Monetary Economics

Lecture 3: classical building blocks, part two

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This class

- Money demand and price level determination
- Reading: Gali (2008), chapter 2 sections 2.4–2.5

Real variables

- Real variables all independent of nominal variables

- Equilibrium labor (in log deviations)

$$\hat{n}_t = \psi_{na}\,\hat{a}_t$$

- Equilibrium output and consumption

$$\hat{c}_t = \hat{y}_t = \psi_{ya}\,\hat{a}_t$$

- Equilibrium real wage

$$\hat{w}_t - \hat{p}_t = \psi_{wa}\hat{a}_t$$

- Equilibrium real interest rate

$$r_t = \rho + \sigma\psi_{ya}\mathbb{E}_t\{\Delta a_{t+1}\}$$

Nominal variables

- What about nominal variables p_t, π_t, i_t etc?
- To solve for them, need *money demand* and *money supply*

Money-in-the-utility function

- Household preferences now include *real money* M/P

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

- Intertemporal preferences

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right)\right\}, \qquad 0<\beta<1$$

- Flow budget constraint at every date and state

$$P_tC_t + Q_tB_t + M_t \leq B_{t-1} + M_{t-1} + W_tN_t - T_t$$

Household intertemporal optimisation

- Lagrangian with nonnegative, stochastic, multipliers $\{\lambda_t\}$

$$L = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \left[\beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right) \right.\right.$$
$$\left.\left. + \quad \lambda_t \left(B_{t-1} + M_{t-1} + W_t N_t - T_t - P_t C_t - Q_t B_t - M_t\right) \right] \right\}$$

- First order conditions

$$\begin{aligned}
C_t : \quad & \beta^t U_{c,t} = \lambda_t P_t \\
N_t : \quad & -\beta^t U_{n,t} = \lambda_t W_t \\
\\
B_t : \quad & \lambda_t Q_t = \mathbb{E}_t \{\lambda_{t+1}\} \\
M_t : \quad & \lambda_t = \mathbb{E}_t \{\lambda_{t+1}\} + \beta^t U_{m,t} \frac{1}{P_t}
\end{aligned}$$

As usual, these hold at every date and state

Household first order conditions

- Money demand

$$\frac{U_{m,t}}{U_{c,t}} = 1 - Q_t = 1 - \exp(-i_t)$$

- Labor supply

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

- Intertemporal consumption Euler equation

$$Q_t = \mathbb{E}_t \left\{ \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

Standard functional forms

- Separable utility function

$$U\left(C,\,\frac{M}{P},\,N\right)=\frac{C^{1-\sigma}}{1-\sigma}+\frac{(M/P)^{1-\nu}}{1-\nu}-\frac{N^{1+\varphi}}{1+\varphi}$$

- Money demand

$$\frac{U_{m,t}}{U_{c,t}}=\frac{(M_t/P_t)^{-\nu}}{C_t^{-\sigma}}=1-\exp(-i_t)$$

or

$$\frac{M_t}{P_t}=C_t^{\sigma/\nu}\left[1-\exp(-i_t)\right]^{-1/\nu}$$

- Take logs

$$m_t-p_t=\frac{\sigma}{\nu}c_t-\frac{1}{\nu}\log\left[1-\exp(-i_t)\right]$$

Log-linear money demand

- Approximate last term in i_t around some steady-state $\bar{\imath}$ to get

$$m_t - p_t \approx \frac{\sigma}{\nu} c_t - \eta i_t + \text{constant}$$

where $\eta > 0$ is the *interest semi-elasticity* of money demand (a coefficient from the linearisation, depends on $\nu, \bar{\imath}$)

- Treating this as exact and writing it in log deviations

$$\hat{m}_t - \hat{p}_t = \frac{\sigma}{\nu} \hat{c}_t - \eta i_t$$

- Money supply: central bank sets $\hat{m}_t$, then price level $\hat{p}_t$ and interest rate i_t determined endogenously ($\hat{c}_t$ already determined)

Price level determination

- Fisher equation

$$i_t = r_t + \mathbb{E}_t\{\pi_{t+1}\}$$

- Money demand

$$\hat{m}_t - \hat{p}_t = \frac{\sigma}{\nu}\hat{c}_t - \eta i_t$$

with $\{\hat{m}_t\}$ process given exogenously and real variables independent of nominal

- For simplicity, set real variables to steady state

- *Stochastic difference equation* in endogenous process $\{\hat{p}_t\}$

$$\hat{p}_t = \frac{\eta}{1+\eta}\mathbb{E}_t\{\hat{p}_{t+1}\} + \frac{1}{1+\eta}\hat{m}_t$$

Equilibrium price level

- Solve the stochastic difference equation by iterating forward

$$\hat{p}_t = \frac{1}{1+\eta}\mathbb{E}_t\left\{\sum_{k=0}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \hat{m}_{t+k}\right\}$$

- We typically work with a process for *money growth*

$$\Delta\hat{m}_t \equiv \hat{m}_t - \hat{m}_{t-1}$$

- So rewrite price level in terms of money growth. Key steps

$$(1+\eta)\hat{p}_t = \hat{m}_t + \mathbb{E}_t\left\{\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \left[\Delta\hat{m}_{t+k} + \hat{m}_{t+k-1}\right]\right\}$$

$$\Rightarrow$$

$$\hat{p}_t = \hat{m}_t + \mathbb{E}_t\left\{\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^k \Delta\hat{m}_{t+k}\right\}$$

Equilibrium price level

- AR(1) money growth example

$$\Delta \hat{m}_{t+1} = \phi_m \, \Delta \hat{m}_t + \varepsilon_{m,t+1} \quad \varepsilon_{m,t+1} \sim \text{IID and } N(0, \sigma_\varepsilon^2)$$

with persistence $0 < \phi_m < 1$

- Impulse response function with AR(1) money growth

$$\mathbb{E}_t\{\Delta \hat{m}_{t+k}\} = \phi_m^k \Delta \hat{m}_t, \qquad k \geq 1$$

- So price level evaluates to

$$\hat{p}_t = \hat{m}_t + \eta \frac{\phi_m}{1 + \eta(1 - \phi_m)} \Delta \hat{m}_t$$

responds more than one-for-one to money shocks on impact

- With solution for $\hat{p}_t$, can now determine other nominal variables

Inflation

- Implies inflation

$$\pi_t = \Delta \hat{p}_t = \frac{1+\eta}{1+\eta(1-\phi_m)} \Delta \hat{m}_t - \frac{\eta \phi_m}{1+\eta(1-\phi_m)} \Delta \hat{m}_{t-1}$$

- Note that in ‘steady-state’ $\Delta \hat{m}_t = \Delta \hat{m}_{t-1}$ so

$$\pi_t = \Delta \hat{m}_t$$

- Long-run inflation rate determined by long-run money growth rate

Nominal interest rates

- Can calculate from either Fisher equation or money demand

- Easier from money demand

$$i_t = \frac{\hat{p}_t - \hat{m}_t}{\eta} = \frac{\phi_m}{1 + \eta(1 - \phi_m)} \Delta \hat{m}_t$$

- Nominal interest rates and money growth positively correlated (i.e., no *liquidity effect*)

- Long-run nominal interest rate likewise determined by long-run money growth

Velocity

- Classical *exchange equation*, implicitly defines *velocity*

$$MV = PC$$

- For this example, log velocity is just

$$\hat{v}_t = \hat{p}_t - \hat{m}_t = \eta i_t = \frac{\eta \phi_m}{1 + \eta(1 - \phi_m)} \Delta \hat{m}_t$$

- Moves with interest rates, positively correlated with money growth

- Long-run velocity again determined by long-run money growth

Next class

- The basic new Keynesian model, part one
  - imperfect competition and price setting
- Reading: Gali (2008), chapter 3 sections 3.0–3.1 and appendix 3.1