PhD Topics in Macroeconomics

Lecture 1: introduction and course overview

Chris Edmond

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Introduction

- A course ‘the whole family can enjoy’

- Research on firm heterogeneity takes place at the intersection of macro, trade, IO, labor etc

- Papers typically integrate micro-data and theory

  – data guiding model development, *and*

  – data examined through the lens of models

Introduction

- We will begin with classic papers on firm dynamics *per se*
- Then turn to more recent applications/extensions, including
  - innovation and aggregate growth
  - misallocation and cross-country income differences
  - heterogeneous firms and international trade

Course requirements

- Four problem sets, 10% each
- Two referee report, 10% each, due Monday Oct 20th
- Research proposal presentation, 40%, beginning Monday Oct 27th

Overview

- *Firm dynamics: basic models*, 4 lectures

  Hopenhayn (1992 Ecma), Hopenhayn/Rogerson (1993 JPE)

  Problem set #1 based on this material

- *Innovation and firm dynamics*, 4 lectures

  Review of quality ladder models, then:

  Klette/Kortum (2004 JPE), Lentz/Mortensen (2005 IER, 2008 Ecma), Atkeson/Kehoe (2007 AER)

  Problem set #2 based on this material

Overview

- *Misallocation*, 4 lectures

  Restuccia/Rogerson (2008 RED), Hsieh/Klenow (2009 QJE), Peters (2013wp), Buera/Shin (2013 JPE), Midrigan/Xu (2014 AER)

  Problem set #3 based on this material

- *Heterogeneous firms and international trade*, 6 lectures

  Review of monopolistic competition and trade, then:

  Melitz (2003 Ecma), Chaney (2008 AER), Eaton/Kortum (2002 Ecma) Bernard/Eaton/Jensen/Kortum (2003 AER), Eaton/Kortum/Kramarz (2011 Ecma)

  Problem set #4 based on this material

Overview

- *Aggregate gains from trade*, 3 lectures

  Arkolakis/Costinot/Rodriguez-Clare (2012 AER),
  Arkolakis/Costinot/Donaldson/Rodriguez-Clare (2012wp),
  Edmond/Midrigan/Xu (2014wp)

- *'Thoughts on Piketty'*, 3 lectures

  Piketty (2014, selections), Atkinson/Piketty/Saez (2011 JEL),
  Piketty/Zucman (2014 QJE), Benhabib/Bisin/Zhu (2013wp)

  What's the connection? Pareto distributions, distributional dynamics etc.

Firm dynamics: basic models, part one

Rest of today's class:

- Background facts on firm-size distribution
  - Zipf's law and Gibrat's law as organizing principles
- Simple models of the firm-size distribution
  - statistical models
    - Yule/Simon *preferential attachment*
  - economic models
    - Lucas *span of control*

Two empirical benchmarks

1- *Zipf's law*:

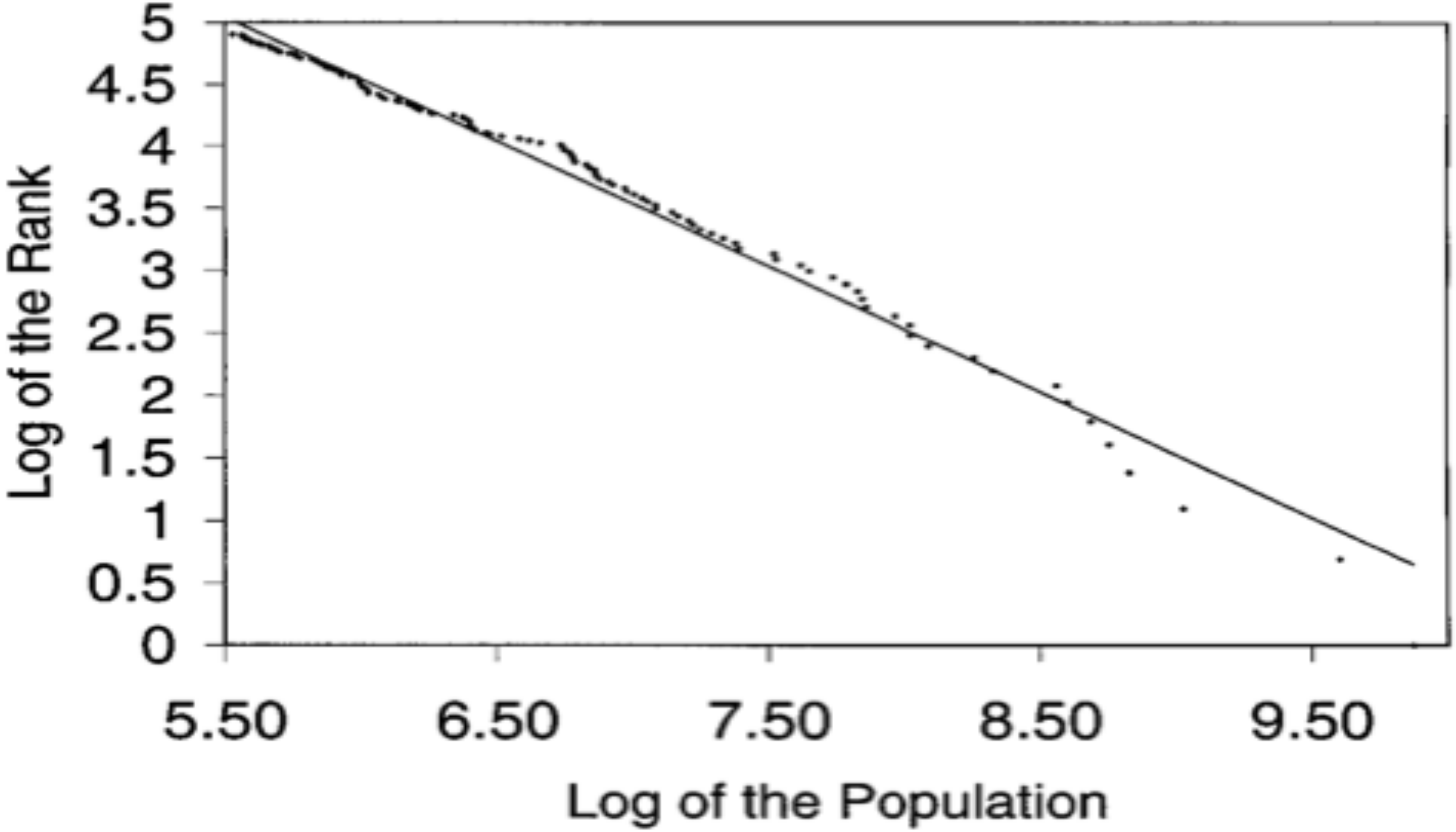
– frequency of observation inversely proportional to rank
(word-use, city-size, firm-size, . . .)

2- *Gibrat's law*:

– individual firm growth rates independent of size
(at least, for large enough firms)

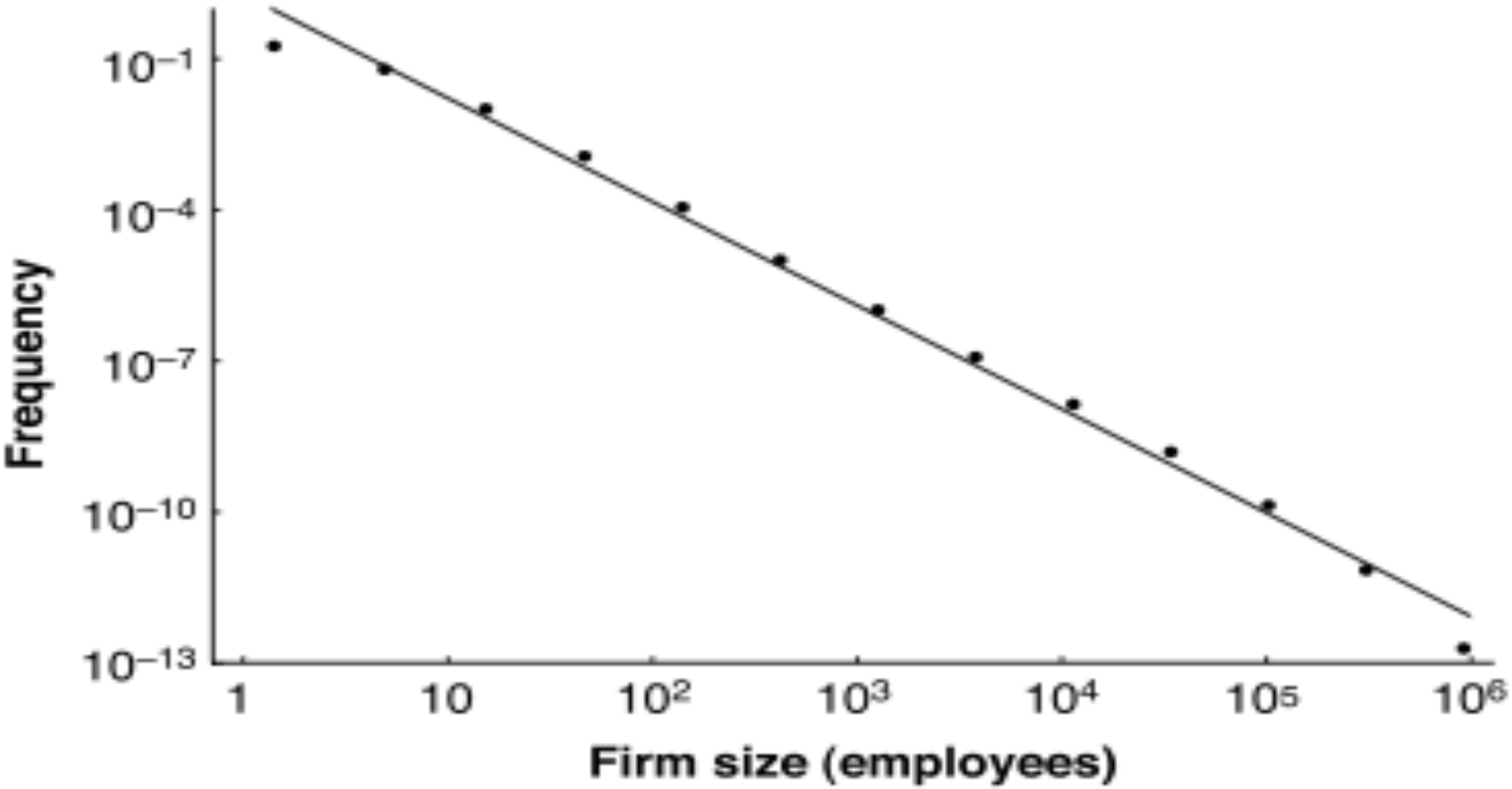
How are these related?

Zipf's law for US city sizes



135 largest US metropolitan areas. Source: Gabaix (1999).

Zipf's law for US firm sizes



US Census Bureau. Source: Axtell (2001).

- Suppose we measure firm size by number of employees, n

- Firm-size distribution well approximated by a *Pareto distribution*

Pareto distribution reminder

- *Survivor function* for standard Pareto distribution

$$\text{Prob}[n' \geq n] = n^{-\xi} \qquad n > 1, \qquad \xi > 0$$

- Associated CDF and PDF

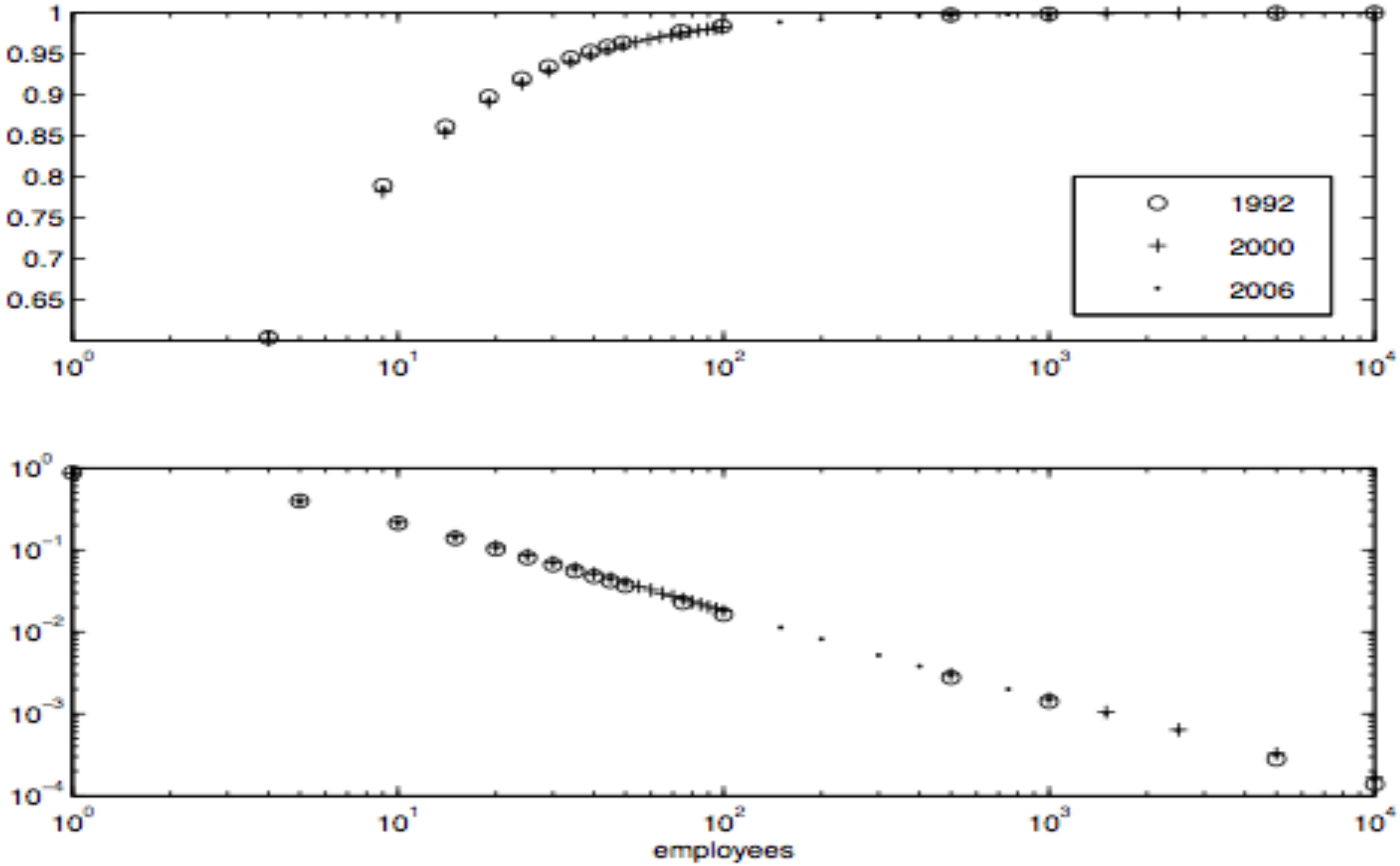
$$\text{Prob}[n' \leq n] := F(n) = 1 - n^{-\xi}, \qquad f(n) = \xi n^{-(\xi+1)}$$

- Finite mean requires $\xi > 1$, finite variance requires $\xi > 2$, finite skewness requires $\xi > 3$, etc

- Zeta distribution is the discrete analogue. Zipf's law is the zeta distribution with $\xi = 1$

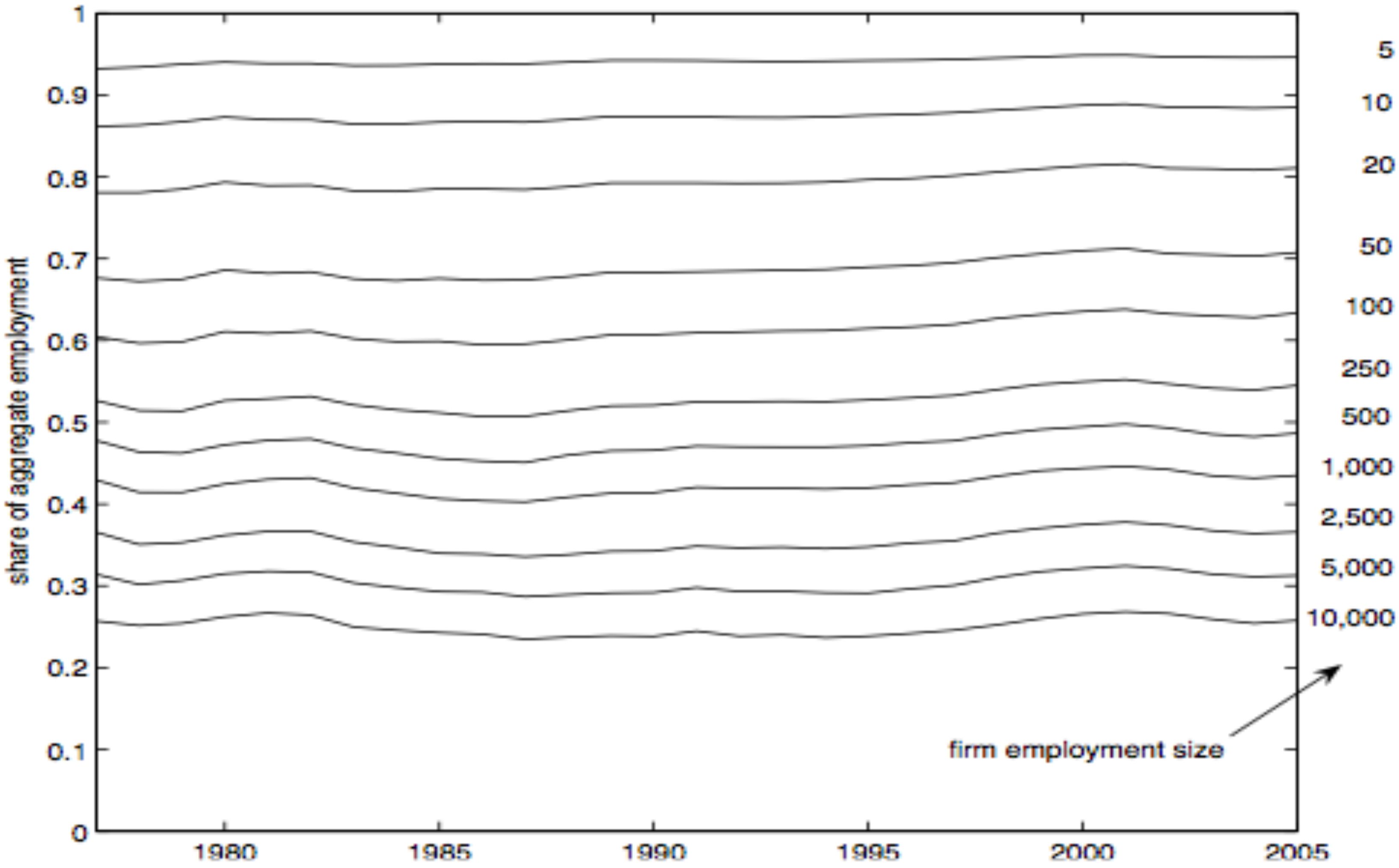
US firm-size distribution

implied $\xi \approx 1.05$



Small Business Administration (SBA) counts. Top panel is CDF, bottom panel is survivor function (1 − CDF). Source: Luttmer (2010).

Repeated cross-sections



Business Dynamics Statistics (BDS) categories. Source: Luttmer (2010).

- In short, US firm-size distribution is both very stable and very skewed

  - there are about 6 million firms

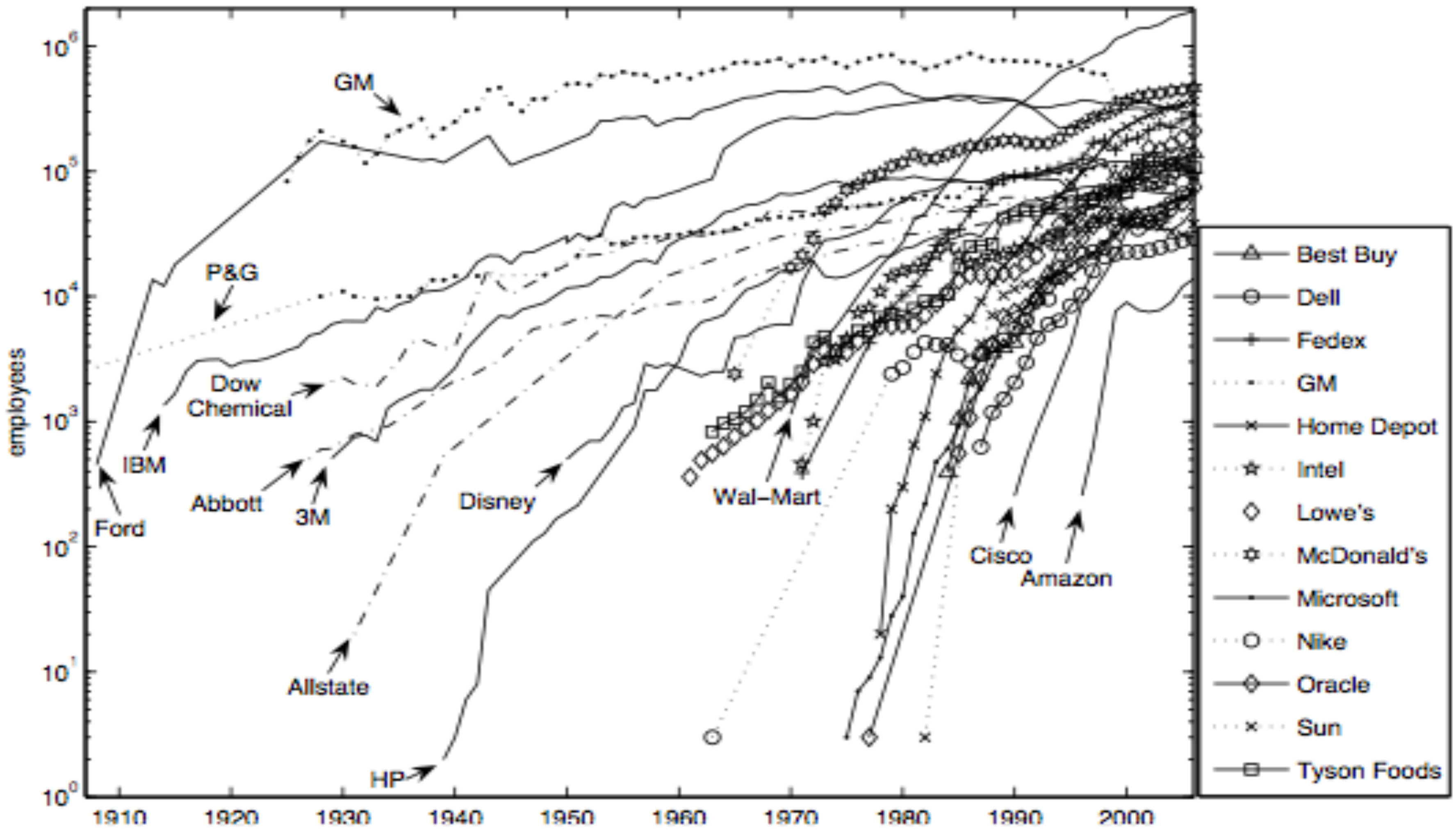
  - around *one-half* of total employment is accounted for by some 18,000 very large firms with more than 500 employees each

  - around *one-quarter* of total employment is accounted for by some 1,000 enormous firms with more than 10,000 employees each

  - but most firms are small, almost 80% of firms have less than 10 employees

- Perhaps surprisingly, a stable firm-size distribution is the fairly natural consequence of random growth at the micro level (almost)

Individual growth histories



Source: Luttmer (2011).

Gibrat's law

- Starting point: Gibrat's law of 'proportional effect'
- Fixed population of units i (cities, firms, ...) of size P_t^i
- Let $S_t^i := P_t^i / \bar{P}_t$ denote normalized size (relative to average size $\bar{P}_t$) and suppose

$$S_{t+1}^i = \gamma_t^i \, S_t^i \qquad \gamma_t^i \quad \sim \text{IID} \quad f(\gamma) \tag{1}$$

  Growth independent of current size, so absolute increment $S_{t+1} - S_t$ approximately proportional to current size S_t
- Let $G_t(x) := \text{Prob}[S_t^i \geq x]$ denote the survivor function

From random growth to Pareto distribution

- Law of motion for distribution

$$G_{t+1}(x) = \text{Prob}[S^i_{t+1} \geq x] = \text{Prob}[S^i_t \geq x/\gamma^i_t]$$
$$= \int_0^\infty G_t\left(\frac{x}{\gamma}\right) f(\gamma)\, d\gamma$$

- Steady state distribution, *if it exists*, satisfies

$$G(x) = \int_0^\infty G\left(\frac{x}{\gamma}\right) f(\gamma)\, d\gamma$$

- Guess-and-verify that $G(x) = kx^{-\xi}$ solves this fixed point problem. Requires:

$$1 = \int_0^\infty \gamma^\xi f(\gamma)\, d\gamma$$

which pins down Pareto exponent ξ in terms of $f(\cdot)$. Coefficient k then pinned down by requiring density $-G'(x)$ integrates to 1

Existence of a steady-state distribution

- Of course there need not be a steady-state distribution

- Suppose $f(\gamma)$ is log-normal, $\ln \gamma \sim N(\mu, \sigma^2)$, then

$$\ln S_t^i - \ln S_0^i \sim N(\mu t, \sigma^2 t)$$

  and there is no steady-state distribution, it keeps fanning out

- But turns out that 'small departures' from this strict version of Gibrat's law give us back a steady-state distribution and moreover give micro foundations for $\xi \approx 1$

  Examples: Gabaix (1999 QJE), Luttmer (2007 QJE)

Simple models of the firm size distribution

(1) Yule/Simon *preferential attachment* model: simple statistical approach giving Pareto-like distribution

(2) Lucas *span of control* model: simple economic model rationalizing Pareto firm size in terms of underlying Pareto distribution of managerial talent

Yule/Simon preferential attachment

- Firms have *discrete* sizes, $n = 1, 2, 3, \ldots$

- A given firm transitions from size n to $n+1$ with constant hazard $\lambda > 0$ per instant time

- Let $P_n(a)$ denote the probability the firm is of size n at age $a \geq 0$. For $n = 1$ we have

  $$P_1(a) = e^{-\lambda a}$$

  For $n = 2$

  $$P_2(a) = (1 - e^{-\lambda a})\, e^{-\lambda a}$$

  And, by induction, for any $n = 1, 2, 3, \ldots$

  $$P_n(a) = (1 - e^{-\lambda a})^{n-1}\, e^{-\lambda a}$$

- This is a *geometric distribution* with parameter $\theta = e^{-\lambda a}$

Yule/Simon distribution

- The cross-sectional firm-size distribution is then given by

$$P_n := \int_0^\infty P_n(a) f(a)\, da$$

where $f(a)$ is the PDF of firm ages

- Suppose age has *exponential distribution* with parameter γ. Then

$$P_n = \int_0^\infty P_n(a)\, \gamma e^{-\gamma a}\, da = \int_0^\infty (1 - e^{-\lambda a})^{n-1}\, e^{-\lambda a}\, \gamma e^{-\gamma a}\, da$$

$$= \frac{\gamma}{\lambda} \int_0^1 (1-\theta)^{n-1} \theta^{\gamma/\lambda}\, d\theta$$

(making change of variables to the geometric paramater $\theta = e^{-\lambda a}$)

Yule/Simon distribution

- Recall the beta and gamma functions

$$B(x,y) := \int_0^1 \theta^{x-1}(1-\theta)^{y-1}\, d\theta, \quad \Gamma(x) := \int_0^\infty \theta^{x-1} e^{-\theta}\, d\theta$$

The gamma function $\Gamma(x)$ is the continuous analogue of the factorial function, $x\Gamma(x) = \Gamma(x+1)$ etc

- These are related by

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

- So the firm-size distribution can be written

$$P_n = (\gamma/\lambda)B\Big((\gamma/\lambda)+1,\, n\Big) = (\gamma/\lambda)\frac{\Gamma((\gamma/\lambda)+1)\Gamma(n)}{\Gamma((\gamma/\lambda)+1+n)}$$

This is the *Yule/Simon distribution* with parameter γ/λ.

Yule/Simon distribution

Two approximations:

i. in limit as $\gamma/\lambda \to 1$, we have

$$P_n \to B(2,n) = \frac{\Gamma(2)\Gamma(n)}{\Gamma(2+n)} = \frac{1!(n-1)!}{(1+n)n(1-n)!} = \frac{1}{(1+n)n}$$

with survivor function

$$\text{Prob}[k \geq n] = \sum_{k=n}^{\infty} \frac{1}{(1+k)k} = \frac{1}{n}$$

a zeta/Pareto distribution with exponent 1, i.e, Zipf's law again

ii. in limit as $n \to \infty$, we have approximation $\Gamma(n+\alpha)/\Gamma(n) \sim n^{\alpha}$ so

$$P_n \to (\gamma/\lambda)\Gamma((\gamma/\lambda)+1)n^{-((\gamma/\lambda)+1)}$$

with survivor function proportional to $n^{-\gamma/\lambda}$, i.e., a zeta/Pareto distribution with exponent γ/λ

Lucas 1978 span of control

- Firm consists of a *production technology* and a *managerial technology*

- Production technology is standard concave CRS

$$y = F(k, n) = f(k/n)n$$

- Managerial technology: manager of talent x produces

$$x\, g(y)$$

units of output where $g(\cdot)$ is strictly increasing, strictly concave

- Manager is a fixed input. DRS of $g(\cdot)$ reflects their limited span of control – best manager can't control everything. Efficient for some resources to be controlled by next-best manager, etc

Lucas 1978 span of control

- Profits for a firm with manager x facing factor prices w, r

$$\pi(x) = \max_{\kappa,n} \left[xg(f(\kappa)n) - r\kappa n - wn \right], \qquad \kappa := k/n$$

- First order conditions

$$xg'(y)f'(\kappa) = r$$

$$xg'(y)f(\kappa) = r\kappa + w$$

- All firms choose same $\kappa = k/n$ ratio, independent of x

$$\frac{f(\kappa) - f'(\kappa)\kappa}{f'(\kappa)} = \frac{w}{r}$$

- Scale $y(x)$ then determined by finding y that solves

$$xg'(y)f'(\kappa) = r$$

which can be used to recover $n(x)$

Implications of Gibrat's law

- Firm growth induced by changes in factor prices

$$\frac{d}{dt}\ln[n(x\,;\,w(t),r(t))] = \frac{n_w(x,w,r)}{n(x,w,r)}w'(t) + \frac{n_r(x,w,r)}{n(x,w,r)}r'(t)$$

- Strong form of Gibrat's law is hypothesis that this derivative is invariant to firm size

$$\frac{\partial}{\partial x}\left[\frac{n_w(x,w,r)}{n(x,w,r)}w'(t) + \frac{n_r(x,w,r)}{n(x,w,r)}r'(t)\right] = 0$$

- For this to hold for all patterns of changing factor prices, must have both

$$\frac{\partial}{\partial x}\frac{n_w(x,w,r)}{n(x,w,r)} = \frac{\partial}{\partial x}\frac{n_r(x,w,r)}{n(x,w,r)} = 0$$

Implications of Gibrat's law

- The condition

$$\frac{\partial}{\partial x} \frac{n_w(x, w, r)}{n(x, w, r)} = 0$$

is implicitly a restriction on the functional form of $g(\cdot)$

- Calculating $n_w(x, w, r)$ and solving the differential equation for $g(\cdot)$, Lucas finds that

$$g(y) = Ay^{\alpha}, \qquad A > 0, \qquad 0 < \alpha < 1$$

is the unique functional form consistent with the strong version of Gibrat's law

Lucas 1978 example

- Suppose the production function $y = n$, the managerial technology xy^α and that managerial talent has the Pareto distribution with CDF $1 - x^{-\xi}$

- First order condition

$$x\alpha y^{\alpha-1} = w \quad \Rightarrow \quad y(x) = n(x) = \left(\frac{\alpha x}{w}\right)^{1/(1-\alpha)}$$

- Hence if managerial talent has Pareto distribution with exponent ξ, then firm-size is also Pareto with exponent $\xi(1-\alpha)$

Managerial selection

- Suppose individual of talent x can opt for wage w or manage and earn income $\pi(x)$
- Indifference condition

$$\pi(x) = xg[f(\kappa)n(x)] - r\kappa n(x) - wn(x) = w$$

- Cutoff managerial type x^* such that only $x > x^*$ actively manage

$$x^* g[f(\kappa)n(x^*)] = w + (r\kappa n(x^*) + wn(x^*))$$

i.e., fixed cost w (opportunity cost of manager) plus variable costs

Next

- Firm dynamics: basic models, part two
  - Hopenhayn (1992): Entry, exit and firm dynamics in long run equilibrium, *Econometrica*.