Monetary Economics: Problem Set #3

Due Thursday September 11th in class

This problem set is marked out of 100 points. The weight given to each part is indicated below. Please contact me asap if you have any questions.

1. **Policy tradeoffs in the new Keynesian model.** Consider a new Keynesian model with output gap and inflation given by

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho\right) + \mathbb{E}_t[\tilde{y}_{t+1}] \tag{1}$$

and

$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa \tilde{y}_t + x_t \tag{2}$$

where $\{x_t\}$ is an exogenous shock. Monetary policy is given by the interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

where $\{v_t\}$ is an exogenous monetary policy shock and is independent of $\{x_t\}$.

   (a) Explain in words the economic interpretation of equations (1) and (2). How could you interpret the x_t shock? (10 points)

To simplify the algebra, assume for the rest of this question that $\sigma = 1$ and that $\{x_t\}$ and $\{v_t\}$ are both IID white noise.

   (b) Solve for equilibrium inflation π_t and the output gap $\tilde{y}_t$ in terms of the shocks v_t and x_t. (20 points)

   (c) Explain how inflation, interest rates and the output gap respond to the v_t and x_t shocks. Give economic intuition for all your answers. (10 points)

   (d) Suppose the central bank chooses the feedback coefficient ϕ_π to minimise the loss function

$$L = \text{Var}[\pi_t] + \text{Var}[\tilde{y}_t] \tag{3}$$

   Solve for the value of ϕ_π that minimises this loss function. Is there a policy "trade-off" here? Explain how your answer depends on the parameter κ and on the variances of the shocks, $\text{Var}[v_t]$ and $\text{Var}[x_t]$. Give economic intuition for your answers. (15 points)

   (e) Does the value of ϕ_π that minimises the loss function (3) satisfy the Taylor principle? Why or why not? (5 points)

2. **Interest rate versus money supply rules.** Consider an economy again described by the equilibrium conditions

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - \mathbb{E}_t[\pi_{t+1}] - r_t^n) + \mathbb{E}_t[\tilde{y}_{t+1}]$$
$$\pi_t = \beta \mathbb{E}_t[\pi_{t+1}] + \kappa \tilde{y}$$

and now also a money demand equation of the form

$$m_t - p_t = y_t - \eta i_t, \qquad \eta > 0$$

where all variables are defined as usual. Both y_t^n and r_t^n evolve exogenously, independent of monetary policy. The central bank seeks to minimize a loss function of the form

$$L = \text{Var}[\pi_t] + \text{Var}[\tilde{y}_t]$$

(a) Explain how the optimal monetary policy outcomes can be implemented by an interest rate feedback rule. (10 points)

(b) Show that a constant money supply will generally not be optimal. (15 points)

(c) Derive a money supply rule that *would* implement the optimal monetary policy. (15 points)