Monetary Economics

Lecture 7: the basic new Keynesian model, part four

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This class

- Equilibrium dynamics, response to shocks
- Reading: Gali (2008), chapter 3 section 3.4

This class

1- Solving the new Keynesian model
(*method of undetermined coefficients*)

2- Equilibrium response to a monetary policy shock

3- Equilibrium response to a productivity shock

New Keynesian model

- New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} + \kappa \tilde{y}_t, \qquad \tilde{y}_t \equiv y_t - y_t^n$$

- Dynamic IS curve

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n\right) + \mathbb{E}_t\{\tilde{y}_{t+1}\}$$

- Monetary policy rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

- Natural output, natural real rate, shocks

$$y_t^n = \psi_{ya}^n a_t, \qquad r_t^n = \rho + \sigma \psi_{ya}^n \mathbb{E}_t\{\Delta a_{t+1}\}, \qquad \text{shocks } \{v_t, a_t\}$$

Solving the dynamic equations

- Linear rational expectations model

- No endogenous state variable (i.e., no capital etc)

- Guess and verify solutions are linear in exogenous shocks

$$\tilde{y}_t = \psi_{yv} v_t + \psi_{ya} a_t$$

and

$$\pi_t = \psi_{\pi v} v_t + \psi_{\pi a} a_t$$

for some $\psi_{yv}, \psi_{ya}, \psi_{\pi v}, \psi_{\pi a}$ to be determined in equilibrium

- *Method of undetermined coefficients*

Shocks and expectations

- Independent AR(1) processes for policy and productivity shocks

$$v_{t+1} = \rho_v v_t + \epsilon^v_{t+1}, \qquad \epsilon^v_{t+1} \sim \text{ IID and } N(0, \sigma^2_{\epsilon v})$$

and

$$a_{t+1} = \rho_a a_t + \epsilon^a_{t+1}, \qquad \epsilon^a_{t+1} \sim \text{ IID and } N(0, \sigma^2_{\epsilon a})$$

where $0 \leq \rho_v < 1$ and $0 \leq \rho_a < 1$

⇒ conditional expectations in terms of unknown coefficients

$$\mathbb{E}_t \{\tilde{y}_{t+1}\} = \psi_{yv}\rho_v v_t + \psi_{ya}\rho_a a_t$$

and

$$\mathbb{E}_t \{\pi_{t+1}\} = \psi_{\pi v}\rho_v v_t + \psi_{\pi a}\rho_a a_t$$

- Natural rate determined by expected productivity growth

$$r^n_t - \rho = \sigma\psi^n_{ya}\mathbb{E}_t\{\Delta a_{t+1}\} = -\sigma\psi^n_{ya}(1-\rho_a)a_t \equiv -\psi^n_{ra}a_t$$

Solve for unknown coefficients

- Substitute for endogenous variables in equilibrium conditions

- Dynamic aggregate supply equation

$$\psi_{\pi v}v_t + \psi_{\pi a}a_t = \beta(\psi_{\pi v}\rho_v v_t + \psi_{\pi a}\rho_a a_t) + \kappa(\psi_{yv}v_t + \psi_{ya}a_t)$$

Collecting terms

$$0 = (\beta\psi_{\pi v}\rho_v + \kappa\psi_{yv} - \psi_{\pi v})v_t + (\beta\psi_{\pi a}\rho_a + \kappa\psi_{ya} - \psi_{\pi a})a_t$$

In equilibrium, has to hold *for any realization of shocks* v_t, a_t

- Therefore

$$\beta\psi_{\pi v}\rho_v + \kappa\psi_{yv} - \psi_{\pi v} = 0$$

and

$$\beta\psi_{\pi a}\rho_a + \kappa\psi_{ya} - \psi_{\pi a} = 0$$

- Two linear equations in four unknown coefficients $\psi_{yv}, \psi_{ya}, \psi_{\pi v}, \psi_{\pi a}$

Solve for unknown coefficients

- Substitute policy rule and natural rate into dynamic IS equation

$$\tilde{y}_t = -\frac{1}{\sigma}\left(\phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t - \mathbb{E}_t\{\pi_{t+1}\} + \psi^n_{ra} a_t\right) + \mathbb{E}_t\{\tilde{y}_{t+1}\}$$

Again substituting for endogenous variables and collecting terms

$$\begin{aligned} 0 \quad = \quad & [1 + (\phi_\pi - \rho_v)\psi_{\pi v} + (\sigma(1-\rho_v) + \phi_y)\psi_{yv}]v_t \\ & + [\psi^n_{ra} + (\phi_\pi - \rho_a)\psi_{\pi a} + (\sigma(1-\rho_a) + \phi_y)\psi_{ya}]a_t \end{aligned}$$

- Therefore

$$1 + (\phi_\pi - \rho_v)\psi_{\pi v} + (\sigma(1-\rho_v) + \phi_y)\psi_{yv} = 0$$

and

$$\psi^n_{ra} + (\phi_\pi - \rho_a)\psi_{\pi a} + (\sigma(1-\rho_a) + \phi_y)\psi_{ya} = 0$$

- Another two linear equations in the four unknown coefficients

Equilibrium coefficients

- Solve for response to monetary policy shock

$$\psi_{\pi v} = -\frac{\kappa}{(1-\beta\rho_v)(\sigma(1-\rho_v)+\phi_y)+\kappa(\phi_\pi-\rho_v)} \equiv -\kappa\Lambda_v$$

and

$$\psi_{yv} = -(1-\beta\rho_v)\Lambda_v$$

- Symmetrically for productivity shocks

$$\psi_{\pi a} = -\frac{\psi_{ra}^n\kappa}{(1-\beta\rho_a)(\sigma(1-\rho_a)+\phi_y)+\kappa(\phi_\pi-\rho_a)} \equiv -\kappa\Lambda_a$$

and

$$\psi_{ya} = -(1-\beta\rho_a)\Lambda_a$$

- Note $\phi_\pi > \rho_v$ implies $\Lambda_v > 0$ and $\phi_\pi > \rho_a$ implies $\Lambda_a > 0$

Equilibrium dynamics

- In short

$$\begin{pmatrix} \pi_t \\ \tilde{y}_t \end{pmatrix} = \begin{pmatrix} \psi_{\pi v} & \psi_{\pi a} \\ \psi_{yv} & \psi_{ya} \end{pmatrix} \begin{pmatrix} v_t \\ a_t \end{pmatrix}$$

- Bivariate vector autoregression for shocks

$$\begin{pmatrix} v_{t+1} \\ a_{t+1} \end{pmatrix} = \begin{pmatrix} \rho_v & 0 \\ 0 & \rho_a \end{pmatrix} \begin{pmatrix} v_t \\ a_t \end{pmatrix} + \begin{pmatrix} \epsilon^v_{t+1} \\ \epsilon^a_{t+1} \end{pmatrix}$$

$\Rightarrow$ equilibrium vector autoregression for endogenous variables

$$\begin{pmatrix} \pi_{t+1} \\ \tilde{y}_{t+1} \end{pmatrix} = \Psi \begin{pmatrix} \rho_v & 0 \\ 0 & \rho_a \end{pmatrix} \Psi^{-1} \begin{pmatrix} \pi_t \\ \tilde{y}_t \end{pmatrix} + \Psi \begin{pmatrix} \epsilon^v_{t+1} \\ \epsilon^a_{t+1} \end{pmatrix}$$

Econometrics

- Residuals from an estimated VAR are not structural

- Residuals are linear combinations of underlying structural shocks

$$= \begin{pmatrix} \psi_{\pi v} & \psi_{\pi a} \\ \psi_{yv} & \psi_{ya} \end{pmatrix} \begin{pmatrix} \epsilon_{t+1}^{v} \\ \epsilon_{t+1}^{a} \end{pmatrix}$$

- Straightforward to calculate moments implied by model (variances, covariances, impulse responses etc)

Recover other variables

- Nominal interest rate

$$i_t = \rho + (\phi_\pi \psi_{\pi v} + \phi_y \psi_{yv} + 1)v_t + (\phi_\pi \psi_{\pi a} + \phi_y \psi_{ya})a_t$$

- Subtract expected inflation to get real interest rate

$$r_t = \rho + ((\phi_\pi - \rho_v)\psi_{\pi v} + \phi_y \psi_{yv} + 1)v_t + ((\phi_\pi - \rho_a)\psi_{\pi a} + \phi_y \psi_{ya})a_t$$

- Output

$$y_t = y_t^n + \tilde{y}_t = \psi_{yv} v_t + (\psi_{ya}^n + \psi_{ya})a_t$$

- Employment

$$n_t = \frac{1}{1-\alpha}(y_t - a_t) = \frac{1}{1-\alpha}(\psi_{yv} v_t + (\psi_{ya}^n + \psi_{ya} - 1)a_t)$$

Monetary policy shock

- For simplicity, set $a_t = 0$. Assume sufficient reactivity $\phi_\pi > \rho_v$

- Inflation falls on impact (in response to contractionary shock)

$$\frac{\partial \pi_t}{\partial v_t} = -\kappa \Lambda_v < 0$$

- Output and output gap fall on impact

$$\frac{\partial y_t}{\partial v_t} = \frac{\partial \tilde{y}_t}{\partial v_t} = -(1 - \beta \rho_v)\Lambda_v < 0$$

- Employment falls on impact

$$\frac{\partial n_t}{\partial v_t} = \frac{1}{1-\alpha} \frac{\partial y_t}{\partial v_t} = -\frac{1 - \beta \rho_v}{1-\alpha} \Lambda_v < 0$$

Monetary policy shock

- Natural rate of interest unaffected

$$\frac{\partial r_t^n}{\partial v_t} = 0$$

- Nominal interest rate is ambiguous

$$\frac{\partial i_t}{\partial v_t} = \phi_\pi \frac{\partial \pi_t}{\partial v_t} + \phi_y \frac{\partial \tilde{y}_t}{\partial v_t} + 1$$

(depends on ρ_v, how persistent shock is)

- Real rate of interest increases

$$\frac{\partial r_t}{\partial v_t} = (\phi_\pi - \rho_v) \frac{\partial \pi_t}{\partial v_t} + \phi_y \frac{\partial \tilde{y}_t}{\partial v_t} + 1 > 0$$

Monetary policy shock

- Money growth on impact is also ambiguous

$$\frac{\partial m_t}{\partial v_t} = \frac{\partial p_t}{\partial v_t} + \frac{\partial y_t}{\partial v_t} - \eta \frac{\partial i_t}{\partial v_t}$$

  (again, depends on ρ_v, how persistent shock is)

- If nominal rate rises in response, surely money growth falls
  - "liquidity effect"

- All variables return to steady state at rate governed by $\rho_v^t \to 0$

Monetary policy shock

Quantitative example

$$\begin{aligned}
\beta &= 0.99 \text{ quarterly} \\
\sigma &= 1 \text{ log utility} \\
\varphi &= 1 \text{ unit Frisch elasticity} \\
\alpha &= 1/3 \text{ labor's share } 2/3 \\
\varepsilon &= 6 \text{ static markup } 20\% \\
\theta &= 2/3 \text{ price stickiness 3 quarters} \\
\eta &= 4 \text{ coefficient of M2 velocity on } i \text{ in quarterly data}
\end{aligned}$$

$$\begin{aligned}
\phi_\pi &= 1.5 \\
\phi_y &= 0.5/4 \text{ quarterly}
\end{aligned}$$

$$\begin{aligned}
\rho_v &= 0.5 \\
\epsilon_0^v &= 0.25 \text{ basis points } (1\% \text{ annualized})
\end{aligned}$$

Policy shock: quantitative example

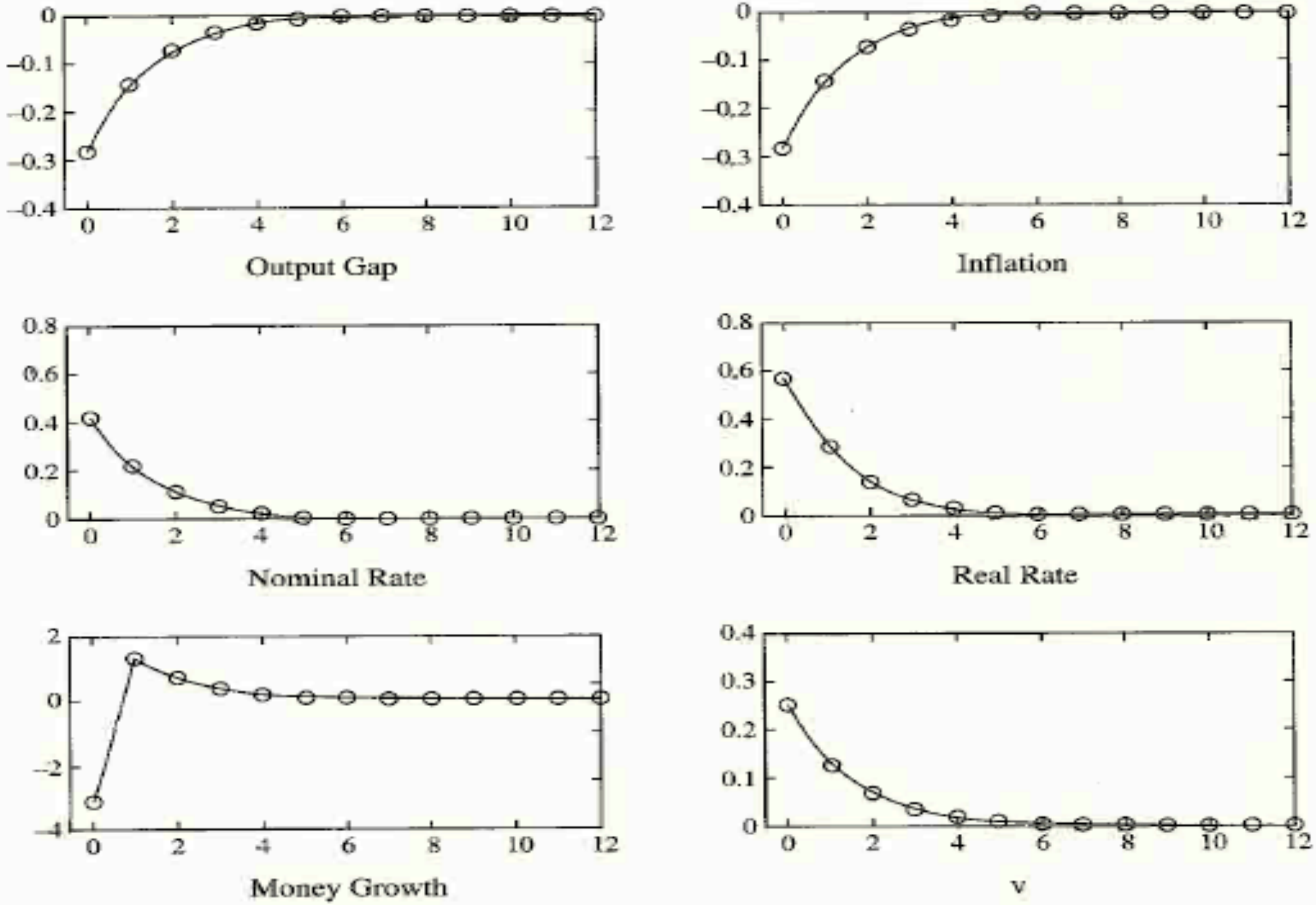

Figure 3.1 Effects of a Monetary Policy Shock (Interest Rate Rule)

Productivity shock

- For simplicity, set $v_t = 0$. Assume sufficient reactivity $\phi_\pi > \rho_a$

- Inflation falls on impact (in response to increase in productivity)

$$\frac{\partial \pi_t}{\partial a_t} = -\kappa \Lambda_a < 0$$

(more slack in the economy)

- Natural rate falls on impact

$$\frac{\partial r_t^n}{\partial a_t} = -\psi_{ra}^n < 0$$

- Output gap falls on impact

$$\frac{\partial \tilde{y}_t}{\partial a_t} = -\psi_{ra}^n (1 - \beta \rho_a) \Lambda_a < 0$$

Productivity shock

- Natural output rises

$$\frac{\partial y_t^n}{\partial a_t} = \psi_{ya}^n > 0$$

- So output response is ambiguous

$$\frac{\partial y_t}{\partial a_t} = \frac{\partial y_t^n}{\partial a_t} + \frac{\partial \tilde{y}_t}{\partial a_t} = \psi_{ya}^n \left[1 - \sigma(1-\rho_a)(1-\beta\rho_a)\Lambda_a\right]$$

- Employment response is similarly ambiguous

$$\frac{\partial n_t}{\partial a_t} = \frac{1}{1-\alpha}\left(\frac{\partial y_t}{\partial a_t} - 1\right)$$

(if $\sigma = 1$, then $\psi_{ya}^n = 1$ and employment *falls*)

Productivity shock

- Policy accommodates productivity shock

$$\frac{\partial i_t}{\partial a_t} = \phi_\pi \frac{\partial \pi_t}{\partial a_t} + \phi_y \frac{\partial \tilde{y}_t}{\partial a_t} < 0$$

- Real rate also falls on impact

$$\frac{\partial i_t}{\partial a_t} = (\phi_\pi - \rho_a) \frac{\partial \pi_t}{\partial a_t} + \phi_y \frac{\partial \tilde{y}_t}{\partial a_t} < 0$$

- Money growth is ambiguous

$$\frac{\partial m_t}{\partial a_t} = \frac{\partial p_t}{\partial a_t} + \frac{\partial y_t}{\partial a_t} - \eta \frac{\partial i_t}{\partial a_t}$$

Productivity shock

Quantitative example

$$\begin{aligned}
\beta &= 0.99 \text{ quarterly} \\
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\end{aligned}$$

$$\begin{aligned}
\phi_\pi &= 1.5 \\
\phi_y &= 0.5/4 \text{ quarterly}
\end{aligned}$$

$$\begin{aligned}
\rho_a &= 0.9 \\
\epsilon_0^a &= 0.25 \text{ basis points } (1\% \text{ annualized})
\end{aligned}$$

Productivity shock: quantitative example

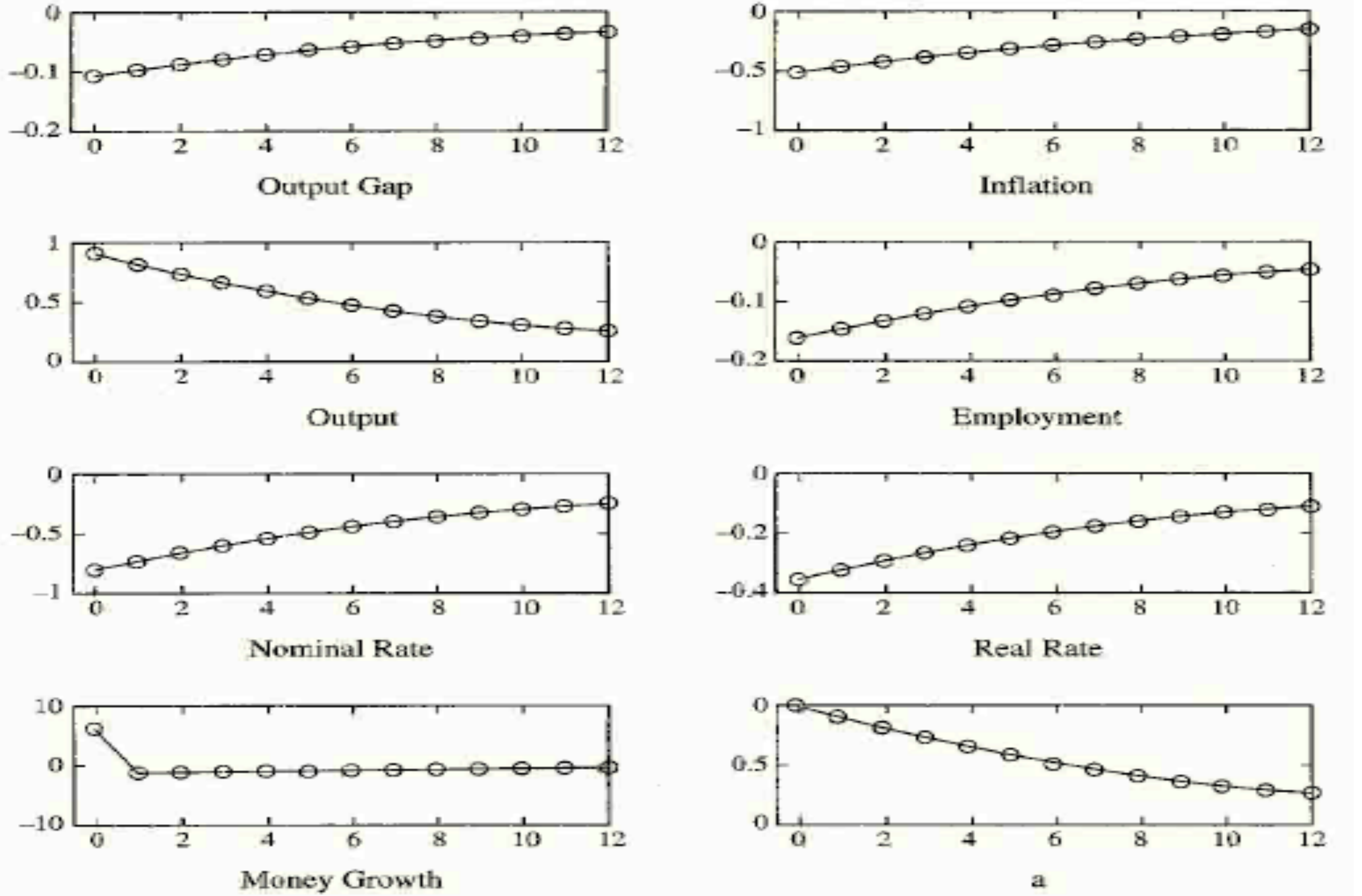

Figure 3.2 Effects of a Technology Shock (Interest Rate Rule)

Next class

- Monetary policy in the new Keynesian model, part one
  - efficient allocations, sources of distortion, optimal policy
- Reading: Gali, chapter 4 section 4.1–4.2