PhD Topics in Macroeconomics

Lecture 15: heterogeneous firms and trade, part three

Chris Edmond

2nd Semester 2014

This lecture

Chaney (2008) on intensive and extensive margins of trade

1- Open economy model, many asymmetric countries

2- Intensive vs. extensive margins of trade

3- Implications for gravity equations

Chaney model: key features

- Many asymmetric countries, asymmetric trade costs
- No free entry, number potential firms proportional to country size
- Pareto distribution for firm productivity
- Numeraire good, costlessly traded

Relative to Melitz (2003), extra structure allows model to be solved in closed form, despite much richer patterns of asymmetry

Chaney model: overview

- Countries $i = 1, \ldots, N$ of sizes L_i

- Labor is only factor of production, inelastic supply

- Sectors $s = 0, 1, \ldots, S$. Sector $s = 0$ is *competitive numeraire*. Sectors $s = 1, \ldots, S$ are *monopolistically competitive*

- Sector-specific variable trade costs $\tau_{ij,s} \geq 1$ for each bilateral pair. Sector-specific fixed trade costs $f_{ij,s} \geq 0$ for each bilateral pair

  Trade costs need not be symmetric

- Numeraire is costless to trade, $\tau_{ij,0} = 1$ and $f_{ij,0} = 0$

Preferences

- Cobb-Douglas across sectors

$$\log U = \mu_0 \log C_0 + \sum_{s=1}^{S} \mu_s \log C_s, \qquad \sum_{s=0}^{S} \mu_s = 1$$

- CES within sectors $s = 1, \ldots, S$

$$C_s = \left(\int_{\Omega_s} c_s(\omega)^{\frac{\sigma_s - 1}{\sigma_s}} d\omega \right)^{\frac{\sigma_s}{\sigma_s - 1}}, \qquad \sigma_s > 1$$

- Budget constraint

$$C_0 + \sum_{s=1}^{S} \int_{\Omega_s} p_s(\omega) c_s(\omega) \, d\omega \leq Y$$

Demand

- Sectoral demand

$$C_s = \mu_s \frac{Y}{P_s}$$

- Demand for variety ω in sector $s = 1, \ldots, S$

$$c_s(\omega) = \left(\frac{p_s(\omega)}{P_s}\right)^{-\sigma_s} C_s$$

Price index

- Sectoral price index implicitly defined by

$$P_s C_s = \int_{\Omega_s} p_s(\omega) c_s(\omega)\, d\omega$$

so

$$P_s = \left(\int_{\Omega_s} p_s(\omega)^{1-\sigma_s}\, d\omega \right)^{\frac{1}{1-\sigma_s}}$$

Competitive numeraire sector

- Numeraire good produced by competitive firms
- Country-specific labor productivity A_i in numeraire sector
- Real wage in units of the numeraire

$$w_i = A_i$$

(if country i produces the numeraire, i.e., if μ_0 is large enough)

Trade barriers and technology

- Variable $\tau_{ij,s}$ and fixed $f_{ij,s}$ trade costs (in units of labor)

- For firm in sector s with productivity draw a, labor used to deliver y units of output from country i to country j is

$$l_{ij,s}(y,a) = f_{ij,s} + \frac{\tau_{ij,s}}{a}\,y$$

- Firm productivity is Pareto with sector-specific shape parameter ξ_s

$$G_s(a) := \text{Prob}[a' \leq a \,|\, s] = 1 - a^{-\xi_s}$$

and we will need $\xi_s > \sigma_s - 1$ for various moments to be defined

Pricing

- Isoelastic demand with elasticity $\sigma_s > 1$, same for all countries

- Price set by firm in country i for market in j is

$$p_{ij,s}(a) = \frac{\sigma_s}{\sigma_s - 1} \frac{\tau_{ij,s} w_i}{a}$$

- Hence sectoral price index in *destination country j*

$$P_{j,s} = \left(\sum_{i=1}^{N} n_{i,s} \int_1^{\infty} p_{ij,s}(a)^{1-\sigma_s} \, dG_s(a) \right)^{\frac{1}{1-\sigma_s}}, \qquad n_{i,s} := \int_{\Omega_{i,s}} d\omega$$

with $p_{ij,s}(a) = +\infty$ for producers that do not sell in j

Exogenous number of potential producers

- No free entry into production
- Measure of producers per sector proportional to country income

$$n_{i,s} = w_i L_i \qquad \text{for all } s$$

- Since no free entry, will be positive profits in equilibrium

$$\Pi_i = \sum_{s=1}^{S} \sum_{j=1}^{N} \int_1^{\infty} \pi_{ij,s}(a)\, dG_s(a)$$

where

$$\pi_{ij,s}(a) = \max\left[0\,,\ \left(p_{ij,s}(a) - \frac{\tau_{ij,s} w_i}{a}\right) y_{ij,s}(a) - f_{ij,s} w_i\right]$$

- Ownership structure matters

Ownership structure

- Income Y_i in country i (in units of the numeraire), consists of labor income $w_i L_i$ plus profit income

- Global profit income pooled and paid out as *dividends* π per share

- Assume that each representative worker has w_i shares. Then total income in country i is

$$Y_i = w_i L_i + \pi w_i L_i$$

where dividends per share are

$$\pi := \frac{\sum_{j=1}^{N} \Pi_j w_j L_j}{\sum_{j=1}^{N} w_j L_j}$$

Profits

- Conditional on operating, profits are

$$\pi_{ij,s}(a) = \left(p_{ij,s}(a) - \frac{\tau_{ij,s}w_i}{a}\right)y_{ij,s}(a) - f_{ij,s}w_i$$

$$= \left(p_{ij,s}(a) - \frac{\tau_{ij,s}w_i}{a}\right)\left(\frac{p_{ij,s}(a)}{P_{j,s}}\right)^{-\sigma_s}\left(\frac{\mu_s Y_{j,s}}{P_{j,s}}\right) - f_{ij,s}w_i$$

$$= B_{ij,s}a^{\sigma_s-1} - f_{ij,s}w_i$$

where

$$B_{ij,s} := \left(\frac{\sigma_s - 1}{\sigma_s}\frac{P_{j,s}}{\tau_{ij,s}w_i}\right)^{\sigma_s-1}\left(\frac{\mu_s Y_{j,s}}{\sigma_s}\right)$$

- From here on, drop s subscript to simplify notation

Cutoff productivity

- For cutoff firm

$$\pi_{ij}(a) = 0 \qquad \Leftrightarrow \qquad B_{ij} a^{\sigma-1} = f_{ij} w_i$$

- Solves for

$$a_{ij}^* = \left(\frac{f_{ij} w_i}{B_{ij}}\right)^{\frac{1}{\sigma-1}} = \overline{a}\,\frac{\tau_{ij} w_i}{P_j}\left(\frac{f_{ij} w_i}{Y_j}\right)^{\frac{1}{\sigma-1}} \tag{1}$$

where $\overline{a}$ is the first of many tedious constants

$$\overline{a} := \frac{\sigma}{\sigma-1}\left(\frac{\sigma}{\mu}\right)^{\frac{1}{\sigma-1}}$$

- For each country i only producers with $a > a_{ij}^*$ export to j. Exporting to j depends on price level in j (and trade costs)

Solving for P_j

- Price level in j depends on which firms enter that market

$$P_j = \Big(\sum_{k=1}^{N} n_k \int_{a_{kj}^*}^{\infty} p_{kj}(a)^{1-\sigma} \, dG(a) \Big)^{\frac{1}{1-\sigma}}$$

so that

$$P_j^{1-\sigma} = \sum_{k=1}^{N} n_k \int_{a_{kj}^*}^{\infty} \Big(\frac{\sigma}{\sigma - 1} \frac{\tau_{kj} w_k}{a} \Big)^{1-\sigma} \, dG(a)$$

$$= \sum_{k=1}^{N} n_k \Big(\frac{\sigma}{\sigma - 1} \tau_{kj} w_k \Big)^{1-\sigma} \Big(\int_{a_{kj}^*}^{\infty} a^{\sigma-1} \, dG(a) \Big)$$

Aside on the Pareto

- Need to calculate the integral

$$H(x) := \int_x^\infty a^{\sigma-1}\, dG(a) \tag{2}$$

- With Pareto this is

$$H(x) = \int_x^\infty a^{\sigma-1}\, \xi a^{-(\xi+1)}\, da$$

$$= -\frac{\xi}{\xi-(\sigma-1)} a^{-(\xi-(\sigma-1))}\Big|_x^\infty$$

$$= +\frac{\xi}{\xi-(\sigma-1)} x^{-(\xi-(\sigma-1))}, \qquad \text{assuming } \xi > \sigma - 1$$

Solving for P_j (cont.)

- So price index satisfies

$$P_j^{1-\sigma} = \sum_{k=1}^{N} n_k \left(\frac{\sigma}{\sigma - 1} \tau_{kj} w_k \right)^{1-\sigma} \frac{\xi}{\xi - (\sigma - 1)} (a_{kj}^*)^{-(\xi - (\sigma - 1))}$$

where

$$a_{kj}^* = \bar{a} \, \frac{\tau_{kj} w_k}{P_j} \left(\frac{f_{kj} w_k}{Y_j} \right)^{\frac{1}{\sigma - 1}}$$

- Plug in a_{kj}^* and solve for P_j ...

Solution for P_j

- After some horrible algebra, we get

$$P_j = \overline{P}\,\theta_j\,Y_j^{-(\frac{1}{\sigma-1}-\frac{1}{\xi})} \tag{3}$$

where $\overline{P}$ is another tedious constant

$$\overline{P} := \Big[\frac{\sigma}{\mu}\Big(\frac{\xi}{\xi-(\sigma-1)}\Big)\Big(\sum_{k=1}^{N} w_k L_k\Big)\Big]^{-1/\xi}\,\overline{a}$$

- Index of *multilateral resistance* to trade flows

$$\theta_j := \Big[\sum_{k=1}^{N} \alpha_k (\tau_{kj} w_k)^{-\xi} (f_{kj} w_k)^{-(\frac{\xi}{\sigma-1}-1)}\Big]^{-1/\xi}$$

a measure of the *'tyranny of distance'* — due to fixed and variable trade costs — weighted by shares of world income

$$\alpha_k := \frac{Y_k}{Y} = \frac{(1+\pi) w_k L_k}{(1+\pi)\sum_{i=1}^{N} w_i L_i} = \frac{w_k L_k}{\sum_{i=1}^{N} w_i L_i}$$

Cutoff productivity revisited

- Plugging (3) back into formula (1) for a^*_{ij} gives

$$a^*_{ij} = \left(\frac{\overline{a}}{\overline{\overline{P}}}\right)\left(\frac{\tau_{ij} w_i}{\theta_j}\right)\left(f_{ij} w_i\right)^{\frac{1}{\sigma-1}} Y_j^{-\frac{1}{\xi}} \tag{4}$$

- Can now use this to derive implications for trade flows

Micro trade flows

- Exports from i to j by a firm of type $a \geq a^*_{ij}$ are

$$x_{ij}(a) = p_{ij}(a)y_{ij}(a)$$

$$= \left(\frac{p_{ij}(a)}{P_j}\right)^{1-\sigma} \mu Y_j$$

$$= \overline{x} \left(\frac{\tau_{ij} w_i}{\theta_j}\right)^{1-\sigma} Y_j^{\frac{\sigma-1}{\xi}} a^{\sigma-1}$$

with yet another tedious constant

$$\overline{x} := \mu \left(\frac{\sigma}{\sigma - 1} \frac{1}{\overline{\overline{P}}}\right)^{1-\sigma}$$

- Micro trade flows

$$x_{ij}(a) = \overline{x} \left(\frac{\tau_{ij} w_i}{\theta_j} \right)^{1-\sigma} Y_j^{\frac{\sigma-1}{\xi}} a^{\sigma-1}$$

- Elasticity with respect to variable trade costs

$$-\frac{\partial \log x_{ij}}{\partial \log \tau_{ij}} = \sigma - 1$$

same as Krugman (1980). A partial elasticity, holds θ_j constant

- Elasticity with respect to destination country income

$$\frac{\partial \log x_{ij}}{\partial \log Y_j} = \frac{\sigma - 1}{\xi} < 1$$

- Micro-level trade flows are similar to what standard monopolistic competition model would predict

Macro trade flows

- Aggregate exports from i to j are

$$X_{ij} := n_i \int_1^\infty x_{ij}(a)\, dG(a)$$

$$= n_i \int_{a_{ij}^*}^\infty \overline{x} \left(\frac{\tau_{ij} w_i}{\theta_j}\right)^{1-\sigma} Y_j^{\frac{\sigma-1}{\xi}}\, a^{\sigma-1}\, dG(a)$$

$$= \overline{x}\, n_i \left(\frac{\tau_{ij} w_i}{\theta_j}\right)^{1-\sigma} Y_j^{\frac{\sigma-1}{\xi}} \left(\int_{a_{ij}^*}^\infty a^{\sigma-1}\, dG(a)\right)$$

- Evaluate using $H(x)$ from (2) above and expression for a_{ij}^* from (4)

Macro trade flows

- Gives

$$X_{ij} = \overline{X}\, n_i \left(\frac{\tau_{ij} w_i}{\theta_j}\right)^{-\xi} (f_{ij} w_i)^{-(\frac{\xi}{\sigma-1}-1)}\, Y_j$$

with yet another tedious constant

$$\overline{X} := \bar{x}\, \frac{\xi}{\xi-(\sigma-1)} \left(\frac{\overline{a}}{\overline{\overline{P}}}\right)^{-(\xi-(\sigma-1))}$$

- Recall $n_i = w_i L_i = Y_i/(1+\pi)$ to turn this into a *gravity equation*

- To get a simple expression, recognise that

$$\overline{X}\frac{Y}{1+\pi} = \mu$$

Gravity equation

- Can then write

$$X_{ij} = \mu \left(\frac{\tau_{ij} w_i}{\theta_j} \right)^{-\xi} (f_{ij} w_i)^{-\left(\frac{\xi}{\sigma - 1} - 1\right)} \frac{Y_i Y_j}{Y}$$

- Elasticity with respect to variable trade costs

$$\boxed{-\frac{\partial \log X_{ij}}{\partial \log \tau_{ij}} = \xi, \qquad \text{independent of } \sigma \text{ !}}$$

- Elasticity with respect to fixed trade costs

$$\boxed{-\frac{\partial \log X_{ij}}{\partial \log f_{ij}} = \frac{\xi}{\sigma - 1} - 1, \qquad \text{decreasing in } \sigma \text{ !}}$$

Both larger in sectors where ξ is large (dispersion in a is small)

- Macro-level trade flows completely different to micro-level flows.

Decomposition

- Recall

$$X_{ij} = n_i \int_{a_{ij}^*}^{\infty} x_{ij}(a)\, dG(a)$$

- Total differential

$$\frac{dX_{ij}}{n_i} = \left(\int_{a_{ij}^*}^{\infty} \frac{\partial x_{ij}(a)}{\partial \tau_{ij}}\, dG(a) \right) d\tau_{ij} - \left(x_{ij}(a_{ij}^*) G'(a_{ij}^*) \frac{\partial a_{ij}^*}{\partial \tau_{ij}} \right) d\tau_{ij}$$

$$+ \left(\int_{a_{ij}^*}^{\infty} \frac{\partial x_{ij}(a)}{\partial f_{ij}}\, dG(a) \right) df_{ij} - \left(x_{ij}(a_{ij}^*) G'(a_{ij}^*) \frac{\partial a_{ij}^*}{\partial f_{ij}} \right) df_{ij}$$

- Sum of intensive margin and extensive margin effects

Variable trade cost effects

- Elasticity with respect to variable trade costs

$$-\frac{\partial \log X_{ij}}{\partial \log \tau_{ij}} = \underbrace{(\sigma - 1)}_{\substack{\text{standard intensive margin} \\ \text{from Krugman}}} + \underbrace{(\xi - (\sigma - 1))}_{\text{new extensive margin}} = \xi$$

- Higher σ amplifies intensive margin effect of τ_{ij} but dampens extensive margin effect of τ_{ij}

- Exactly cancel so that net effect is that elasticity with respect to variable trade costs is independent of σ

- That said, since $\xi > \sigma - 1$, actual elasticity must be greater than in Krugman model (trade flows more responsive)

- Different structural interpretation of estimated trade elasticities

Fixed trade cost effects

- Elasticity with respect to fixed trade costs

$$-\frac{\partial \log X_{ij}}{\partial \log f_{ij}} = \underbrace{0}_{\text{no intensive margin}} + \underbrace{\frac{\xi}{\sigma - 1} - 1}_{\text{extensive margin}} = \frac{\xi}{\sigma - 1} - 1$$

- Higher σ also dampens extensive margin effect of f_{ij}

- But now this is the only effect, so elasticity with respect to fixed trade costs is decreasing in σ

Intuition

- Consider sector with very differentiated goods (low σ)

- Intensive margin effect

  - demand insensitive to trade costs
  - intensive margin elasticity small when σ low
  - this is only effect in Krugman model

- Extensive margin effect

  - market shares insensitive to trade costs
  - less productive firms still have relatively high market share, despite relatively high price

  - as trade costs (τ or f) fall, some relatively unproductive firms enter
  - σ low, so entrants relatively large compared to existing exporters

  - extensive margin elasticity large when σ low!

Structure of gravity equations

- Define *composite trade friction*

$$\rho_{ij} := (\tau_{ij} w_i) \, (f_{ij} w_i)^{\left(\frac{1}{\sigma-1} - \frac{1}{\xi}\right)}$$

- The gravity equation can be written

$$X_{ij} = \mu \, \frac{\rho_{ij}^{-\varepsilon}}{\sum_{k=1}^{N} \alpha_k \rho_{kj}^{-\varepsilon}} \, \frac{Y_i Y_j}{Y}, \qquad \textit{trade elasticity } \varepsilon = \xi$$

- Krugman model likewise has gravity equation

$$X_{ij} = \mu \, \frac{\tilde{\rho}_{ij}^{-\tilde{\varepsilon}}}{\sum_{k=1}^{N} \alpha_k \tilde{\rho}_{kj}^{-\tilde{\varepsilon}}} \, \frac{Y_i Y_j}{Y}, \qquad \textit{trade elasticity } \tilde{\varepsilon} = \sigma - 1$$

  with trade friction $\tilde{\rho}_{ij} := \tau_{ij} w_i$

- Eaton-Kortum (2002) has similar gravity representation

Next

- Heterogeneous firms and international trade, part four
- Technology and trade frictions in Ricardian models with heterogeneous firms
  - Eaton and Kortum (2002): Technology, geography and trade, *Econometrica*.