PhD Topics in Macroeconomics

Lecture 17: aggregate gains from trade, part one

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This lecture

Aggregate gains from trade in standard 'quantitative trade models' as summarised by Arkolakis, Costinot and Rodríguez-Clare (2012)

1- Simple example based on Armington (1969) model

2- General ACR result, sufficient conditions

3- How Eaton/Kortum (2002) and other models fit within this general framework

Gravity equations

- Many trade models give rise to a gravity equation of the form

$$X_{ij} = \frac{\rho_{ij}^{-\varepsilon}}{\sum_k \rho_{ik}^{-\varepsilon} s_k} \frac{X_i X_j}{X}$$

varying in details of *trade friction* ρ_{ij} and *trade elasticity* ε

	trade friction ρ_{ij}	trade elasticity ε
Armington/Krugman	$\tau_{ij} w_i$	$\sigma - 1$
Melitz/Chaney	$(\tau_{ij} w_i)(w_j f_{ij})^{\left(\frac{1}{\sigma-1}-\frac{1}{\xi}\right)}$	ξ (Pareto)
Eaton/Kortum	$\tau_{ij} \Phi_j^{1/\xi}$	ξ (Fréchet)

- These models also turn out to have similar welfare implications

Arkolakis, Costinot and Rodríguez-Clare (2012)

- There is a class of 'quantitative trade models' for which the gains from trade can be written

$$\frac{C'}{C} = \left(\frac{\lambda'}{\lambda}\right)^{-\frac{1}{\varepsilon}}$$

  where C is real consumption/income, λ is the share of spending on domestic goods, and $\varepsilon > 0$ is the trade elasticity

- Examples of this class include: Armington (1969), Krugman (1980), Eaton/Kortum (2002), Melitz (2003), Chaney (2008) etc

- Gains summarized by two pieces of information, λ'/λ and ε

- The change from (λ, C) to (λ', C') may be brought about by any 'foreign shock', not just changes in trade costs

Example: US import share ≈ 0.07, so $\lambda \approx 0.93$. Welfare change from move to *autarky* ($\lambda' = 1$) is then

$$\frac{C'}{C} = (1/0.93)^{-1/\varepsilon}$$

Range ε estimates 5 to 10. So welfare changes range from $C'/C = 0.9856$ (loss $\approx -1.4\%$) to $C'/C = 0.9928$ (loss $\approx -0.7\%$)

$$C'/C = (\lambda'/\lambda)^{-1/\varepsilon}$$

- So *conditional* on λ'/λ and ε, any two models in this class have the same welfare calculation
- In that sense, models share aggregate welfare implications even though typically differ in micro details (margins of adjustment, reallocations) and differ in structural interpretation of elasticity ε
- Models may predict different λ'/λ, to the extent they do welfare implications will also differ
- Hence importance of move to autarky, since all models will start with $\lambda = 0.93$ (say) and finish with $\lambda' = 1$

First take: Armington (1969) model

- This version: Anderson (1979)/Anderson and Van Wincoop (2003)

- Goods differentiated by country [*Armington assumption*], each country completely specialized in its good, supply of goods fixed

- Representative consumer in each country has CES preferences

$$C_j = \left(\sum_{i=1}^{n} c_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1$$

- Multiplicative trade costs τ_{ij}

- Perfect competition

Country j consumer problem

- Choose consumption bundle c_{ij} for $i = 1, ..., n$ to max

$$C_j = \left(\sum_{i=1}^{n} c_{ij}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1$$

subject to

$$\sum_{i=1}^{n} p_{ij} c_{ij} = P_j C_j =: X_j$$

- Standard residual demand curves and price index

$$c_{ij} = \left(\frac{p_{ij}}{P_j} \right)^{-\sigma} C_j$$

$$P_j = \left(\sum_{i=1}^{n} p_{ij}^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

- Or, in terms of expenditure

$$X_{ij} := p_{ij}c_{ij} = \left(\frac{p_{ij}}{P_j}\right)^{1-\sigma} P_jC_j$$

$$= \left(\frac{p_{ij}}{P_j}\right)^{1-\sigma} X_j$$

- Multiplicative trade costs passed through to importer and price equal marginal cost imply

$$p_{ij} = \tau_{ij}p_i, \qquad p_i = w_i$$

so

$$X_{ij} = \left(\frac{\tau_{ij}w_i}{P_j}\right)^{1-\sigma} X_j, \qquad P_j = \left(\sum_{i=1}^{n}(\tau_{ij}w_i)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

Equilibrium

- Market clearing condition for the good produced by each country

$$\sum_{j=1}^{n} X_{ij} = X_i$$

- Equilibrium problem is to find market clearing wages w_i subject to

$$X_{ij} = \left(\frac{\tau_{ij} w_i}{P_j}\right)^{1-\sigma} X_j, \qquad P_j = \left(\sum_{i=1}^{n} (\tau_{ij} w_i)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

Gravity representation

- Rewriting the market clearing condition

$$w_i^{1-\sigma} \sum_{k=1}^{n} \left(\frac{\tau_{ik}}{P_k} \right)^{1-\sigma} X_k = X_i$$

- Plugging this into the expenditure of country j

$$X_{ij} = \frac{\left(\tau_{ij}/P_j\right)^{1-\sigma}}{\sum_k \left(\tau_{ik}/P_k\right)^{1-\sigma} s_k} \frac{X_i X_j}{X}$$

where $X := \sum_k X_k$ and $s_k := X_k/X$

Welfare in the Armington model

- Recall $C_j = X_j / P_j$

- National income accounting and balanced trade

$$w_j L_j = L_j = X_j = \sum_{i=1}^{n} X_{ij}$$

where we take $w_j = 1$ to be the numeraire

- Consider shock that leaves L_j unchanged. Then, in log-deviations

$$\widehat{X}_j = \widehat{w}_j + \widehat{L}_j = 0$$

$$\Rightarrow \widehat{C}_j = \widehat{X}_j - \widehat{P}_j = -\widehat{P}_j$$

Gain in real consumption/income is simply fall in price level

Change in price index

- Recall price index is

$$P_j = \left(\sum_{i=1}^{n} (\tau_{ij} w_i)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

- In log-deviations

$$\widehat{P}_j = \sum_{i=1}^{n} \left(\frac{\tau_{ij} w_i}{P_j} \right)^{1-\sigma} \left[\widehat{\tau}_{ij} + \widehat{w}_i \right]$$

- Share of spending on country i goods by country j

$$\lambda_{ij} := \frac{X_{ij}}{X_j} = \left(\frac{\tau_{ij} w_i}{P_j} \right)^{1-\sigma}$$

- Hence

$$\widehat{C}_j = -\widehat{P}_j = -\sum_{i=1}^{n} \lambda_{ij} \left[\widehat{\tau}_{ij} + \widehat{w}_i \right]$$

Change in relative imports

- Relative spending

$$\frac{\lambda_{ij}}{\lambda_{jj}} = \frac{X_{ij}}{X_{jj}} = \left(\frac{\tau_{ij} w_i}{\tau_{jj} w_j}\right)^{1-\sigma}$$

- Since $w_j = 1$ and τ_{jj} unchanged by assumption

$$\widehat{\lambda}_{ij} - \widehat{\lambda}_{jj} = (1-\sigma)\Big[\widehat{\tau}_{ij} + \widehat{w}_i\Big]$$

- Plugging this change back into the price index

$$\widehat{C}_j = -\frac{1}{1-\sigma}\sum_{i=1}^{n} \lambda_{ij}\Big[\widehat{\lambda}_{ij} - \widehat{\lambda}_{jj}\Big]$$

$$= -\frac{1}{\sigma-1}\widehat{\lambda}_{jj}$$

- Indeed satisfies ACR formula with trade elasticity $\varepsilon = \sigma - 1 > 0$

Key elements

- Welfare gains only through aggregate terms-of-trade (fall in P_j)
- Bilateral terms-of-trade changes inferred through changes in relative spending (X_{ij}/X_{jj})
- Can aggregate bilateral terms-of-trade changes to get final result

General ACR result: Sufficient conditions

- **Microeconomic structure**
  - countries $i = 1, ..., n$
  - one factor (labor), immobile and inelastic supply L_i
  - CES preferences over measure N_i goods, indexed $\omega \in \Omega$
  - technology for ω in country i represented by cost function

$$\sum_{j=1}^{n} \left[\tau_{ij} w_i \alpha_{ij}(\omega) y_j + f_{ij}(w_i, w_j, \omega) \mathbb{1}\{y_j > 0\} \right]$$

i.e., good specific constant marginal cost + fixed exporting cost

$$f_{ij}(w_i, w_j, \omega) := \bar{f}_{ij} h_{ij}(w_i, w_j) \phi_{ij}(\omega)$$

  - market structure: either (i) perfect competition or (ii) monopolistic competition with either (a) free entry given entry cost f_i^e, or (b) fixed number of goods

- **Macroeconomic structure**

(R1) *Balanced trade.* For any importer j

$$\sum_{i=1}^{n} X_{ij} = \sum_{k=1}^{n} X_{jk}, \qquad X_{ij} := \int x_{ij}(\omega)\, d\omega$$

(R2) *Aggregate profits a constant share of aggregate revenue*

(R3) *CES 'import demand system'* (weak version). Let

$$\varepsilon_j^{ik} := -\frac{\partial \log(X_{ij}/X_{jj})}{\partial \log \tau_{kj}}$$

denote j's substitution between goods from i and k. Assumption is

$$\varepsilon_j^{ik} = \begin{cases} \varepsilon & \text{if } i = k \\ 0 & \text{otherwise} \end{cases}$$

i.e., relative demand separable across exporters

To obtain stronger ‘ex ante’ results, need:

(R3′) *CES ‘import demand system’* (strong version). Import spending can be written

$$X_{ij} = \frac{\chi_{ij}(\tau_{ij}w_i)^{-\varepsilon}}{\sum_k \chi_{kj}(\tau_{kj}w_k)^{-\varepsilon}} X_j$$

where the χ_{ij} terms are independent of τ_{ij}

Example: Ricardian model

- Perfect competition

- Good-specific *unit labor requirements* $\alpha_i(\omega)$ [inverse productivity]

- Country j buys from i all the goods that satisfy

$$\tau_{ij} w_i \alpha_i(\omega) < \tau_{kj} w_k \alpha_k(\omega) \tag{1}$$

- Gives spending

$$X_{ij} = \frac{\int (\tau_{ij} w_i \alpha_i)^{1-\sigma} g_i(\alpha_i\,;\,\mathbf{w}, \boldsymbol{\tau})\, d\alpha_i}{\sum_k \int (\tau_{kj} w_k \alpha_k)^{1-\sigma} g_k(\alpha_k\,;\,\mathbf{w}, \boldsymbol{\tau})\, d\alpha_k} X_j$$

where $g_i(\alpha_i\,;\,\mathbf{w}, \boldsymbol{\tau})$ denotes density of goods satisfying ineq. (1)

- Import demand system elasticities

$$\varepsilon_j^{ik} := -\frac{\partial \log(X_{ij}/X_{jj})}{\partial \log \tau_{kj}} = \begin{cases} \sigma - 1 + \gamma_{jj}^i - \gamma_{ij}^i & \text{if } i = k \\ \gamma_{jj}^k - \gamma_{ij}^k & \text{if } i \neq k \end{cases}$$

- Elasticity $\sigma - 1 > 0$ governs the *intensive margin* of trade

- Elasticity γ_{ij}^k governs the *extensive margin* of trade

$$\gamma_{ij}^k := -\frac{\partial}{\partial \log \tau_{kj}} \left[\int \alpha_i^{1-\sigma} g_i(\alpha_i\,; \mathbf{w}, \boldsymbol{\tau})\, d\alpha_i \right]$$

- As in the Armington model

$$\widehat{C}_j = -\widehat{P}_j = -\sum_{i=1}^{n} \lambda_{ij} \left[\widehat{\tau}_{ij} + \widehat{w}_i\right]$$

- Changes in relative spending

$$\widehat{\lambda}_{ij} - \widehat{\lambda}_{jj} = (1 - \sigma + \gamma_{ij}^{i} - \gamma_{jj}^{i}) \left[\widehat{\tau}_{ij} + \widehat{w}_i\right] + \sum_{k \neq i,j} \left(\gamma_{ij}^{k} - \gamma_{jj}^{k}\right) \left[\widehat{\tau}_{kj} + \widehat{w}_k\right]$$

- Hence

$$\widehat{C}_j = -\sum_{i=1}^{n} \lambda_{ij} \left[\widehat{\tau}_{ij} + \widehat{w}_i\right]$$

$$= \sum_{i=1}^{n} \lambda_{ij} \left\{ \frac{\widehat{\lambda}_{ij} - \widehat{\lambda}_{jj}}{1 - \sigma + \gamma_{ij}^{i} - \gamma_{jj}^{i}} - \sum_{k \neq i,j} \left(\frac{\gamma_{ij}^{k} - \gamma_{jj}^{k}}{1 - \sigma + \gamma_{ij}^{i} - \gamma_{jj}^{i}} \right) \left[\widehat{\tau}_{kj} + \widehat{w}_k\right] \right\}$$

- But by the CES import demand system assumption **R3** we know $\gamma_{ij}^{k} - \gamma_{jj}^{k} = 0$ for all $i \neq k, j$ and $\sigma - 1 + \gamma_{jj}^{i} - \gamma_{ij}^{i} = \varepsilon > 0$ (constant)

- Hence, as before

$$\widehat{C}_j = -\frac{1}{\varepsilon}\widehat{\lambda}_{jj}$$

- Note

  - **R1** and **R2** trivially satisfied here

  - **R3** does all the work, permitting changes in relative prices to be inferred from changes in relative spending which can then be aggregated into changes in the price index

Example: Eaton/Kortum model

- Eaton/Kortum special case where $1/\alpha_i(\omega)$ have Fréchet density

$$\varphi(\boldsymbol{\alpha}) = \xi \prod_{i=1}^{n} T_i \alpha_i^{\xi-1} e^{-T_i \alpha_i^{\xi}}, \qquad T_i > 0, \quad \xi > \sigma - 1$$

- Can then show that trade elasticity is given by

$$\varepsilon_{ij}^{k} = \begin{cases} \xi & \text{if } i = k \\ 0 & \text{otherwise} \end{cases}$$

- Moreover as we have seen, for the Eaton/Kortum model

$$X_{ij} = \frac{T_i(\tau_{ij} w_i)^{-\xi}}{\sum_k T_k(\tau_{kj} w_k)^{-\xi}} X_j$$

and hence strong version of CES demand system **R3′** also satisfied

Discussion

- Eaton/Kortum features *production* gains from trade that are completely absent from Armington model

- But this does not imply aggregate gains are larger

- Instead, *composition* of gains is different

  – new gains on extensive margin
  – but smaller gains on intensive margin

- Moreover, structural interpretation of trade elasticity is different (now technology ξ, rather than preference $\sigma - 1$)

Next

- Aggregate gains from trade, part two
- Gains from trade in models with variable markups
  - Arkolakis, Costinot, Donaldson and Rodríguez-Clare (2012): The elusive pro-competitive effects of trade, Yale University working paper