PhD Topics in Macroeconomics

Lecture 14: heterogeneous firms and trade, part two

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This lecture

Melitz (2003) model of monopolistic competition and heterogeneous firms

1- Empirical motivation

2- Closed economy model

3- Symmetric open economy with trade costs

Empirical motivation

- Extensive cross-sectional dispersion in productivity
  (even within narrowly defined industries)

- Exporting is relatively rare
  (even within so-called export industries)

- Exporters are systematically larger, more productive, more skill-intensive, more capital-intensive, pay higher wages
  (even within narrowly defined industries, even controlling for size)

Productivity dispersion (BEJK, AER 2003)

Table 2: Plant-Level Productivity Facts

Productivity measure (value added per worker)	Variability (standard deviation of log productivity)	Advantage of exporters (exporter less nonexporter avg. log productivity, %)
Unconditional	0.75	33
Within 4-digit industries	0.66	15
Within capital intensity bins	0.67	20
Within production labor share bins	0.73	25
Within industries (capital bins)	0.60	9
Within industries (prod. labor bins)	0.64	11

The statistics are calculated from all plants in the 1992 Census of Manufactures. The "within" measures subtract the mean value of log productivity for each category. There are 450 4-digit industries, 500 capital-intensity bins (based on total assets per worker), 500 production labor share bins (based on payments to production workers as a share of total labor cost). When appearing within industries there are 10 capital-intensity bins or 10 production labor share bins.

Exporting is rare (BJRS, JEP 2007)

Exporting By U.S. Manufacturing Firms, 2002

NAICS industry	*Percent of firms*	*Percent of firms that export*	*Mean exports as a percent of total shipments*
311 Food Manufacturing	6.8	12	15
312 Beverage and Tobacco Product	0.7	23	7
313 Textile Mills	1.0	25	13
314 Textile Product Mills	1.9	12	12
315 Apparel Manufacturing	3.2	8	14
316 Leather and Allied Product	0.4	24	13
321 Wood Product Manufacturing	5.5	8	19
322 Paper Manufacturing	1.4	24	9
323 Printing and Related Support	11.9	5	14
324 Petroleum and Coal Products	0.4	18	12
325 Chemical Manufacturing	3.1	36	14
326 Plastics and Rubber Products	4.4	28	10
327 Nonmetallic Mineral Product	4.0	9	12
331 Primary Metal Manufacturing	1.5	30	10
332 Fabricated Metal Product	19.9	14	12
333 Machinery Manufacturing	9.0	33	16
334 Computer and Electronic Product	4.5	38	21
335 Electrical Equipment, Appliance	1.7	38	13
336 Transportation Equipment	3.4	28	13
337 Furniture and Related Product	6.4	7	10
339 Miscellaneous Manufacturing	9.1	2	15
Aggregate manufacturing	**100**	**18**	**14**

Exporters are different (BJRS, JEP 2007)

Exporter Premia in U.S. Manufacturing, 2002

	Exporter premia		
	(1)	*(2)*	*(3)*
Log employment	1.19	0.97	
Log shipments	1.48	1.08	0.08
Log value-added per worker	0.26	0.11	0.10
Log TFP	0.02	0.03	0.05
Log wage	0.17	0.06	0.06
Log capital per worker	0.32	0.12	0.04
Log skill per worker	0.19	0.11	0.19
Additional covariates	None	Industry fixed effects	Industry fixed effects, log employment

Sources: Data are for 2002 and are from the U.S. Census of Manufactures.
Notes: All results are from bivariate ordinary least squares regressions of the firm characteristic in the first column on a dummy variable indicating firm's export status. Regressions in column 2 include industry fixed effects. Regressions in column 3 include industry fixed effects and log firm employment as controls. Total factor productivity (TFP) is computed as in Caves, Christensen, and Diewert (1982). "Capital per worker" refers to capital stock per worker. "Skill per worker" is nonproduction workers per total employment. All results are significant at the 1 percent level.

Melitz model: key features

- Heterogeneous firms
  - exogenous productivity differences a
- Monopolistic competition with CES demand
- Increasing returns to scale in production
  - fixed cost of operating f
  - constant marginal cost $1/a$
- Fixed and variable trade costs
  - not all firms export, only high productivity firms can cover fixed costs of operating f *and* fixed costs of exporting f_x

Closed economy: consumers

- Representative consumer, inelastic labor supply L

- CES consumption aggregate over symmetric varieties

$$C = \left(\int_{\Omega} c(\omega)^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1$$

- Budget constraint

$$PC = \int_{\Omega} p(\omega) c(\omega) \, d\omega \leq wL$$

Demand and price index

- Demand for each variety

$$c(\omega) = \left(\frac{p(\omega)}{P}\right)^{-\sigma} C, \qquad \omega \in \Omega$$

with demand elasticity $\sigma > 1$. Equivalently, spending is

$$x(\omega) := p(\omega)c(\omega) = \left(\frac{p(\omega)}{P}\right)^{1-\sigma} X, \qquad X := PC$$

- Usual CES price index

$$P = \left(\int_{\Omega} p(\omega)^{1-\sigma}\, d\omega\right)^{\frac{1}{1-\sigma}}$$

Heterogeneous firms

- Exogenous productivity differences a

- Labor $l(y, a)$ required to produce y units of output

$$l(y, a) = f + \frac{y}{a}$$

- With constant elasticity demand, set price a constant markup over marginal cost

$$p(a) = \frac{\sigma}{\sigma - 1} \frac{w}{a}$$

with wage w from here on normalized to $w = 1$

- Firm profit is then

$$\pi(a) = x(a) - l(y(a), a) = \frac{x(a)}{\sigma} - f$$

- Plugging price $p(a)$ into spending $x(a)$ we have

$$x(a) = \left(\frac{\frac{\sigma}{\sigma-1}\frac{1}{a}}{P}\right)^{1-\sigma} X = \left(\frac{\sigma-1}{\sigma} aP\right)^{\sigma-1} X$$

and hence profits are

$$\pi(a) = \frac{x(a)}{\sigma} - f = \left(\frac{\sigma-1}{\sigma} aP\right)^{\sigma-1} \frac{X}{\sigma} - f$$

- Levels of $x(a), y(a), \pi(a)$ depend on aggregate X, P

- But relative spending, output depend only on a and elasticity σ

$$\frac{y(a_1)}{y(a_2)} = \left(\frac{p(a_1)}{p(a_2)}\right)^{-\sigma} = \left(\frac{a_1}{a_2}\right)^{\sigma}, \quad \text{and} \quad \frac{x(a_1)}{x(a_2)} = \left(\frac{a_1}{a_2}\right)^{\sigma-1}$$

Aggregation

- Equilibrium characterized by n producers and distribution $\mu(a)$ of productivity levels $a \in [0, \infty)$ of operating producers

- Price index

$$P = \left(\int_\Omega p(\omega)^{1-\sigma} \, d\omega \right)^{\frac{1}{1-\sigma}}$$

- All varieties ω with same a set same price $p(a)$, hence

$$P = \left(n \int_0^\infty p(a)^{1-\sigma} \, \mu(a) \, da \right)^{\frac{1}{1-\sigma}}, \qquad n := \int_\Omega d\omega$$

- Aggregate productivity index

$$A = \left(\int_0^\infty a^{\sigma-1} \, \mu(a) \, da \right)^{\frac{1}{\sigma-1}}$$

- With this productivity index A, have that

$$P = p(A)n^{\frac{1}{1-\sigma}}, \qquad Y = y(A)n^{\frac{\sigma-1}{\sigma}}$$

and likewise

$$X = x(A)n, \qquad \Pi = \pi(A)n$$

- Given index A, model equivalent to Krugman (1980) with homogeneous firms all having $a = A$

- But here A depends on $\mu(a)$, which is endogenous

Entry and exit

- After entry, draw $a \sim g(a)$ on $[0, \infty)$

- May immediately exit on learning a draw. If operate, IID probability δ of exogenous exit each period

- Hence

$$v(a) = \max\left[0\,,\, \sum_{t=0}^{\infty} \beta^t \pi(a)\right], \qquad \beta := 1 - \delta$$

or

$$v(a) = \max\left[0\,,\, \frac{\pi(a)}{\delta}\right]$$

- One-time fixed cost $f_e/\delta > 0$ to enter (in units of labor). Equivalent per-period cost f_e

Cutoff productivity

- Let a^* denote lowest productivity such that firm will operate

$$v(a^*) = 0 \qquad \Leftrightarrow \qquad \pi(a^*) = 0$$

- Firms with draws $a < a^*$ immediately exit, all others operate until hit by exogenous δ shock

- Hence productivity distribution is

$$\mu(a) = \frac{g(a)}{1 - G(a^*)}, \qquad a \geq a^*$$

  and zero otherwise

- Here $1 - G(a^*)$ is ex ante probability of 'successful' entry

Aggregate productivity

- Write aggregate productivity as function of cutoff a^*, namely

$$A(a^*) = \Big(\int_{a^*}^{\infty} a^{\sigma-1} \, \frac{g(a)}{1 - G(a^*)} \, da \Big)^{\frac{1}{\sigma-1}}$$

  for cutoff a^* still to be determined

Free entry condition

- Expected value of entry

$$\int_0^\infty v(a)g(a)\,da$$

- Free entry then implies

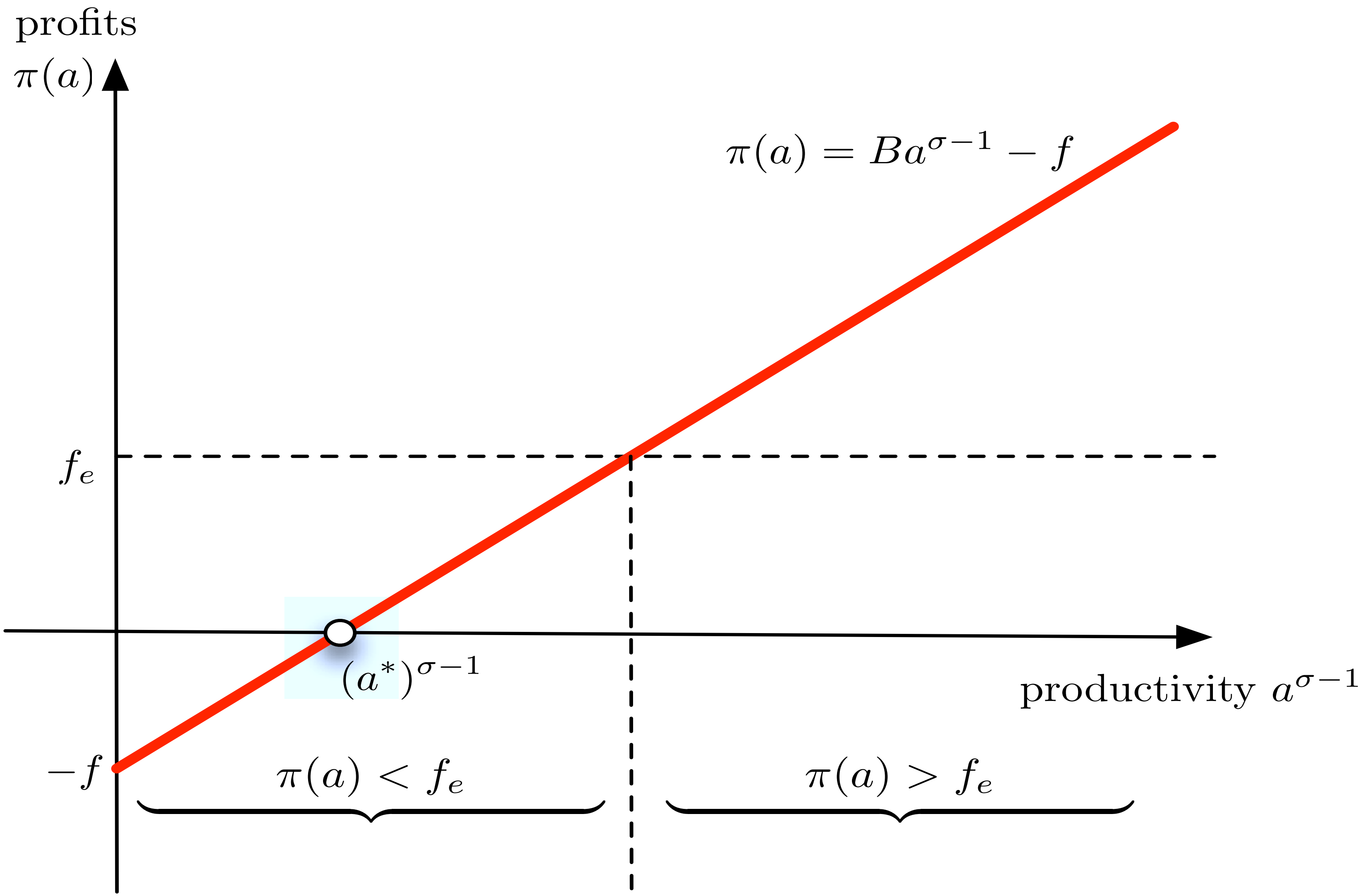
$$\int_0^\infty v(a)g(a)\,da \leq \frac{f_e}{\delta}$$

or

$$\int_0^\infty \max\left[0\,,\ \frac{\pi(a)}{\delta}\right]g(a)\,da \leq \frac{f_e}{\delta}$$

or

$$\int_{a^*}^\infty \pi(a)g(a)\,da \leq f_e$$


profits
$\pi(a)$
$\pi(a) = Ba^{\sigma-1} - f$
f_e
$(a^*)^{\sigma-1}$
productivity $a^{\sigma-1}$
$-f$
$\pi(a) < f_e$
$\pi(a) > f_e$

Solving for a^*

- Write profits

$$\pi(a) = Ba^{\sigma-1} - f, \qquad B := \left(\frac{\sigma-1}{\sigma}P\right)^{\sigma-1}\frac{X}{\sigma}$$

- So free entry condition can be written

$$\int_{a^*}^{\infty}\left[Ba^{\sigma-1} - f\right]g(a)\,da \leq f_e$$

But B is endogenous, a measure of market demand

- Use cutoff rule $\pi(a^*) = 0$ to eliminate dependence on B, gives

$$J(a^*) := \int_{a^*}^{\infty}\left[\left(\frac{a}{a^*}\right)^{\sigma-1} - 1\right]g(a)\,da \leq \frac{f_e}{f} \qquad (*)$$

with $J(a^*)$ strictly decreasing in a^*, unique solution

Pareto example

- Suppose a is Pareto on $[a_{\min}, \infty)$ with shape parameter ξ

$$G(a) = 1 - \left(\frac{a}{a_{\min}}\right)^{-\xi}$$

- Then $J(a)$ evaluates to

$$J(a) = \frac{\sigma - 1}{\xi - (\sigma - 1)} \left(\frac{a}{a_{\min}}\right)^{-\xi}$$

(for which we need the parameter restriction $\xi > \sigma - 1$)

- Since $J(a^*) = f_e/f$, can solve for

$$a^* = \left(\frac{\sigma - 1}{\xi - (\sigma - 1)} \frac{f}{f_e}\right)^{1/\xi} a_{\min}$$

- Truncating below just gives another Pareto

Recover other aggregate variables

- In any case, with a^* solved for from $(*)$, can recover

$$B(a^*) = (a^*)^{1-\sigma} f$$

and

$$A(a^*) = \left(\int_{a^*}^{\infty} a^{\sigma-1} \, \frac{g(a)}{1 - G(a^*)} \, da \right)^{\frac{1}{\sigma-1}}$$

- Number of producers then determined by market clearing

$$n(a^*) = \frac{X}{x(A)} = \frac{L}{\sigma B(a^*) A(a^*)^{\sigma-1}}$$

- Price index

$$P(a^*) = n(a^*)^{\frac{1}{1-\sigma}} p(A(a^*)) = \frac{\sigma}{\sigma - 1} n(a^*)^{\frac{1}{1-\sigma}} \frac{1}{A(a^*)}$$

- Consumption per worker (welfare) equals real wage

$$\frac{C}{L} = \frac{1}{P(a^*)} = \frac{\sigma - 1}{\sigma} n(a^*)^{\frac{1}{\sigma-1}} A(a^*)$$

Open economy

- Symmetric countries $i = 1, \ldots, N+1$, each of size L
- Variable trade costs $\tau > 1$ of the usual iceberg type
- But now also *fixed trade cost*, $f_x > 0$ to export (in units of labor)

  For convenience, export cost expressed as per-period cost.
  Equivalent one-time cost of f_x/δ

Pricing and spending

- Symmetry implies wage $w_i = w$ for all i, again normalize $w = 1$

- Domestic and export prices

$$p_d(a) = \frac{\sigma}{\sigma - 1}\frac{1}{a}, \qquad p_x(a) = \tau p_d(a)$$

- Revenues from domestic market and each export market

$$x_d(a) = \left(\frac{\sigma - 1}{\sigma} aP\right)^{\sigma - 1} X, \qquad x_x(a) = \tau^{1-\sigma} x_d(a)$$

- Again by symmetry P, X will be same in every country

Revenues and profits

- Any exporting firm will also produce for domestic market, but not all firms will export

- Profits from domestic market and each export market

$$\pi_d(a) = \frac{x_d(a)}{\sigma} - f, \qquad \pi_x(a) = \frac{x_x(a)}{\sigma} - f_x$$

- Total profits then

$$\pi(a) = \pi_d(a) + \max\left[0\,,\; N\pi_x(a)\right]$$

with firm value then

$$v(a) = \max\left[0\,,\; \frac{\pi(a)}{\delta}\right]$$

Productivity cutoffs

- Cutoff for 'successful' entry a^*, solves

$$v(a^*) = 0 \qquad \Leftrightarrow \qquad \pi(a^*) = 0$$

- Cutoff for export status a_x^*, solves

$$\pi_x(a_x^*) = 0$$

- Two cases:

  (i) if $a^* = a_x^*$, all firms that operate also export

  (ii) if $a^* < a_x^*$, some firms produce only for domestic market while others produce for domestic market and export

Case (ii) happens if $\boxed{\tau^{\sigma-1} f_x > f}$, i.e., if selection into exporting is tougher. No such partition if $f_x = 0$

Solving the open economy model

- Supposing case (ii), then

$$\pi(a^*) = \pi_d(a^*) = 0, \qquad \text{and} \qquad \pi_x(a_x^*) = 0$$

- Write profits as

$$\pi_d(a) = Ba^{\sigma-1} - f, \qquad \text{and} \qquad \pi_x(a) = B(a/\tau)^{\sigma-1} - f_x$$

- Eliminate market conditions B between them to get

$$a_x^* = \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} a^*$$

- Now use free entry to pin down a^*

Solving the open economy model

- With export option, free entry condition is

$$\int_0^\infty v(a)g(a)\,da \leq \frac{f_e}{\delta}$$

where

$$v(a) = \max\left[0\,,\ \frac{\pi_d(a) + \max[0\,,\ N\pi_x(a)]}{\delta}\right]$$

- So in terms of the cutoff a^* for domestic operations

$$\int_{a^*}^\infty \Big[\pi_d(a) + \max[0\,,\ N\pi_x(a)]\Big]g(a)\,da \leq f_e$$

Solving the open economy model

- And since $a^* < a_x^*$, can write this

$$\int_{a^*}^{\infty} \pi_d(a) g(a)\, da + N \int_{a_x^*}^{\infty} \pi_x(a) g(a)\, da \leq f_e$$

- Plugging in for $\pi_d(a)$ and $\pi_x(a)$ have

$$\int_{a^*}^{\infty} \left[B a^{\sigma-1} - f \right] g(a)\, da$$
$$+ N \int_{a_x^*}^{\infty} \left[B (a/\tau)^{\sigma-1} - f_x \right] g(a)\, da \leq f_e$$

- As in closed economy, eliminate the market condition B term using cutoffs a^*, a_x^*

$$\int_{a^*}^{\infty} \left[\left(\frac{a}{a^*} \right)^{\sigma-1} - 1 \right] g(a)\, da$$
$$+ N f_x \int_{a_x^*}^{\infty} \left[\left(\frac{a}{a_x^*} \right)^{\sigma-1} - 1 \right] g(a)\, da \leq \frac{f_e}{f}$$

Solving the open economy model

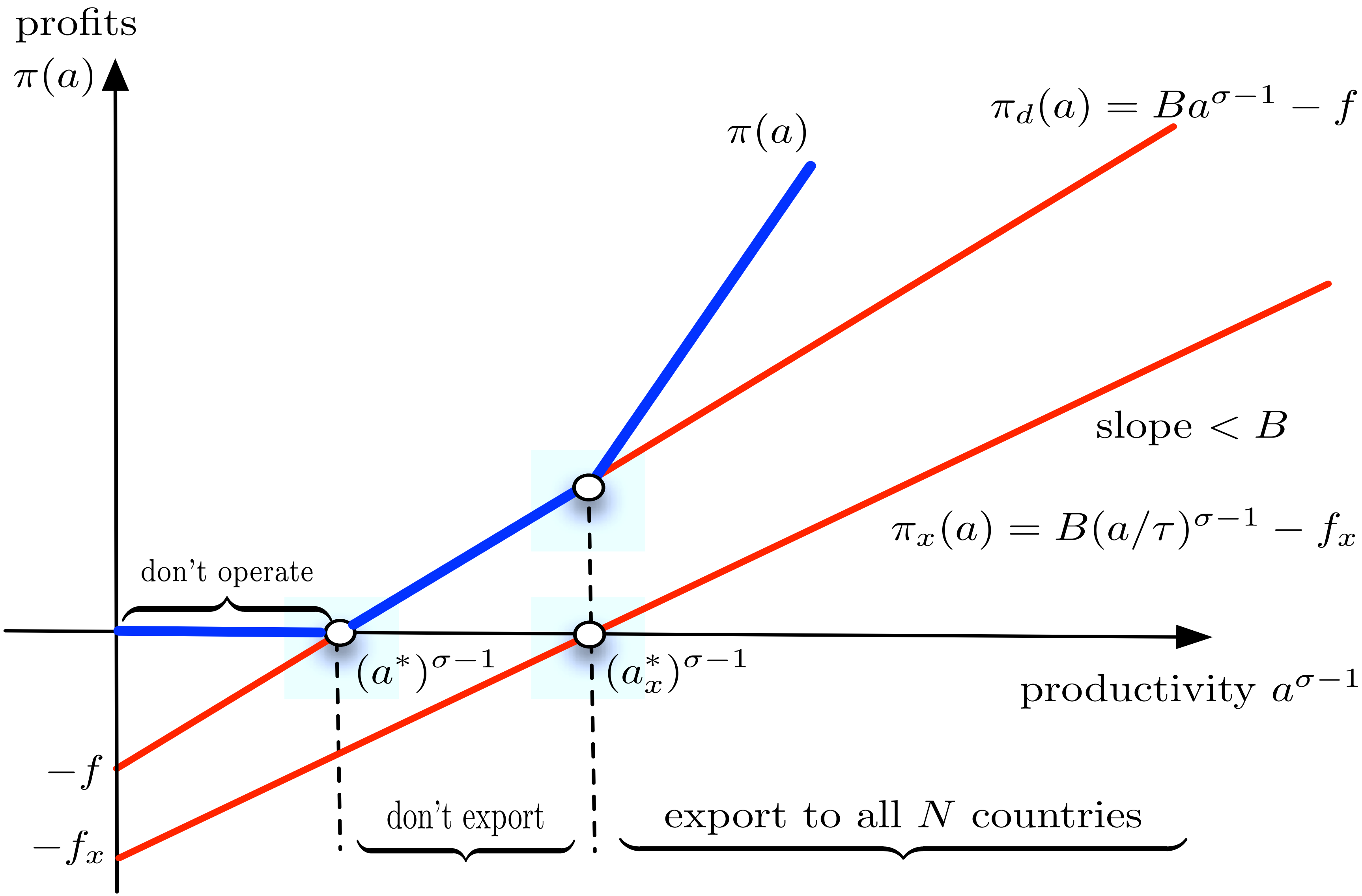
- In short, have a system in a^*, a_x^*

$$J(a^*) + N\frac{f_x}{f}J(a_x^*) \leq \frac{f_e}{f}, \qquad a_x^* = \tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} a^* \qquad (**)$$

where again

$$J(x) := \int_x^\infty \left[\left(\frac{a}{x}\right)^{\sigma-1} - 1\right] g(a)\, da, \qquad J'(x) < 0$$

- From $(**)$, any change in τ moves a^* and a_x^* in opposite directions



profits
$\pi(a)$
$\pi(a)$
$\pi_d(a) = Ba^{\sigma-1} - f$
slope $< B$
$\pi_x(a) = B(a/\tau)^{\sigma-1} - f_x$
don't operate
$(a^*)^{\sigma-1}$
$(a_x^*)^{\sigma-1}$
productivity $a^{\sigma-1}$
$-f$
$-f_x$
don't export
export to all N countries

Summary

- Firms $a < a^*$ exit
- Firms $a > a^*$ operate, of these $a > a_x^*$ also export
- Only firms $a > a_x^*$ can cover *both* fixed costs of operating and fixed costs of exporting
- As in the data, exporters are more productive and larger in terms of revenue and employment

Pareto example revisited

- Again suppose a is Pareto on $[a_{\min}, \infty)$ with shape $\xi > \sigma - 1$

- Then $J(a)$ evaluates to

$$J(a) = \frac{\sigma - 1}{\xi - (\sigma - 1)} \left(\frac{a}{a_{\min}} \right)^{-\xi}, \qquad \xi > \sigma - 1$$

- Recall key condition

$$J(a^*) + N \frac{f_x}{f} J(a_x^*) \leq \frac{f_e}{f}, \qquad a_x^* = \tau \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma - 1}} a^* \qquad (**)$$

- Solves for

$$a^* = \left(\frac{\sigma - 1}{\xi - (\sigma - 1)} \frac{f + N \left[\tau \left(\frac{f_x}{f} \right)^{\frac{1}{\sigma - 1}} \right]^{-\xi} f_x}{f_e} \right)^{1/\xi} a_{\min}$$

Trade liberalization

- Compare cutoffs between closed economy and open economy
  - closed economy (autarky)

$$J(a^*_{aut}) = \frac{f_e}{f}$$

  - open economy

$$J(a^*) + N\frac{f_x}{f}J(a^*_x) = \frac{f_e}{f}$$

- Open economy cutoff $a^* > a^*_{aut}$ (since $J'(x) < 0$)
- Higher aggregate productivity (greater selection)
- Lower price index and higher real wages in each country

Reallocation due to trade liberalization

- Let B_{aut} denote domestic market demand under autarky
- Let B denote domestic market demand in open economy

$$B < B_{aut}$$

(else free entry condition cannot hold)

- But combined market demand, domestic + export market, is

$$(1 + N\tau^{1-\sigma})B > B_{aut}$$

(else free entry condition cannot hold in both equilibria)

Reallocation due to trade liberalization

- Domestic sales under autarky

$$x_{aut}(a) = \sigma B_{aut} a^{\sigma-1}$$

- Domestic sales with trade

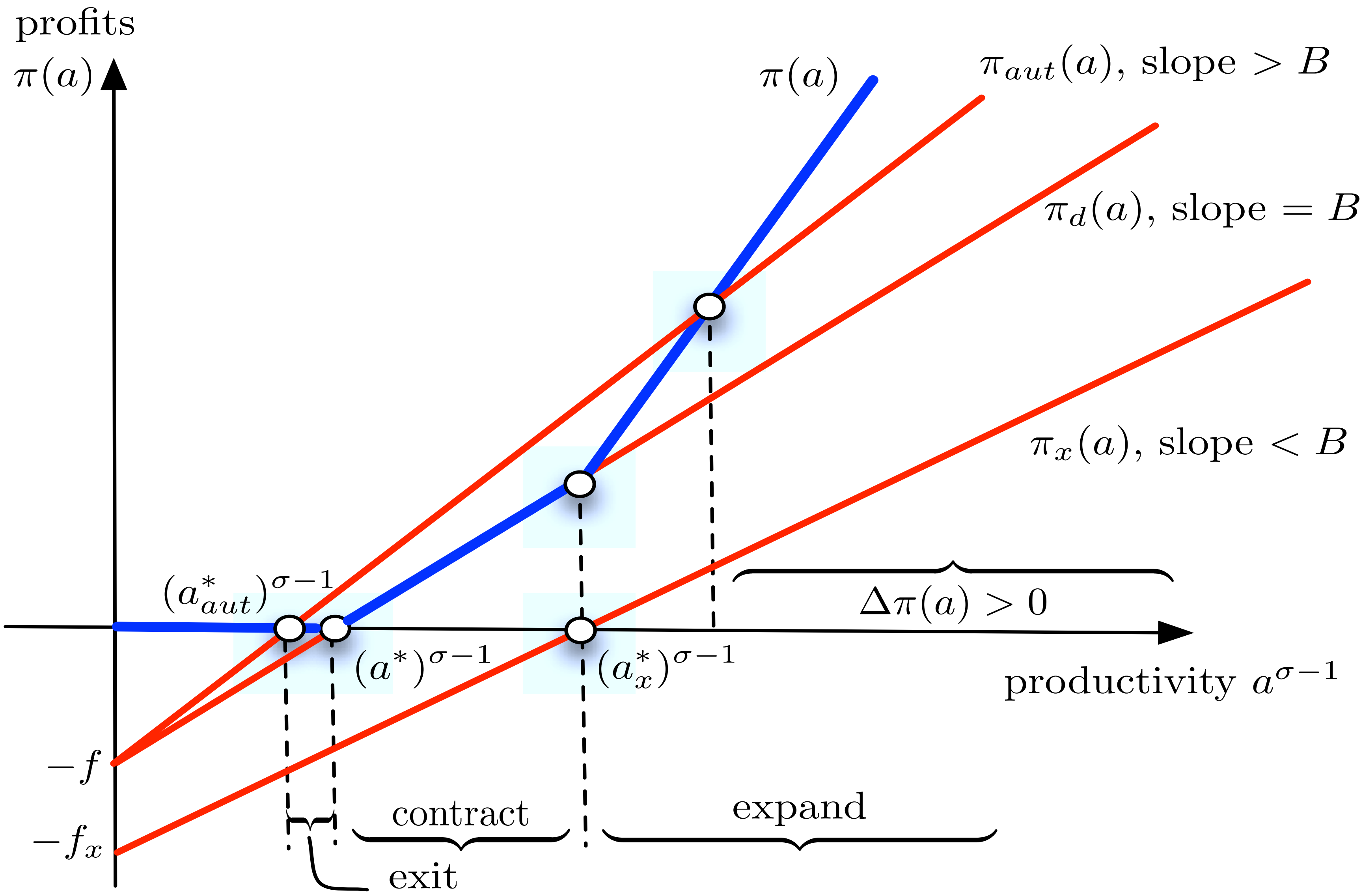
$$x_d(a) = \sigma B a^{\sigma-1}, \qquad B < B_{aut}$$

- Total sales with trade

$$x(a) = \sigma(1 + N\tau^{\sigma-1}) B a^{\sigma-1}$$

- In short: (i) all $a \in (a^*_{aut}, a^*)$ *exit*, (ii) all $a \in (a^*, a^*_x)$ *contract*, and (iii) all $a > a^*_x$ *expand*

  Reallocation of labor from (i) and (ii) towards (iii).



profits
$\pi(a)$
$\pi(a)$
$\pi_{aut}(a)$, slope $> B$
$\pi_d(a)$, slope $= B$
$\pi_x(a)$, slope $< B$
$(a^*_{aut})^{\sigma-1}$
$(a^*)^{\sigma-1}$
$(a^*_x)^{\sigma-1}$
$\Delta\pi(a) > 0$
productivity $a^{\sigma-1}$
$-f$
$-f_x$
contract
expand
exit

Profits

- Ex ante expected profits unchanged, still f_e

- Ex post profits of surviving firms higher on average

$$\bar{\pi} := \int_{a^*}^{\infty} \pi(a) \frac{g(a)}{1 - G(a^*)} \, da = \frac{f_e}{1 - G(a^*)}$$

(increasing in a^*)

- Change in profits for exporting firms $a > a_x^*$

$$\pi(a) - \pi_{aut}(a) = \frac{1}{\sigma}\Big(x_d(a) + N x_x(a) - x_{aut}(a)\Big) - N f_x$$
$$= \left(\frac{a}{a^*}\right)^{\sigma-1}\left(1 + N\tau^{1-\sigma} - \left(\frac{a^*}{a^*_{aut}}\right)^{\sigma-1}\right) f - N f_x$$

Change is increasing in a, negative for $a = a_x^*$. Can have more market share but less profit. Increasing profit dispersion

Next

- Heterogeneous firms and international trade, part three
- Implications for trade flows
  - Chaney (2008): Distorted gravity: The intensive and extensive margins of international trade, *American Economic Review*.