that solves the market-clearing condition

# Speeding up the last step

- Because of the linear law of motion for $\mu$, the stationary distribution is linearly homogeneous in $m$

- In terms of the discretized system above

$$\boldsymbol{\mu} = \boldsymbol{\Psi}\boldsymbol{\mu} + m\mathbf{g} \qquad \Rightarrow \qquad \boldsymbol{\mu} = m(\mathbf{I} - \boldsymbol{\Psi})^{-1}\mathbf{g}$$

  where $\mathbf{I}$ is an identity matrix

- Two implications

  – no need to use simulations to find stationary distribution $\boldsymbol{\mu}$, just set up coefficient matrix $\boldsymbol{\Psi}$ (implied by $a^*(p^*)$) and calculate directly

  – only invert $(\mathbf{I} - \boldsymbol{\Psi})$ once, then just rescale by $m$

# Comparative statics

- Increase in entry cost $k_e$
  - increases expected discounted profits
  - decreases exit threshold $a^*$
    - less selection, incumbents make more profits, more continue
    - increases average age of firms
  - decreases entrants $m^*$
  - decreases entry/exit rate $m^*/\mu^*(\mathbb{R})$
  - increases price $p^*$

# Comparative statics

- Ambiguous implications for firm-size distribution
  - *price effect*, higher $k_e$ increases price $p^*$

    hence incumbents increase output $y(a, p^*)$ and employment $n(a, p^*)$
  - *selection effect*, higher $k_e$ reduces productivity threshold $a^*$

    hence more incumbent firms are relatively-low productivity firms

# Static version for intuition

- Once-and-for-all productivity draw $a \sim G(a)$, once-and-for-all endogenous exit

- Static profit maximization problem

$$\pi(a, p) := \max_n \Big[ pan^\alpha - n - k \Big], \qquad (w = 1 \text{ is numeraire})$$

- Implies employment, output

$$n(a, p) = (\alpha pa)^{\frac{1}{1-\alpha}}, \qquad y(a, p) = an(a, p)^\alpha$$

and profits

$$\pi(a, p) = (1 - \alpha)\alpha^{\frac{\alpha}{1-\alpha}} (pa)^{\frac{1}{1-\alpha}} - k$$

# Exit and entry conditions

- Exit threshold $a^*$ satisfies

$$\pi(a^*, p) = 0$$

such that firms immediately exit for all $a < a^*$

- Value of a firm given once-and-for-all choices

$$v(a, p) = \max\left[0\,,\, \sum_{t=0}^{\infty} \beta^t \pi(a, p)\right] = \max\left[0, \frac{\pi(a, p)}{1 - \beta}\right]$$

- Free entry condition

$$k_e = \beta \int v(a, p^*)\, dG(a) = \beta \int_{a^*}^{\infty} \frac{\pi(a, p^*)}{1 - \beta}\, dG(a)$$

Two conditions in two unknowns $a^*, p^*$

# Implications for selection

- Substituting the profit function into the free entry condition

$$(1-\beta)k_e = \beta \int_{a^*}^{\infty} \left[ (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} (p^* a)^{\frac{1}{1-\alpha}} - k \right] dG(a)$$

- Using the exit condition for $a^*$ to eliminate $p^*$ gives

$$(1-\beta)\frac{k_e}{k} = \beta \int_{a^*}^{\infty} \left[ \left(\frac{a}{a^*}\right)^{\frac{1}{1-\alpha}} - 1 \right] dG(a)$$

- Increase in $k_e$ (or decrease in $k$) reduces cutoff $a^*$ and increases $p^*$

$\Rightarrow$ Larger entry barriers *weaken the selection effect and allow more unproductive firms to operate*

- Let's now turn to an application of these ideas

# Aw, Chung, Roberts (2003)

- Comparison of Taiwanese and Korean manufacturers
- Background: at aggregate level, Taiwan and Korea similar export-oriented economies, but interesting micro-level differences
- Goal:
  - interpret relationships between market concentration, producer turnover and productivity through the lens of a Hopenhayn model
  - compare these relationships between Taiwan and Korea

# Overview

- Korean industries characterized by
  - more market concentration
  - more cross-sectional productivity dispersion
  - less producer turnover
  - more low-productivity producers operating
  - more output attributable to high-productivity producers
  - larger prod. differentials between surviving and failing producers

- Suggestive of Hopenhayn model where entry sunk costs $k_e$ are higher in Korea than Taiwan. Remember
  - higher $k_e$ discourages entry
  - higher $k_e$ protects incumbents

  $\Rightarrow$ reduces productivity threshold $a^*$, weakens selection effect on existing producers, more incumbent firms are low-productivity firms

# Large firms account for more output in Korea

*Distribution of Gross Output by Employment Categories*

| | Taiwan (1986) | South Korea (1988) |
|---|---|---|
| Textiles | | |
| 5–99 workers | 0.235 | 0.193 |
| 100–499 workers | 0.400 | 0.223 |
| 500+ workers | 0.365 | 0.553 |
| Apparel | | |
| 5–99 workers | 0.340 | 0.285 |
| 100–499 workers | 0.401 | 0.383 |
| 500+ workers | 0.259 | 0.332 |
| Chemicals | | |
| 5–99 workers | 0.430 | 0.140 |
| 100–499 workers | 0.233 | 0.361 |
| 500+ workers | 0.337 | 0.499 |
| Plastics | | |
| 5–99 workers | 0.324 | 0.352 |
| 100–499 workers | 0.342 | 0.263 |
| 500+ workers | 0.334 | 0.385 |
| Fabricated metals | | |
| 5–99 workers | 0.640 | 0.330 |
| 100–499 workers | 0.272 | 0.352 |
| 500+ workers | 0.088 | 0.317 |
| Electrical machinery/electronics | | |
| 5–99 workers | 0.184 | 0.105 |
| 100–499 workers | 0.262 | 0.168 |
| 500+ workers | 0.554 | 0.727 |
| Transport equipment | | |
| 5–99 workers | 0.225 | 0.070 |
| 100–499 workers | 0.237 | 0.114 |
| 500+ workers | 0.538 | 0.816 |

# Productivity dispersion is higher in Korea

*Cross-sectional Productivity Dispersion* (average values over three census years)

| Industry | Taiwan (1981–91) | South Korea (1983–93) |
|---|---|---|
| Textiles | | |
| Standard Deviation | 0.266 | 0.380 |
| Range ($10^{th}$–$90^{th}$ Percentile) | 0.635 | 0.893 |
| Mean *TFP* Growth | 0.159 | 0.095 |
| Apparel | | |
| Standard Deviation | 0.273 | 0.376 |
| Range ($10^{th}$–$90^{th}$ Percentile) | 0.643 | 0.910 |
| Mean *TFP* Growth | 0.039 | 0.217 |
| Chemicals | | |
| Standard Deviation | 0.193 | 0.013 |
| Range ($10^{th}$–$90^{th}$ Percentile) | 0.255 | 0.400 |
| Mean *TFP* Growth | 0.576 | 0.936 |
| Plastics | | |
| Standard Deviation | 0.237 | 0.314 |
| Range ($10^{th}$–$90^{th}$ Percentile) | 0.553 | 0.713 |
| Mean *TFP* Growth | 0.119 | 0.120 |
| Fabricated Metals | | |
| Standard Deviation | 0.244 | 0.330 |
| Range ($10^{th}$–$90^{th}$ Percentile) | 0.575 | 0.791 |
| Mean *TFP* Growth | 0.052 | 0.083 |
| Electrical Machinery/Electronics | | |
| Standard Deviation | 0.236 | 0.326 |
| Range ($10^{th}$–$90^{th}$ Percentile) | 0.546 | 0.756 |
| Mean *TFP* Growth | 0.173 | 0.061 |
| Transport Equipment | | |
| Standard Deviation | 0.240 | 0.310 |
| Range ($10^{th}$–$90^{th}$ Percentile) | 0.550 | 0.722 |
| Mean *TFP* Growth | −0.020 | 0.115 |

# Less turnover in Korea

*Industry Output Volatility Rates*

| | Taiwan | | South Korea | |
|---|---|---|---|---|
| Industry | 1981–6 | 1986–91 | 1983–8 | 1988–93 |
| Textiles | 0.847 | 0.829 | 0.335 | 0.656 |
| Apparel | 1.00 | 0.881 | 0.750 | 1.10 |
| Chemicals | 0.540 | 0.655 | 0.265 | 0.158 |
| Plastics | 0.719 | 0.886 | 0.618 | 0.558 |
| Fabricated Metals | 0.788 | 0.918 | 0.786 | 0.858 |
| Electrical Machinery/Electronics | 0.602 | 0.640 | 0.370 | 0.454 |
| Transport Equipment | 0.451 | 0.510 | 0.432 | 0.194 |

Volatility is defined as (entry rate + exit rate) − |entry rate − exit rate| = 2 Min(entry rate, exit rate). Entry and exit rates are measured using the value of output accounted for by the entering and exiting firms. See Dunne and Roberts (1991) or Roberts (1996) for details.

# Exiting firms have low productivity

| | Taiwan | | | | |
|---|---|---|---|---|---|
| | | | | F-statistic: | |
| | $\alpha_1^x - \alpha_1^S$ | $\alpha_2^x - \alpha_2^S$ | $\beta_2^x - \beta_2^S$ | No exit differential (a) | Equal exit differential (b) |
| Textiles | −0.078** (0.013) | 0.028 (0.021) | −0.028* (0.013) | 13.41** | 9.93** |
| Apparel | −0.072** (0.018) | 0.007 (0.031) | −0.010 (0.018) | 5.52** | 4.06* |
| Chemicals | −0.074** (0.012) | −0.041* (0.018) | −0.054** (0.011) | 22.90** | 1.52 |
| Plastics | −0.039** (0.009) | 0.015 (0.014) | −0.011 (0.008) | 6.76** | 5.41** |
| Fabricated metals | −0.034** (0.010) | −0.026 (0.014) | −0.026** (0.007) | 9.55** | 0.22 |
| Electric/ electronics | −0.070** (0.010) | −0.049** (0.016) | −0.020* (0.009) | 20.73** | 7.07** |
| Transport equipment | −0.055** (0.015) | −0.041 (0.024) | −0.023 (0.013) | 6.09** | 1.2 |

*significant at the 0.05 level.
**significant at the 0.01 level.

Mean productivity difference between exiting and surviving firms

# Exiting firms have low productivity

| South Korea | | | | |
|---|---|---|---|---|
| | | | F-statistic: | |
| $\alpha_1^x - \alpha_1^S$ | $\alpha_2^x - \alpha_2^S$ | $\beta_2^x - \beta_2^S$ | No exit differ-ential (a) | Equal exit differ-ential (b) |
| −0.051** (0.017) | −0.115** (0.025) | 0.036 (0.020) | 11.12** | 12.17** |
| −0.119** (0.020) | −0.028 (0.037) | −0.035 (0.021) | 13.55** | 5.13** |
| −0.263** (0.023) | −0.136** (0.039) | −0.046 (0.027) | 47.37** | 18.81** |
| −0.126** (0.017) | −0.028 (0.026) | −0.003 (0.014) | 18.29** | 15.42** |
| −0.129** (0.017) | −0.085** (0.027) | −0.010 (0.017) | 23.63** | 12.78** |
| −0.131** (0.018) | −0.007 (0.030) | −0.028 (0.016) | 19.04** | 11.35** |
| −0.130** (0.026) | 0.015 (0.041) | 0.004 (0.021) | 8.30** | 8.97** |

Lower productivity of exiting firms especially pronounced in Korea

# Korean firms more likely to enter lower tail

*Cumulative Distribution of Current Productivity Conditional on Past Productivity*
$F(\phi_{t+1} = 0|\phi_t)$

| | Taiwan Quartile for $\phi_t$ | | | | Korea Quartile for $\phi_t$ | | | |
|---|---|---|---|---|---|---|---|---|
| | 1st | 2nd | 3rd | 4th | 1st | 2nd | 3rd | 4th |
| Textiles | 0.638 | 0.508 | 0.421 | 0.333 | 0.876 | 0.809 | 0.627 | 0.353 |
| Apparel | 0.660 | 0.539 | 0.463 | 0.311 | 0.859 | 0.700 | 0.541 | 0.471 |
| Chemicals | 0.654 | 0.512 | 0.420 | 0.268 | 0.836 | 0.689 | 0.551 | 0.340 |
| Plastics | 0.607 | 0.498 | 0.383 | 0.357 | 0.784 | 0.685 | 0.565 | 0.491 |
| Fabricated metals | 0.548 | 0.483 | 0.412 | 0.334 | 0.846 | 0.686 | 0.574 | 0.412 |
| Electrical machinery | 0.602 | 0.541 | 0.461 | 0.458 | 0.791 | 0.700 | 0.611 | 0.470 |
| Transportation equipment | 0.632 | 0.516 | 0.408 | 0.394 | 0.789 | 0.685 | 0.560 | 0.497 |

In Korea, more likely to move down but not more likely to exit

# Summary/bigger picture

- Productivity dispersion higher and turnover lower in Korea, consistent with weaker selection effects (greater barriers to entry)
- What accounts for these weaker selection effects? If larger barriers to entry, do these reflect policy settings, or features of market structure (e.g., credit market frictions, supplier networks), or both?
- More generally, what accounts for differences in selection effects across industries and countries? Are these micro-level differences an important determinant of cross-country differences in aggregate productivity and income?

# Next

- Firm dynamics: basic models, part three
- Computing the Hopenhayn model, further details and practicalities