Monetary Economics

Lecture 6: the basic new Keynesian model, part three

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This class

- The new Keynesian Phillips curve
- Reading: Gali, chapter 3 sections 3.2 and appendix 3.2–3.3

Recall: log-linear pricing formulas

- Law of motion for price level

$$\hat{p}_t = \theta \hat{p}_{t-1} + (1-\theta)\hat{p}_t^* \qquad \Leftrightarrow \qquad \pi_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1})$$

- Price chosen by firms that get the opportunity

$$\hat{p}_t^* = (1-\theta\beta)\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k \widehat{\psi}_{t,t+k} \right\}$$

Obtained by approximation around a zero inflation steady state

Can also add back in steady-state terms

Algebra for new Keynesian Phillips curve

Step (i): reset price $\hat{p}_t^*$ in terms of future real marginal cost and future prices

Step (ii): inflation in terms of real marginal cost and expected inflation

$$\pi_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}\Theta\widehat{mc}_t + \beta\mathbb{E}_t\left\{\pi_{t+1}\right\}$$

Step (iii): real marginal cost in terms of output gap $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$

Step (iv): inflation in terms of output gap and expected inflation

$$\pi_t = \kappa\tilde{y}_t + \beta\mathbb{E}_t\left\{\pi_{t+1}\right\}$$

Algebra for new Keynesian Phillips curve

- **Step (i)**: reset price in terms of future real marginal cost and future prices

- Begin with nominal total cost

$$\Psi(Y) = W \left(\frac{Y}{A}\right)^{\frac{1}{1-\alpha}}$$

- Nominal marginal cost

$$\psi \equiv \Psi'(Y) = \frac{1}{1-\alpha} \frac{W}{A} \left(\frac{Y}{A}\right)^{\frac{\alpha}{1-\alpha}}$$

- Real marginal cost in log deviations

$$\widehat{mc} \equiv \widehat{\psi} - \hat{p} = \hat{w} - \hat{p} - \frac{1}{1-\alpha}(\hat{a} - \alpha \hat{y})$$

Real marginal cost

- Real marginal cost in period $t+k$ *given price set at t*

$$\widehat{mc}_{t,t+k} \equiv \widehat{\psi}_{t,t+k} - \hat{p}_{t+k} = \hat{w}_{t+k} - \hat{p}_{t+k} - \frac{1}{1-\alpha}(\hat{a}_{t+k} - \alpha \hat{y}_{t,t+k})$$

- Real marginal cost in period $t+k$ *given price set at $t+k$*

$$\widehat{mc}_{t+k} \equiv \widehat{\psi}_{t+k} - \hat{p}_{t+k} = \hat{w}_{t+k} - \hat{p}_{t+k} - \frac{1}{1-\alpha}(\hat{a}_{t+k} - \alpha \hat{y}_{t+k})$$

so

$$\widehat{mc}_{t,t+k} - \widehat{mc}_{t+k} = \frac{\alpha}{1-\alpha}(\hat{y}_{t,t+k} - \hat{y}_{t+k})$$

- From demand curve

$$\hat{y}_{t,t+k} = -\varepsilon(\hat{p}_t^* - \hat{p}_{t+k}) + \hat{y}_{t+k}$$

- Therefore

$$\widehat{mc}_{t,t+k} = \widehat{mc}_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(\hat{p}_t^* - \hat{p}_{t+k})$$

- Back to formula for reset prices

$$\hat{p}_t^* = (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\widehat{mc}_{t,t+k} + \hat{p}_{t+k}\right]\right\}$$

so

$$\hat{p}_t^* = (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\widehat{mc}_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(\hat{p}_t^* - \hat{p}_{t+k}) + \hat{p}_{t+k}\right]\right\}$$

- Solve for $\widehat{p}_t^*$ to get

$$\hat{p}_t^* = (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k\left[\Theta\widehat{mc}_{t+k}+\hat{p}_{t+k}\right]\right\}$$

where

$$0 < \Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \leq 1$$

This is the end of Step (i).

- **Step (ii)**: inflation in terms of real marginal cost and expected inflation. Begin by breaking up the sum so that

$$\hat{p}_t^* = (1-\theta\beta)\left[\Theta\widehat{mc}_t+\hat{p}_t\right]+(1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=1}^{\infty}(\theta\beta)^k\left[\widehat{mc}_{t+k}+\hat{p}_{t+k}\right]\right\}$$

or

$$\hat{p}_t^* = (1-\theta\beta)\left[\Theta\widehat{mc}_t+\hat{p}_t\right]+\theta\beta\mathbb{E}_t\left\{\hat{p}_{t+1}^*\right\}$$

- Now subtract $\widehat{p}_{t-1}$ from both sides and rearrange

$$\hat{p}_t^* - \hat{p}_{t-1} = (1-\theta\beta)\Theta\widehat{mc}_t + \theta\beta\mathbb{E}_t\left\{\hat{p}_{t+1}^* - \hat{p}_t\right\} + \hat{p}_t - \hat{p}_{t-1}$$

- Recall that the law of motion for the price level implies

$$\pi_t = (1-\theta)(\hat{p}_t^* - \hat{p}_{t-1})$$

- This simplifies to

$$\pi_t = \frac{(1-\theta)(1-\theta\beta)}{\theta}\Theta\widehat{mc}_t + \beta\mathbb{E}_t\left\{\pi_{t+1}\right\}$$

Inflation is discounted sum of future real marginal costs

$$\pi_t = \lambda\mathbb{E}_t\left\{\sum_{k=0}^{\infty}\beta^k\widehat{mc}_{t+k}\right\}, \qquad \lambda \equiv \frac{(1-\theta)(1-\theta\beta)}{\theta}\Theta > 0$$

- This is the end of Step (ii).

Flexible price equilibrium

- **Step (iii)**: real marginal cost in terms of output gap. Begin by solving for flexible price equilibrium to get natural level of output

- Resource constraint

$$C_t = Y_t = A_t N_t^{1-\alpha}$$

- Household labor supply

$$N_t^{\varphi} C_t^{\sigma} = \frac{W_t}{P_t}$$

- Firm price setting (in symmetric equilibrium)

$$P_t = \frac{\varepsilon}{\varepsilon - 1} \left[\frac{1}{1-\alpha} \frac{W_t}{A_t} \left(\frac{Y_t}{A_t} \right)^{\frac{\alpha}{1-\alpha}} \right]$$

equivalently

$$(1-\alpha) A_t N_t^{-\alpha} = \frac{\varepsilon}{\varepsilon - 1} \frac{W_t}{P_t}$$

Flexible price equilibrium

- Reduce this to two equations in two unknowns

$$C_t = A_t N_t^{1-\alpha}$$

and

$$N_t^{\varphi} C_t^{\sigma} = \frac{\varepsilon - 1}{\varepsilon}(1-\alpha) A_t N_t^{-\alpha}$$

- Take logs and solve to get flexible price equilibrium values

$$n_t = \frac{1}{(1-\alpha)\sigma + \alpha + \varphi} \log\left[\frac{\varepsilon - 1}{\varepsilon}(1-\alpha)\right] + \frac{1-\sigma}{(1-\alpha)\sigma + \alpha + \varphi} a_t$$

and

$$y_t = \frac{1}{(1-\alpha)\sigma + \alpha + \varphi} \log\left[\frac{\varepsilon - 1}{\varepsilon}(1-\alpha)\right] + \frac{1+\varphi}{(1-\alpha)\sigma + \alpha + \varphi} a_t$$

(this is the *natural level of output*).

Output and real marginal cost

- Irrespective of pricing behavior, use labor supply condition to write real marginal costs

$$\widehat{mc}_t = \hat{w}_t - \hat{p}_t + \alpha \hat{n}_t - \hat{a}_t$$

$$= \varphi \hat{n}_t + \sigma \hat{c}_t + \alpha \hat{n}_t - \hat{a}_t$$

- Using production function and market clearing

$$\widehat{mc}_t = \frac{(1-\alpha)\sigma + \alpha + \varphi}{1-\alpha} \hat{y}_t - \frac{1+\varphi}{1-\alpha} \hat{a}_t$$

Output gap and real marginal cost

- But natural output in log deviations is

$$\hat{y}_t^n = \frac{1+\varphi}{(1-\alpha)\sigma + \alpha + \varphi}\hat{a}_t$$

- Therefore can write

$$\widehat{mc}_t = \frac{(1-\alpha)\sigma + \alpha + \varphi}{1-\alpha}(\hat{y}_t - \hat{y}_t^n)$$

Real marginal cost is proportional to the output gap. This is the end of Step (iii).

Inflation and output gap

- **Step (iv)**: inflation in terms of output gap and expected inflation
- Just plug the formula above back into the expression relating inflation and real marginal cost on Slide 13 above to get

$$\pi_t = \beta \mathbb{E}_t \left\{ \pi_{t+1} \right\} + \kappa \tilde{y}_t, \qquad \tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$$

where

$$\kappa \equiv \frac{\sigma(1-\alpha)+\alpha+\varphi}{1+\alpha(\varepsilon-1)} \frac{(1-\theta)(1-\theta\beta)}{\theta} > 0$$

Rest of the model

- From the log linearized Euler equation

$$i_t = \rho + \sigma \mathbb{E}_t\{\Delta y_{t+1}\} + \mathbb{E}_t\{\pi_{t+1}\}$$

- Define the *natural rate of interest*

$$r_t^n = \rho + \sigma \mathbb{E}_t\{\Delta y_{t+1}^n\} = \rho + \sigma \psi_{ya} \mathbb{E}_t\{\Delta a_{t+1}\}$$

- Therefore

$$i_t = r_t^n + \sigma \mathbb{E}_t\{\Delta \tilde{y}_{t+1}\} + \mathbb{E}_t\{\pi_{t+1}\}$$

- And so at last, the *dynamic IS curve*

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n\right) + \mathbb{E}_t\{\tilde{y}_{t+1}\}$$

Canonical three equation NK model

- New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} + \kappa \tilde{y}_t, \qquad \tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$$

- Dynamic IS curve or Euler equation

$$\tilde{y}_t = -\frac{1}{\sigma} \left(i_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n \right) + \mathbb{E}_t\{\tilde{y}_{t+1}\}$$

- Monetary policy rule, say

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

Next class

- The basic new Keynesian model, part four
  - equilibrium dynamics, response to policy shocks
- Reading: Gali, chapter 3 section 3.4