Monetary Economics

Lecture 5: the basic new Keynesian model, part two

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This class

- Price setting with sticky prices
- Reading: Gali, chapter 3 sections 3.2 and appendix 3.2–3.3

Calvo price stickiness

- Firms have IID random opportunities to change prices
  - with probability θ, firm keeps current price
  - with probability $1 - \theta$, firm gets to re-optimize price
- Implies average duration of price is $1/(1 - \theta)$ periods
- Discrete time version of Calvo (1983)

Law of motion for price level

- Recall: ideal price index

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon} \, dj \right)^{\frac{1}{1-\varepsilon}}$$

- In *symmetric equilibrium* all firms that get opportunity set same price P_t^*

- Law of motion for price level

$$P_t = \left(\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

- Now need to characterize P_t^*

Expected discounted profits to a firm

- Let $V_{t,t+k}$ be nominal profit in period $t+k$ given price set at t

- Expected discounted nominal profits

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \theta^k \mathcal{Q}_{t,t+k} V_{t,t+k} \right\}$$

where $\mathcal{Q}_{t,t+k}$ is the nominal *stochastic discount factor*

$$\mathcal{Q}_{t,t+k} = \beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}}$$

Expected discounted profits to a firm

- Nominal profit in period $t+k$ given price P_t^* set at t

$$V_{t,t+k} = P_t^* Y_{t,t+k} - \Psi_{t+k}\left(Y_{t,t+k}\right)$$

where

$$Y_{t,t+k} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} Y_{t+k}$$

- Choose single number P_t^* to maximize

$$\mathbb{E}_t\left\{\sum_{k=0}^{\infty}\theta^k \mathcal{Q}_{t,t+k} V_{t,t+k}\right\}$$

- First order condition

$$\mathbb{E}_t\left\{\sum_{k=0}^{\infty}\theta^k \mathcal{Q}_{t,t+k} \frac{\partial V_{t,t+k}}{\partial P_t^*}\right\} = 0$$

Expected discounted profits to a firm

- Calculating the marginal profit in period $t+k$ given P_t^* and simplifying

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k U_{c,t+k} P_{t+k}^{\varepsilon-1} \psi_{t,t+k} Y_{t+k} \right\}}{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k U_{c,t+k} P_{t+k}^{\varepsilon-1} Y_{t+k} \right\}}$$

where $\psi_{t,t+k} \equiv \Psi'_{t+k}\left(Y_{t,t+k}\right)$ is nominal marginal cost

- In general, this is not quite a solution (why?)

Log linearization tricks

- Products

$$\begin{aligned} Z &= X^{\alpha} Y^{\beta} \\ \Rightarrow \hat{z} &= \alpha \hat{x} + \beta \hat{y} \end{aligned}$$

- Sums

$$\begin{aligned} Z &= \alpha X + \beta Y \\ \Rightarrow \bar{Z} \hat{z} &\approx \alpha \bar{X} \hat{x} + \beta \bar{Y} \hat{y} \end{aligned}$$

Log linearization tricks

- Smooth functions

$$
\begin{aligned}
Z &= f(X, Y) \\
\Rightarrow \bar{Z}\hat{z} &\approx f_X(\bar{X}, \bar{Y})\bar{X}\hat{x} + f_Y(\bar{X}, \bar{Y})\bar{Y}\hat{y}
\end{aligned}
$$

Log-linear formulas

- Law of motion for price level

$$P_t = \left(\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

- Log-linear approximation (around zero-inflation steady state)

$$p_t = \theta p_{t-1} + (1-\theta)p_t^*$$

implies

$$\pi_t = p_t - p_{t-1} = (1-\theta)(p_t^* - p_{t-1})$$

Log-linear formulas

- Price setting

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k U_{c,t+k} P_{t+k}^{\varepsilon-1} \psi_{t,t+k} Y_{t+k} \right\}}{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k U_{c,t+k} P_{t+k}^{\varepsilon-1} Y_{t+k} \right\}}$$

- Log-linear approximation (around zero-inflation steady state)

$$p_t^* = \mu + (1 - \theta\beta)\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k [mc_{t,t+k} + p_{t+k}] \right\}$$

with log markup $\mu \equiv \log \mathcal{M}$ and $\mathcal{M} \equiv \varepsilon/(\varepsilon - 1)$ and where $mc_{t,t+k}$ is log *real marginal cost*

Next class

- The basic new Keynesian model, part three
  - the new Keynesian Phillips curve
- Reading: Gali, chapter 3 sections 3.2 and appendix 3.2–3.3

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Appendix to Lecture 5

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Log linearization of the pricing formulas

(1) Law of motion for the price level

$$P_t = \left(\theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

- approximating the law of motion for the price level around a zero inflation steady state (constant price level)

$$\frac{1}{1-\varepsilon}\bar{P}\hat{p}_t \approx \theta\frac{1}{1-\varepsilon}\bar{P}\hat{p}_{t-1} + (1-\theta)\frac{1}{1-\varepsilon}\bar{P}^*\hat{p}_t^*$$

(ignoring the constants, which always cancel from both sides)

- in steady state $\bar{P} = \bar{P}^*$, so cancelling common terms we have

$$\hat{p}_t \approx \theta\hat{p}_{t-1} + (1-\theta)\hat{p}_t^*$$

and since the steady state is the same for all terms, also

$$p_t \approx \theta p_{t-1} + (1-\theta)p_t^*$$

(2) Optimal P_t^* satisfies

$$P_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k U_{c,t+k} P_{t+k}^{\varepsilon-1} \psi_{t,t+k} Y_{t,t+k} \right\}}{\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k U_{c,t+k} P_{t+k}^{\varepsilon-1} Y_{t,t+k} \right\}}$$

- rewrite this as

$$\begin{aligned} & \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k U_{c,t+k} P_{t+k}^{\varepsilon-1} Y_{t,t+k} P_t^* \right\} \\ = \quad & \frac{\varepsilon}{\varepsilon - 1} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k U_{c,t+k} P_{t+k}^{\varepsilon-1} \psi_{t,t+k} Y_{t,t+k} \right\} \end{aligned}$$

- approximating the left hand side gives the terms

$$\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k \bar{U}_c \bar{P}^{\varepsilon-1} \bar{Y} \bar{P}^* \left[\hat{u}_{c,t+k} + (\varepsilon - 1)\hat{p}_{t+k} + \hat{y}_{t,t+k} + \hat{p}_t^* \right] \right\}$$

(steady state values, $\bar{U}_c, \bar{P}$ etc are common to all terms in the sum)

- approximating the right hand side gives the terms

$$\frac{\varepsilon}{\varepsilon - 1} \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k \bar{U}_c \bar{P}^{\varepsilon-1} \bar{\psi} \bar{Y} \left[\hat{u}_{c,t+k} + (\varepsilon - 1)\hat{p}_{t+k} + \hat{\psi}_{t,t+k} + \hat{y}_{t,t+k} \right] \right\}$$

- and in steady state

$$\bar{P}^* = \frac{\varepsilon}{\varepsilon - 1}\bar{\psi}$$

- so we can cancel all the common terms from each side to write

$$\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k \hat{p}_t^*\right\} \approx \mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k \hat{\psi}_{t,t+k}\right\}$$

- pulling the common term $\hat{p}_t^*$ through the sum and expectation

$$\hat{p}_t^* \sum_{k=0}^{\infty}(\theta\beta)^k \approx \mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k \hat{\psi}_{t,t+k}\right\}$$

- calculating the sum we finally have

$$\hat{p}_t^* \approx (1-\theta\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta\beta)^k \hat{\psi}_{t,t+k}\right\}$$

(and add back in steady state terms to get version in lecture notes)