Monetary Economics: Problem Set #1

Due Thursday August 7th in class

This problem set is marked out of 100 points. The weight given to each part is indicated below. Please contact me asap if you have any questions.

1. **A classical real economy.** A representative household maximises utility

$$U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}, \qquad \sigma, \varphi > 0$$

subject to the budget constraint

$$PC \leq WN$$

A representative perfectly competitive firm has the linear production function

$$Y = AN$$

   (a) Solve for the equilibrium levels of consumption C, labor N, output Y and the real wage W/P in terms of productivity A and the other parameters. (20 points)

   (b) Suppose log productivity $a = \log A$ fluctuates randomly with variance $\text{Var}\{a\} = 1$. Calculate the variances of log consumption c, log labor n, log output y, and the log real wage $w - p$. Which of these variables is more volatile than productivity? Which is less volatile? Which of these variables is positively correlated with productivity? Which is negatively correlated? How do your answers depend on the preference and parameters σ and φ? Give economic intuition for all your answers. (20 points)

2. **Real interest rates in the classical model.** Consider a classical model with the following (log-linearised) household optimality conditions:

$$c_t = -\frac{1}{\sigma}(r_t - \rho) + \mathbb{E}_t[c_{t+1}], \qquad \rho, \sigma > 0 \tag{1}$$

and

$$\sigma c_t + \varphi n_t = w_t - p_t, \qquad \varphi > 0 \tag{2}$$

Perfectly competitive firms choose labor demand to maximise profits subject to the (log-linear) production function

$$y_t = a_t + n_t$$

where $\{a_t\}$ is log productivity which follows an AR(1) process

$$a_{t+1} = \rho_a a_t + \epsilon_{t+1}, \qquad 0 \leq \rho_a < 1$$

where $\{\epsilon_t\}$ is an IID white noise shock.

(a) Explain in words the economic interpretation of equations (1) and (2). (15 points)

(b) Solve for the equilibrium levels of (log) consumption c_t, employment n_t and output y_t in terms of productivity a_t and the exogenous parameters. Briefly explain the effects of a positive productivity shock on each of these endogenous variables. Give intuition for all your answers. (15 points)

(c) Solve for the equilibrium real interest rate r_t in terms of the productivity process and other parameters. Does an increase in productivity increase or decrease the real interest rate? Does a higher value of σ increase or decrease the sensitivity of the real interest rate to a productivity shock? Explain. (15 points)

(d) Suppose the productivity process is instead a random walk with drift $\gamma > 0$

$$a_{t+1} = \gamma + a_t + \epsilon_{t+1}$$

Does an increase in productivity increase or decrease the real interest rate? Does a higher value of σ increase or decrease the sensitivity of the real interest rate to a productivity shock? Explain the differences, if any, between your answers for parts (c) and (d). (15 points)