Monetary Economics: Problem Set #6

Solutions

This problem set is marked out of 100 points. The weight given to each part is indicated below. Please contact me asap if you have any questions.

1. **Optimal insurance and deposit contracts**. Consider the Diamond-Dybvig model. There are three dates $\{0, 1, 2\}$ and a unit mass of ex ante identical investors and a single bank. Each of the investors has an endowment of 1 to invest at date $T = 0$. The type of each investor is revealed at date $T = 1$. A fraction $t = 0.20$ are *impatient* and consume only at $T = 1$. The remaining fraction are *patient* and indifferent between consuming at either $T = 1$ or $T = 2$. An individual's realised type is her own private information.

   Funds invested for *two* periods earn a gross return $R = 1.65$ (an *illiquid project*). Funds invested for only one period earn a gross return of 1 (i.e., the investor just gets their funds back).

   Each investor has the CRRA utility function

   $$U(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

   with coefficient of relative risk aversion $\gamma = 3$.

   (a) Set up the optimisation problem the solution of which gives the efficient amount of risk-sharing (optimal insurance) between impatient and patient investors. (10 points)

   (b) Using the numerical values given, solve the optimisation problem for the payments (c_1^*, c_2^*) to impatient and patient investors. (15 points)

   (c) Explain how the optimal insurance scheme can be implemented by a liquid deposit contract with the bank that pays returns (r_1, r_2) on dates $T = 1$ and $T = 2$ respectively. What values would the returns (r_1, r_2) have to be? (15 points)

   (d) Calculate the ex ante expected utility to an investor who enters into this deposit contract. Is this higher or lower than the ex ante expected utility of an investor who just invests and holds the illiquid asset? Explain. How would your answer change (if at all) if the investors were risk neutral (e.g., $U(c) = c$)? Explain. (15 points)

   (e) Explain the *sequential service constraint* facing the bank if it offers deposit contracts. Explain why the bank is prone to a run. If the return on the deposit contract paid in the first period r_1 is the value calculated in part (c), what is the maximum number of withdrawals f^* beyond which any individual patient investor will find it optimal to withdraw? [the "*tipping point*"] (15 points)

SOLUTIONS:

(a) The optimization problem is to choose c_1, c_2 both nonnegative to maximize

$$tU(c_1) + (1-t)U(c_2)$$

subject to the resource constraint

$$tc_1 + (1-t)\frac{c_2}{R} \leq 1$$

and the incentive constraint

$$U(c_1) \leq U(c_2)$$

(b) Guessing that the incentive constraint is slack, the first order condition for the problem is

$$U'(c_1) = U'(c_2)R$$

Since marginal utility is of the form $U'(c) = c^{-\gamma}$ for $\gamma > 0$ (in fact $\gamma = 3$ but we'll get to that in a minute) we can write the first order condition as

$$c_1^{-\gamma} = c_2^{-\gamma}R \qquad \Leftrightarrow \qquad c_2 = R^{\frac{1}{\gamma}}c_1 > c_1$$

since $R > 1$ and $\gamma > 0$. Therefore $U(c_2) > U(c_1)$ and the incentive constraint is indeed slack. Combining this with the resource constraint gives

$$tc_1 + (1-t)R^{\frac{1}{\gamma}-1}c_1 = 1$$

and solving for c_1 gives

$$c_1^* = \frac{1}{t + (1-t)R^{\frac{1-\gamma}{\gamma}}}$$

and therefore

$$c_2^* = \frac{R^{\frac{1}{\gamma}}}{t + (1-t)R^{\frac{1-\gamma}{\gamma}}}$$

Plugging in $t = 0.20$, $R = 1.65$ and $\gamma = 3$ then gives

$$c_1^* = \frac{1}{0.20 + (1-0.20)(1.65)^{-2/3}} = 1.2938 > 1$$

and

$$c_2^* = \frac{1.65^{1/3}}{0.20 + (1-20)(1.65)^{-2/3}} = 1.5288 < R = 1.65$$

(c) For the deposit contract, we take 1 from all investors and pay r_1 to early withdrawals and r_2 to investors who keep leave their deposits in place for two periods. In the meantime, the bank takes the deposits and uses them for the project that delivers R per unit but only if funds are in place for two periods. If the fraction of early withdrawals f just equals the fraction of patient types $f = t$ then we can implement the optimal insurance arrangement by setting $r_1 = c_1^*$ and $r_2 = c_2^*$ as given in part (b) above. That is, $r_1 = 1.2938 > 1$ and $r_2 = 1.5288 < R = 1.65$.

(d) An investor in autarky (who invests and holds the illiquid project but who has to pull funds out at date 1 if they're unlucky and turn out to be impatient) has consumption $c_1 = 1$ and $c_2 = R = 1.8$ and so their expected utility is

$$EU_{autarky} = 0.20U(1) + 0.80U(1.65) = -0.2469$$

using $U(c) = c^{-2}/(-2)$. But an investor who has the deposit contract (with $f = t$) has expected utility

$$EU_{deposit} = 0.20U(1.2938) + 0.80U(1.5288) = -0.2309$$

and so they prefer the deposit arrangement, at least if only the patient types withdraw early. By contrast, a risk neutral investor with $U(c) = c$ would have

$$EU_{autarky} = 0.20 \times 1 + 0.80 \times 1.65 = 1.52$$

while for the deposit contract

$$EU_{deposit} = 0.20 \times 1.2938 + 0.80 \times 1.5288 = 1.4818$$

and so the risk neutral investor prefers autarky (the reduction in return for being patient is too big, it is after all partly to provide insurance to risk averse people and the risk neutral investor doesn't value that). Don't make the mistake of comparing the level of expected utility of the risk averse investor to that of the risk neutral investor (e.g., -0.2469 to 1.52). We can take arbitrary positive monotone increasing transformations of the underlying utility function $U(c)$ (e.g., adding positive constants, multiplying by positive numbers etc) without affecting an individual's rank ordering of outcomes, so interpersonal comparisons of utility levels are not informative.

(e) The sequential service constraint requires that individuals trying to withdraw get paid out depending only on their place in the queue for deposits (so a patient type who arrives before an impatient type gets paid first even though the impatient type has greater need). If the fraction who withdraw early is $f \geq t$ (at least all impatient types withdraw early), then the sequential service constraint can be written

$$r_2(f) = \max\left[0,\ R\frac{1 - fr_1}{1 - f}\right]$$

After early withdrawals there is $1 - fr_1$ remaining in the deposit accounts (or nothing if f is too high). Supposing there's anything left, these funds earn $R(1 - fr_1)$ in total after the second period and this has to be divided amongst the remaining $1 - f$ investors. What level of f is too high (the tipping point)? Well, investors withdraw if $r_2(f) \leq r_1$ or equivalently, after rearranging the equation above, if

$$f \geq f^* \equiv \frac{1}{r_1}\left(\frac{R - r_1}{R - 1}\right)$$

Plugging in the values we have

$$f^* = \frac{1}{1.2938}\left(\frac{1.65 - 1.2938}{1.65 - 1}\right) = 0.4236$$

Any $f > 0.4236$ makes it optimal for an investor to withdraw. Note: that the fraction of individuals that withdraw in a a *pure-strategy Nash equilibrium* are either $f = 0.25$ (only impatient types withdraw) or $f = 1.00$ (all withdraw).

2. **Leverage and balance sheet management**. Consider a bank with an initial balance sheet of:

Assets	Liabilities
Securities 200	Debt 180
	Equity 20

   (a) What is the bank's leverage ratio? Suppose the bank now decides to target a leverage ratio of 15, explain how the bank can expand or contract its balance sheet (as required) to meet this target [*Hint*: assume that the price of debt does not change]. Is this likely to put upwards or downwards pressure on the price of securities? Explain. (15 points)

   (b) Suppose the bank is now operating with a leverage ratio of 15 and the balance sheet calculated in part (a) but that subprime losses mean that the value of its securities are marked down by 5%. Explain how that bank will respond and how its balance sheet will change if it continues to target a leverage ratio of 15? What if it now decides that this is a good time to "de-lever" and to instead have a leverage ratio of 12? Is deleveraging likely to amplify or mitigate the effects of the subprime losses on securities prices? (15 points)

SOLUTIONS:

(a) The bank's leverage ratio is the value of its assets (the securities) divided by its equity, i.e., $200/20 = 10$. If the bank decides to target a leverage ratio of 15 it would have securities of $(20)(15) = 300$. Its balance sheet would then read:

Assets	Liabilities
Securities 300	Debt 280
	Equity 20

To expand the balance sheet in this way, the bank would issue 100 more debt (that is, $280 - 180$) and use the funds raised in this way to buy more securities. Other things equal, this extra demand for securities would drive up the price of securities.

(b) If the value of its securities are marked down by 5%, then the value of securities is now $(300)(0.95) = 285$. The bank's liabilities are still 280, however, so its equity falls from 20 to 5. That is, if the bank was passive its balance sheet would be:

Assets	Liabilities
Securities 285	Debt 280
	Equity 5

This corresponds to a leverage ratio of $285/5 = 57$. If the bank is not passive and tries instead to reestablish a target leverage ratio of 15, then it will contract its balance sheet to:

Assets	Liabilities
Securities 75	Debt 70
	Equity 5

That is, the bank will sell $285 - 75 = 210$ units of securities and use the funds raised to retire 210 units of debt, leaving it with securities worth 75 and debt of 70. Other things equal, the bank's desire to sell securities (increasing supply) would drive down the price of securities yet further — i.e., would amplify the fall in securities prices.

Moreover, if the bank decides now is a good time to de-lever and run a lower leverage ratio of 12 instead of 15 then it will contract its balance sheet even further, to:

Assets	Liabilities
Securities 60	Debt 55
	Equity 5

That is, selling even more securities and using the funds to retire even more debt. This would further amplify the downwards pressure on securities prices.