PhD Topics in Macroeconomics

Lecture 11: misallocation, part three

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2nd Semester 2014

This lecture

Peters (2013) model of endogenous misallocation

1- static implications of variable markups

2- dynamics in a quality ladder model

3- empirical implications using Indonesian data (brief remarks)

Overview

- Background
  - Hsieh/Klenow takes marginal product gaps etc as exogenous
  - firms with higher TFPR are more 'constrained'
- Peters alternative: misallocation through endogenous markups
  - quality ladder model with entry (simplified Klette/Kortum)
  - markups depend on productivity gap between incumbent and rivals
  - incumbent and entrant innovation determines productivity gaps
  - implied markup distribution is Pareto, thicker tails when low entry

Model

- Continuous time $t \geq 0$

- Final output

$$\log Y_t = \int_0^1 \log \left[\sum_{k=0}^{J_t(j)} y_t(j,k) \right] dj$$

horizontally differentiated intermediate goods $j \in [0,1]$, each of which comes in $k \in \{0,1,\dots\}$ vertically differentiated vintages

- Note: *imperfect* horizontal differentiation (elasticity of subs. $= 1$) but *perfect* vertical differentiation (elasticity of subs. $= \infty$)

- Intermediate of productivity a produces with capital and labor

$$y = ak^{\alpha} l^{1-\alpha}, \qquad 0 < \alpha < 1$$

taking input prices r and w as given

Costs and pricing

- Marginal cost of intermediate producer with productivity a

$$\frac{c(w,r)}{a}, \qquad c(w,r) := \left(\frac{w}{1-\alpha}\right)^{1-\alpha}\left(\frac{r}{\alpha}\right)^{\alpha}$$

- Most efficient producer takes whole market and *limit prices*, price equal to marginal cost of *second-best* producer

Markup

- Hence best producer has price

$$p^1 = \frac{c(w,r)}{a^2}$$

where a^2 is the productivity of the second-best producer

- Then best producer has *markup* equal to its relative productivity

$$m := \frac{p^1}{c(w,r)/a^1} = \frac{a^1}{a^2}$$

High productivity differential effectively shields from competition

Aside on demand elasticity

- Suppose we instead had CES demand with elasticity $\sigma > 1$
- Unconstrained monopoly price

$$p^* = \frac{\sigma}{\sigma - 1} \frac{c(w,r)}{a^1}$$

- Then best producer has price equal to the min of the monopoly price and the limit price

$$p^1 = \min\left[p^*, \frac{c(w,r)}{a^2}\right] = \min\left[\frac{\sigma}{\sigma - 1}\left(\frac{a^2}{a^1}\right), 1\right]\frac{c(w,r)}{a^2}$$

- Producer with large enough productivity advantage (a^2/a^1 small) might still be able to set unconstrained monopoly price

Static allocation

For intermediate good j with market taken by $a^1(j)$ producer

$$y(j) = \frac{1}{c(w,r)} \frac{a^1(j)}{m(j)} PY, \qquad m(j) = \frac{a^1(j)}{a^2(j)}$$

$$k(j) = \alpha \frac{c(w,r)}{a^1(j) r} y(j) = \frac{1}{m(j)} \frac{\alpha}{r} PY$$

$$l(j) = (1-\alpha) \frac{c(w,r)}{a^1(j) w} y(j) = \frac{1}{m(j)} \frac{(1-\alpha)}{w} PY$$

$$\pi(j) = \left(p(j) - \frac{c(w,r)}{a^1(j)} \right) y(j) = \left(\frac{m(j)-1}{m(j)} \right) PY$$

TFPR

- Physical productivity of a producer is just $a^1(j)$

- Revenue productivity is

$$p(j)a^1(j) = c(w, r)m(j)$$

- Since $c(w, r)$ is common, all cross-sectional variation in TFPR is coming from markup variation (i.e., relative productivity variation)

- In Hsieh/Klenow all cross-sectional variation in TFPR is coming from (τ_K, τ_Y) variation and high TFPR indicates more distorted ('constrained') firms

Aggregation

- Define aggregate productivity by

$$A := \frac{Y}{K^\alpha L^{1-\alpha}}$$

where K and L are aggregate capital and labor used in production

- Summing input demands over intermediate producers

$$K = \int_0^1 k(j)\,dj = \frac{\alpha}{r} PY \int_0^1 \frac{1}{m(j)}\,dj$$

$$L = \int_0^1 l(j)\,dj = \frac{1-\alpha}{w} PY \int_0^1 \frac{1}{m(j)}\,dj$$

Aggregate TFPR

- Taking the geometric average of K and L

$$K^{\alpha}L^{1-\alpha} = \left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha}{w}\right)^{1-\alpha}\left(\int_0^1 \frac{1}{m(j)}\,dj\right)PY$$

where the coefficient out the front is just $1/c(w,r)$

- Hence aggregate TFPR is

$$PA = \frac{PY}{K^{\alpha}L^{1-\alpha}} = c(w,r)\left(\int_0^1 \frac{1}{m(j)}\,dj\right)^{-1}$$

- Need to decompose this into P and A using price index

Aggregate price index P

- With Cobb-Douglas preferences over the intermediates

$$\log P = \int_0^1 \log p(j)\, dj$$

(this is the limit of the usual CES index as $\sigma \to 1^+$)

- Plugging in for individual prices

$$\log P = \int_0^1 \log\left(m(j)\frac{c(w,r)}{a^1(j)}\right) dj$$

or

$$P = c(w,r)\exp\left(\int_0^1 \log\left(\frac{m(j)}{a^1(j)}\right) dj\right)$$

Aggregate productivity A

- Hence we can write aggregate productivity as

$$A = \frac{\exp\left(\int_0^1 \log\left(\frac{a^1(j)}{m(j)}\right) dj\right)}{\int_0^1 \frac{1}{m(j)}\, dj} = \bar{A}D,$$

product of benchmark productivity $\bar{A}$ and distortion index D

- *Benchmark productivity* (first-best productivity)

$$\bar{A} := \exp\left(\int_0^1 \log a^1(j)\, dj\right)$$

- *Distortion index*

$$D := \exp\left(-\int_0^1 \log m(j)\, dj\right) \Big/ \int_0^1 \frac{1}{m(j)}\, dj$$

TFP distortion index

- Write the TFP distortion index as

$$D = \frac{\exp\Big(-\mathbb{E}[\,\log m\,]\Big)}{\mathbb{E}[\,1/m\,]}$$

- By Jensen's inequality $D \leq 1$ and $= 1$ only if m degenerate

- Is homogeneous degree zero in m: *a pure level shift in markups* *does not* *reduce aggregate productivity*

- Is decreasing in a mean-preserving-spread of $\log m$: *more dispersed markups* *do* *reduce aggregate productivity*

Factor price distortion

- The denominator is the distortion to factor prices, that is

$$r = \bar{r} \exp\Big(\mathbb{E}[\,1/m\,]\Big), \qquad w = \bar{w} \exp\Big(\mathbb{E}[\,1/m\,]\Big)$$

  where $\bar{r}$ and $\bar{w}$ are the factor prices that would obtain in the absence of markup distortions

- Is homogeneous degree one in $1/m$: a pure level shift in markups reduces factor prices

- Is invariant to a mean-preserving-spread of $1/m$

- In short, (i) markup dispersion matters for TFP but not factor prices, (ii) markup level matters for factor prices but not TFP

  Consider monopolistically competitive CES case: common markup $\sigma/(\sigma-1)$ leaves TFP unchanged but reduces factor prices

Quality ladder dynamics

- Firm productivity follows ladder with constant step-size $q > 1$
- If producer has had $n_t(j)$ innovations at t, their productivity is

$$a_t(j) = q^{n_t(j)}$$

- Markup is therefore

$$m_t(j) = \frac{a_t^1(j)}{a_t^2(j)} = \frac{q^{n_t^1(j)}}{q^{n_t^2(j)}} = q^{\Delta_t(j)}, \qquad \Delta := n^1 - n^2 \geq 1$$

- Entry gives access to the current leading technology

$$qa_t^1(j)$$

Innovation and markups

Note contrast:

- *Incumbent innovation*: increases a^1 relative to a^2, increases markup by factor $q > 1$
- *Entrant innovation*: decreases markup, by factor $q^{\Delta - 1}$
- Innovating incumbent may be multiple steps ahead, innovating entrant is only one step ahead
- Why will incumbent innovate? (cf., simple quality ladder model)

Quality gap dynamics

- Let λ denote incumbent innovation rate and let η denote entry rate (both endogenous, will be constant along balanced growth path)

- Let $M_t(\Delta)$ denote measure of intermediates with quality gap Δ

- Law of motion

$$\dot{M}_t(\Delta) = -(\eta + \lambda)M_t(\Delta) + \lambda M_t(\Delta - 1), \qquad \text{for } \Delta \geq 2$$

and

$$\dot{M}_t(1) = -(\eta + \lambda)M_t(1) + \eta$$

Stationary quality gap distribution

- Setting $\dot{M}_t(\Delta) = 0$ for each t we have

$$M(1) = \frac{\eta}{\eta + \lambda}$$

and

$$M(\Delta) = \frac{\lambda}{\eta + \lambda} M(\Delta - 1), \qquad \text{for } \Delta \geq 2$$

- Iterating backwards we get

$$M(\Delta) = \left(\frac{\lambda}{\eta + \lambda}\right)^{\Delta - 1} \frac{\eta}{\eta + \lambda}$$

$$= \left(\frac{\lambda}{\eta + \lambda}\right)^{\Delta} \frac{\eta}{\lambda} = \left(\frac{1}{1 + \theta}\right)^{\Delta} \theta, \qquad \theta := \frac{\eta}{\lambda}$$

Quality gaps and markup distribution

- Cumulative quality gap distribution

$$F_\Delta(n) := \text{Prob}[\Delta \leq n] = \sum_{k=1}^{n} M(k) = 1 - \left(\frac{1}{1+\theta}\right)^n$$

- *Markup distribution is Pareto*

$$F(m) := \text{Prob}[q^\Delta \leq m] = \text{Prob}[\Delta \leq \log m / \log q]$$

$$= 1 - m^{-\xi(\theta)}, \qquad \xi(\theta) := \frac{\log(1+\theta)}{\log q}$$

with shape $\xi(\theta)$ given by *entry intensity* $\theta := \eta/\lambda$ (rate of entrant innovation to incumbent innovation, as in Klette/Kortum)

Distortion index $D := \exp\Big(-\mathbb{E}[\log m]\Big)/\mathbb{E}[1/m]$

- Two statistics to calculate

$$\mathbb{E}[1/m] = \mathbb{E}[q^{-\Delta}] = \sum_{\Delta=0}^{\infty} q^{-\Delta} M(\Delta) = \frac{\theta}{(q-1)+q\theta}$$

and

$$\mathbb{E}[\log m] = \mathbb{E}[\Delta \log q] = (\log q) \sum_{\Delta=0}^{\infty} \Delta M(\Delta) = (\log q)\left(\frac{\theta+1}{\theta}\right)$$

- Then distortion index is

$$D(\theta) = \frac{\exp(-\mathbb{E}[\log m])}{\mathbb{E}[1/m]} = q^{-\frac{\theta+1}{\theta}} \frac{(q-1)+q\theta}{\theta}$$

and determined by entry intensity $\theta = \eta/\lambda$

Continuous approximation

- Approximate formulas treating markups as continuous

$$\mathbb{E}[1/m] \approx \int_1^\infty (1/m)\, dF(m) = \int_1^\infty (1/m)\xi(\theta)(1/m)^{\xi(\theta)+1}\, dm$$
$$= \frac{\xi(\theta)}{\xi(\theta)+1}$$

and

$$\mathbb{E}[\log m] \approx \int_1^\infty (\log m)\, dF(m) = \int_0^\infty z\, \exp(-\xi(\theta)z)\, dz$$
$$= \frac{1}{\xi(\theta)}$$

using that $z = \log m$ has an exponential distribution

- Gives distortion index

$$D(\theta) \approx \exp\left(-\frac{1}{\xi(\theta)}\right)\frac{\xi(\theta)+1}{\xi(\theta)}$$

Effects of higher entry

A higher entry intensity $\theta = \eta/\lambda$:

- Reduces $F(m)$ in FOSD sense ($F(m)$ increasing in θ for all m)
- Increases $\xi(\theta)$ and hence reduces markup dispersion
- Reduces wedge between A and first-best $\bar{A}$, thereby increasing aggregate productivity
- Reduces wedge between factor prices and marginal products

Now need to actually pin down innovation rates η, λ

Innovation and entry costs

- Convex innovation cost function for incumbents

$$C(\lambda, \Delta) = q^{-\Delta}\lambda^{\gamma}, \qquad \gamma > 1$$

- This is the amount of labor required for an incumbent with advantage Δ to generate flow innovation rate λ

- Workers generate ideas with Poisson intensity 1 (normalization), 'blueprint' operational after paying fixed cost $f_e > 0$ units of labor

HJB for incumbents

- Let $V_t(\Delta)$ denote value of a firm with current quality gap Δ

$$(r+\eta)V_t(\Delta) = \pi_t(\Delta) \\ + \max_{\lambda \geq 0} \left[\lambda(V_t(\Delta+1) - V_t(\Delta)) - w_t q^{-\Delta}\lambda^{\gamma}\right] + \dot{V}_t(\Delta)$$

- Along BGP, value function has the form

$$V_t(\Delta) = (v_0 - v_1 q^{-\Delta})e^{gt}$$

for some constants v_0, v_1, g to be determined

- Let $\hat{V}(\Delta), \hat{\pi}(\Delta), \hat{w}$ etc denote variables relative to Y_t on BGP

Rescaled problem

- So relative to the BGP

$$(r + \eta - g)\hat{V}(\Delta) = (1 - q^{-\Delta}) + q^{-\Delta} \max_{\lambda \geq 0} \left[\lambda v_1 \frac{q-1}{q} - \hat{w}\lambda^{\gamma} \right]$$

where $\hat{\pi}(\Delta) = (1 - q^{-\Delta})$ is instantaneous profits

- Hence the incumbent innovation rate λ is independent of Δ and solves the first order condition

$$v_1 \frac{q-1}{q} = \hat{w}\gamma\lambda^{\gamma-1}$$

- Plugging back into the HJB allows us to solve for the coefficients v_0, v_1 (and hence λ) in terms of the aggregates (w, η, etc)

Solution to rescaled problem

- The intercept is

$$v_0 = \frac{1}{r + \eta - g}$$

- The slope v_1 implicitly solves

$$(r + \eta - g)v_1 = 1 - (\gamma - 1)\hat{w}\left(\frac{q-1}{q}\frac{v_1}{\gamma \hat{w}}\right)^{1/(\gamma-1)}$$

- Innovation intensity is then

$$\lambda = \left(\frac{q-1}{q}\frac{v_1}{\gamma \hat{w}}\right)^{1/(\gamma-1)}$$

(all these depend on the aggregate w, η, etc)

General equilibrium

- Representative consumer with preferences over final good

$$U = \int_0^\infty e^{-\rho t} \log C_t \, dt$$

and inelastic labor supply $L > 0$

- Along a BGP we then have

$$r = \rho + g$$

- With constant quality step q and constant innovation rates

$$g = \frac{1}{1-\alpha}(\log q)(\lambda + \eta)$$

- To complete the solution of the model, need to solve for aggregate λ, η etc along this BGP

- Back to the incumbent value function, we now have the intercept

$$v_0 = \frac{1}{r+\eta-g} = \frac{1}{\rho+\eta} =: v_0(\eta)$$

- And the slope $v_1(\eta, \hat{w})$ solves

$$(\rho+\eta)v_1 = 1 - (\gamma-1)\hat{w}\left(\frac{q-1}{q}\frac{v_1}{\gamma\hat{w}}\right)^{1/(\gamma-1)}$$

and then recover $\lambda(\eta, \hat{w})$ from the first order condition

- EXAMPLE: in the special case of $C(\cdot)$ quadratic in λ, i.e., $\gamma = 2$, can solve for v_1 explicitly

$$v_1 = \frac{1}{\rho+\eta+\frac{1}{2}\frac{q-1}{q}}, \qquad \text{(independent of } \hat{w}\text{, in this case)}$$

Equilibrium

- Constants

$$(\eta^* \, , \, \hat{w}^*)$$

consistent with firm optimization, i.e., $v_0(\eta), v_1(\eta, \hat{w}), \lambda(\eta, \hat{w})$, and

(i) free entry condition

$$(v_0 - v_1 q^{-1}) \leq \hat{w} f_e$$

(ii) labor market clearing

$$L_X + L_R + L_S = L$$

- Compute equilibrium by solving fixed point problem in $\hat{w}, \eta$

Then implies innovation intensity $\lambda^* = \lambda(\eta^*, \hat{w}^*)$ etc

- Labor employed in goods production

$$L_X = \sum_{\Delta=1}^{\infty} l_X(\Delta) M(\Delta), \qquad l_X(\Delta) = q^{-\Delta} \frac{1-\alpha}{\hat{w}}$$

$$= \frac{1-\alpha}{\hat{w}} \sum_{\Delta=1}^{\infty} q^{-\Delta} M(\Delta) = \frac{1-\alpha}{\hat{w}\hat{m}}$$

where the *aggregate markup, $\hat{m}$* is given by

$$\hat{m} := \left(\sum_{\Delta=1}^{\infty} q^{-\Delta} M(\Delta) \right)^{-1} = (q-1)\frac{\lambda}{\eta} + q =: m(\lambda, \eta)$$

- Labor employed in research at incumbents

$$L_R = \sum_{\Delta=1}^{\infty} l_R(\Delta) M(\Delta), \qquad l_R(\Delta) = C(\lambda, \Delta) = q^{-\Delta} \lambda^{\gamma}$$

$$= \lambda^{\gamma} \sum_{\Delta=1}^{\infty} q^{-\Delta} M(\Delta) = \frac{\lambda^{\gamma}}{m(\lambda, \eta)}$$

Computing an equilibrium: summary

- Labor employed at startups

$$L_S = \eta f_e$$

- Hence labor market clearing condition is

$$\frac{1}{m\Big(\lambda(\eta,\hat{w}),\eta\Big)}\left(\frac{1-\alpha}{\hat{w}}+\lambda(\eta,\hat{w})^{\gamma}\right)+\eta f_e = L \qquad (*)$$

and the free entry condition is

$$(v_0(\eta)-v_1(\eta,\hat{w})q^{-1}) \leq \hat{w} f_e \qquad (**)$$

- Given $v_0(\eta), v_1(\eta,\hat{w}), \lambda(\eta,\hat{w}), m(\lambda,\eta)$ already determined, now solve these two equations for the two remaining unknowns, $\eta, \hat{w}$

Empirical implications

- Markups proportional to revenue productivity

- Entry vs. markup level, dispersion

Indonesian data

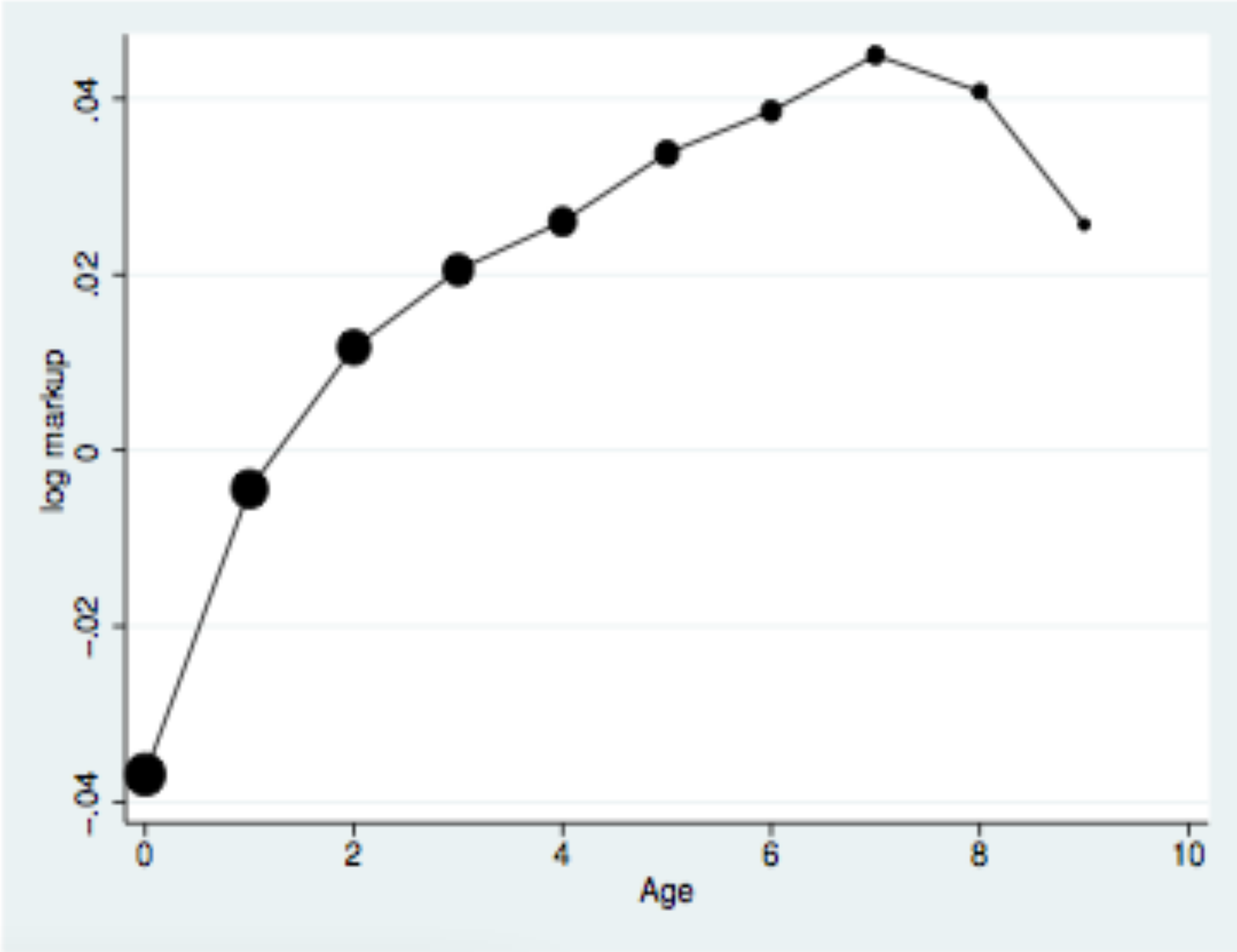
- Manufacturing
  - annual census 1991–2000
  - manufacturing plants > 20 employees
  - revenue, wage bill, productions and non-production workers, capital stock, entry, region
  - trim 1% tails
- Geographic information
  - 240 regencies aggregated to 33 provinces
  - other geographic/regional controls from Village Potential Statistics aggregated to province

Entrants and TFPR

	Dep. Variable: Labor productivity $ln(\frac{py}{wl})$						
	Full Sample					Census Supplement 1996	
Entrant	-0.0424*** (0.00700)						
Exiter		-0.0667*** (0.00738)					
Age			0.00680*** (0.00170)				
Positive Growth				0.252*** (0.00409)			
Exporter					0.0981*** (0.00642)		
Profit growth						0.0775*** (0.0115)	
R&D spending							0.123** (0.0533)
$ln(k/l)$	0.138*** (0.00188)	0.137*** (0.00185)	0.137*** (0.00285)	0.136*** (0.00176)	0.136*** (0.00179)	0.162*** (0.00496)	0.161*** (0.00495)
N	159678	159674	73657	173863	173863	20854	20854
R^2	0.163	0.162	0.163	0.180	0.163	0.210	0.208

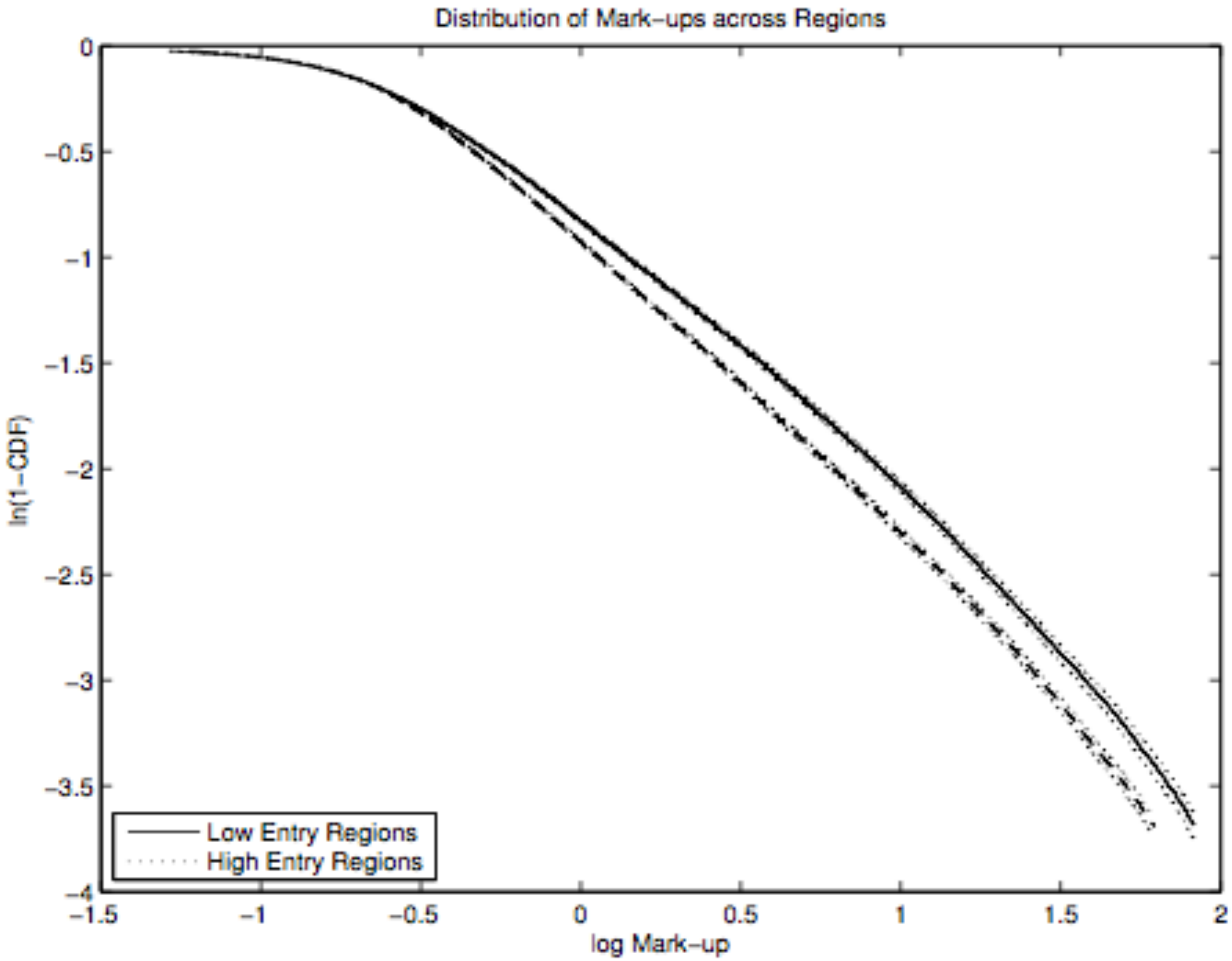
Entrants have lower revenue productivity (markups) than incumbents. Revenue productivity tends to increase with age and to be higher in growing firms.

Lifecycle of TFPR



Revenue productivity (markups) low on entry (relative to average) but increasing with age. Cohort size shrinks with attrition. Size proportional to dot.

Entry and markup distribution



Low entry regions (below median) have thicker tail of markups compared to high entry regions (above median).

Entry and markup distribution

	Avg. log mark-up				
	All products				Differentiated products
Entry	-0.135***	-0.145***	-0.134***	-0.120***	-0.131**
	(0.0409)	(0.0404)	(0.0414)	(0.0415)	(0.0553)
(ln) population		-0.0254	-0.0307*	-0.0147	-0.00581
		(0.0154)	(0.0174)	(0.0164)	(0.0132)
Agricultural employmment share			-0.0772	-0.112	-0.0598
			(0.0655)	(0.101)	(0.0789)
Share of villages with banks				-0.000230	0.00807
				(0.0116)	(0.00942)
Share of villages with BRI branch				-0.0237	-0.00725
				(0.0840)	(0.0928)
Share of villages with accessible markets				-0.549**	-0.607**
				(0.221)	(0.229)
N	4480	4480	4480	4480	2972
R^2	0.006	0.010	0.015	0.036	0.059

Higher entry rates associated with lower average markups (revenue productivity) controlling for various regional characteristics.

Entry and markup distribution

	Other moments of log mark-up distribution				
	Quantiles				Standard
	25%	50%	75%	90%	deviation
Entry	-0.126**	-0.106**	-0.123**	-0.0792	0.0300
	(0.0478)	(0.0400)	(0.0511)	(0.0521)	(0.0345)
(ln) population	-0.0441***	-0.0313*	0.00384	0.0514*	0.0243
	(0.0142)	(0.0171)	(0.0293)	(0.0291)	(0.0150)
Agricultural employmment share	0.0525	-0.128	-0.347**	-0.471***	-0.328***
	(0.0917)	(0.102)	(0.143)	(0.159)	(0.0604)
Share of villages with banks	0.00339	-0.00453	-0.00164	-0.00599	-0.0104
	(0.0101)	(0.0121)	(0.0176)	(0.0200)	(0.00823)
Share of villages with BRI branch	0.0891	-0.00191	-0.167	-0.318*	-0.237*
	(0.0845)	(0.0929)	(0.127)	(0.175)	(0.117)
Share of villages with accessible markets	-0.188	-0.607**	-1.043***	-1.208***	-0.548***
	(0.249)	(0.231)	(0.235)	(0.246)	(0.143)
N	4480	4480	4480	4480	3645
R^2	0.169	0.059	0.169	0.247	0.427

But effect fairly uniform across distribution. Contrary to model, no evidence of higher entry rates being associated with reduction in markup dispersion.

Next

- Misallocation, part four

- Financial frictions, misallocation, and 'growth miracles'

  ◇ Buera and Shin (2013): Financial frictions and the persistence of history: A quantitative exploration, *Journal of Political Economy*.