Monetary Economics: Problem Set #4

Due Thursday September 25th in class

This problem set is marked out of 100 points. The weight given to each part is indicated below. Please contact me asap if you have any questions.

1. **Government purchases in the new Keynesian model.** Consider a basic new Keynesian model with the following (log-linearised) equilibrium conditions: for consumption, a dynamic Euler equation

$$c_t = -\frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho) + \mathbb{E}_t[c_{t+1}]$$

for labor supply

$$w_t - p_t = \sigma c_t + \varphi n_t$$

the production function for firms

$$y_t = n_t$$

If prices were fully flexible, firms would set a constant markup over marginal cost. With sticky prices, there is a new Keynesian Phillips curve in terms of the output gap

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa \tilde{y}_t$$

The government purchases a fraction τ_t of output each period with τ_t varying exogenously.

   (a) Derive a log-linear version of the goods market clearing equation of the form $y_t = c_t + g_t$ where $g_t \equiv -\log(1 - \tau_t)$. (5 points)

   (b) Show that natural output is proportional to government purchases, $y_t^n = \Gamma g_t$, and give an explicit formula for the coefficient Γ. Does a fiscal expansion increase or decrease natural output when prices are fully flexible? By how much? Explain. (10 points)

Now assume that monetary policy is set according to the simple interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t$$

and that government purchases g_t follow an AR(1) process with persistence $0 \leq \rho_g < 1$.

   (c) Show that the output gap $\tilde{y}_t$ satisfies a dynamic IS curve of the form

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) + \mathbb{E}_t[\tilde{y}_{t+1}]$$

   and derive a formula for the natural real rate r_t^n in terms of g_t. (5 points)

   (d) Use the method of undetermined coefficients to solve for the response of the key endogenous variables —output, natural output, employment, inflation, interest rates— to an exogenous increase in government purchases g_t. Give economic intuition for your answers. (20 points)

(e) Explain how the response of output to government purchases depends on the monetary policy coefficient ϕ_π. Does a higher ϕ_π increase or decrease the impact effect of government purchases on output? Similarly, explain how the response of output to government purchases depends on the AR(1) persistence ρ_g. Does a more persistent process increase or decrease the impact effect of government purchases on output? Give economic intuition for your answers. (10 points)

2. **Multipliers, the ZLB, and the duration of fiscal stimulus.** Consider a new Keynesian model with government purchases g_t and shocks Δ_t to the interest rate facing households

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t + \Delta_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) + \mathbb{E}_t[\tilde{y}_{t+1}]$$

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa \tilde{y}_t$$

Monetary policy is set according to an interest rate rule but also faces a *zero lower bound*

$$i_t = \max[0,\, \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t]$$

The natural real rate and natural output are given by

$$r_t^n = \rho - \sigma(1-\Gamma)\mathbb{E}_t[\Delta g_{t+1}], \qquad y_t^n = \Gamma g_t$$

The interest spread shock Δ_t can take on two values Δ_L, Δ_H with $\Delta_H = 0$ and $\Delta_L > 0$. The economy starts in the L state. With probability α it stays in the L state. With probability $1-\alpha$ it transitions to the H state. Once it enters the H ("normal") state it stays there forever. Suppose that $g_H = 0$. We are interested in calculating economic outcomes as a function of fiscal policy g_L in the L ("crisis") state.

(a) To begin with, suppose that $g_L = 0$. Solve for the equilibrium values of inflation, the output gap, and the nominal interest rate in the L state. (5 points)

(b) Now explain how an increase in government purchases to some $g_L > 0$ would affect inflation, expected inflation, output and the nominal and real interest rates in the L state. Explain how your answers depend on the size of the interest spread Δ_L. How large is the government purchases multiplier? Does this value depend on the the size of the fiscal stimulus? (15 points)

Now consider the possibility that government purchases g_L persist at an elevated level after the crisis has abated. To be specific, imagine that there are *three* states, L, S, H. The economy starts in the L state with $\Delta_L > 0$. Suppose the initial interest spread Δ_L is sufficiently high that the ZLB is binding in the L state. With probability α the economy transitions to the S state. In the S ("transitional") state, the crisis is over $\Delta_S = \Delta_H = 0$. In the S state, with probability λ the fiscal stimulus continues with government purchases $g_S = g_L > 0$. With probability $1-\lambda$ the fiscal stimulus ends with $g_S = g_H = 0$.

(c) Let $\pi_S, \tilde{y}_S, i_S$ denote the equilibrium values of inflation, the output gap and the nominal interest rate in the S state. Following the same logic as in part (a), solve for these equilibrium values as a function of the size of the fiscal stimulus $g_L > 0$. Explain how your answers depend on whether the fiscal stimulus continues $g_S = g_L$, or not $g_S = g_H$. (15 points)

(d) Using your results from part (c), now solve for the equilibrium values of inflation, the output gap and the nominal interest rate in the initial L state given that with probability λ the fiscal stimulus continues after the crisis has abated. How does the government purchases multiplier compare to the one you found in part (b)? How does the multiplier vary with the probability of the stimulus continuing? Explain. (15 points)

[*Hint*: this question is based on Woodford's article "Simple Analytics of the Government Expenditure Multiplier" *American Economic Journal: Macroeconomics.* **3**(1): 1–35, especially Section IV.B]