This lecture

• Main reading:
  ◇ Holmström and Tirole, *Inside and outside liquidity*, MIT Press. Chapter 1

• Further reading:
  ◇ Tirole “Lliquidity and all its friends” *Journal of Economic Literature*, 2011

Available from the LMS
Holmström-Tirole overview

*Liquidity*: availability of assets for intertemporal smoothing

**Q.** Is the private supply of liquid assets socially optimal?

**A.** Private supply sufficient to achieve the socially optimal (second best) outcome if *no aggregate risk*

Implementation of this requires financial intermediaries that can *pool idiosyncratic risk*

**Q.** Is there a role for government intervention?

**A.** Yes, especially if there is aggregate risk or impediments to intermediation
Today: basic concepts

1- Credit rationing with fixed investment scale

2- Moral hazard and the wedge between value and pledgeable income

3- Variable investment scale
Credit rationing with fixed investment scale

- Risk neutral entrepreneur with investment opportunity
- Opportunity worth $Z_1$ to entrepreneur but $Z_0 < Z_1$ to investors
- Initial investment $I$ required to implement project
  \[ Z_0 < I < Z_1 \]
  Positive net present value $I < Z_1$, but not self-financing, $Z_0 < I$
- Shortfall $I - Z_0$ must be covered by entrepreneur
- *Entrepreneurial rent* $Z_1 - Z_0$ cannot be pledged to investors (e.g., because private benefits, different beliefs, non-transferability)
Limited pledgeability

Value of project to entrepreneurs is $Z_1$. Value to investors is $Z_0$. Entrepreneurial rent $Z_1 - Z_0$. Investment $I$ required to implement project. Shortfall $I - Z_0$. 
Credit rationing with fixed investment scale

• Let $A > 0$ be entrepreneurial capital committed to project

• Project can proceed if and only if pledgeable income $Z_0$ exceeds financing need $I - A$, i.e.,

\[ A \geq \bar{A} \equiv I - Z_0 \]

• If $A < \bar{A}$, entrepreneur is \textit{credit-rationed}
  
  \begin{itemize}
  \item entrepreneurial rent $Z_1 - Z_0 > 0$ is necessary for credit-rationing (else all positive NPV projects are self-financing)
  \item entrepreneur must also be \textit{capital poor} $A < Z_1 - Z_0$ (else firm can pay ex ante for ex post rents)
  \end{itemize}

\[ \text{NPV} = Z_1 - I \geq Z_1 - Z_0 - A = \text{net entrepreneurial rent} \]

• Positive NPV projects may go unfunded if capital poor
Moral hazard

- Model of wedge between project value and pledgeable income
- Two periods $t = \{0, 1\}$
- Project gross payoff $R$ (success, $s$) or 0 (failure, $f$) at time $t = 1$
- Moral hazard problem: entrepreneur chooses probability of success
  - if diligent, probability of success is high $p_H$
  - if shirks, probability of success is low $p_L < p_H$, obtains private benefit $B$
Moral hazard timing

\[ H \quad p_H \quad R \]
\[ L \quad p_L \quad R \quad +B \text{ (private benefit)} \]
\[ 1 - p_H \quad 0 \]
\[ 1 - p_L \quad 0 \]
Moral hazard constraints

- Project returns shared between entrepreneur and investors

- Payments to entrepreneurs contingent on outcome, $X_s$ or $X_f$

- Individual rationality: investors break even if

$$p_H(R - X_s) + (1 - p_H)(0 - X_f) \geq I - A(> 0)$$

- Incentive compatibility: entrepreneur diligent if

$$p_HX_s + (1 - p_H)X_f \geq p_LX_s + (1 - p_L)X_f + B$$

or

$$X_s - X_f \geq \frac{B}{\Delta p}, \quad \Delta p \equiv p_H - p_L$$

- Limited liability: $X_f, X_s \geq 0$
Moral hazard and pledgeable income

- Limited liability and incentive compatibility together imply an entrepreneurial rent

- Entrepreneurial rent minimised by setting

\[ X_s = \frac{B}{\Delta p}, \quad X_f = 0 \]

- *Pledgeable income* is maximum that can be promised to investors

\[ Z_0 = p_H(R - X_s) = p_H \left( R - \frac{B}{\Delta p} \right) \]
Factors influencing pledgeable income

- Bias towards less risky projects
  (if entrepreneur has portfolio of projects to choose from)

- But diversification across projects increases pledgeable income from portfolio (if projects not perfectly correlated)

- Financial intermediation, loan covenants, costly monitoring etc
Variable investment scale

- Now $I$ is scale of investment, not fixed amount

- Let $\rho_1$ denote expected return per unit investment, $\rho_0$ denote pledgeable return per unit investment

  \[ 0 < \rho_0 < 1 < \rho_1 \]

- Total project payoff $\rho_1 I$, with $\rho_0 I$ pledged to investors, entrepreneurial rent $(\rho_1 - \rho_0) I$

- Entrepreneur’s endowed with capital $A$, $\rho_0 I$ raised from investors, remaining $(1 - \rho_0) I$ covered by own capital

  \[ (1 - \rho_0) I \leq A \]
Equity multiplier

- If this constraint is binding (maximum scale), $I$ is a proportion of own funds

$$I = kA, \quad k \equiv \frac{1}{1 - \rho_0} > 1$$

- A measure of leverage

- Gross payoff to entrepreneur

$$(\rho_1 - \rho_0)I = \frac{\rho_1 - \rho_0}{1 - \rho_0} A \equiv \mu A, \quad \mu > 1$$

where $\mu$ is gross rate of return on own capital (internal rate of return), greater than market return ($=1$)

- Net payoff to entrepreneur

$$U = (\mu - 1)A$$
Internal Rate of Return

Variable investment scale
Slope = internal rate of return

\[
\frac{\rho_1 - 1}{1 - \rho_0}
\]

Fixed investment scale
Jump occurs when own capital sufficient for investing

Firm value

Own capital
NPV vs. pledgeable income

- Consider portfolio of projects distinguished by $\rho_0, \rho_1$

- Rate of return

$$\mu = \frac{\rho_1 - \rho_0}{1 - \rho_0}$$

- Holding $\mu$ fixed

$$\frac{d \rho_1}{d \rho_0} = 1 - \mu < 0$$

- Substitute NPV for more pledgeable income. Each $\rho_0$ is worth $\mu - 1$ units of $\rho_1$
Monetary Economics

Lecture 23b: inside and outside liquidity, part two

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2nd Semester 2014
(not examinable)
This lecture

• Inside and outside liquidity, part two

• Further reading:
  ◇ Tirole “Illiquidity and all its friends” *Journal of Economic Literature*, 2011

• Available from LMS
1- Holmström-Tirole model
  - binary shocks
  - continuous shocks

2- Implementing the optimal (second-best) contract

3- Idiosyncratic vs. aggregate risk
Holmström-Tirole setup

- Three dates $t = \{0, 1, 2\}$
- Firm has endowment $A$, chooses investment scale $I$ at $t = 0$
- Liquidity shock $\rho \geq 0$ realised at $t = 1$
  - continuation scale $i(\rho) \leq I$
  - required reinvestment $\rho i(\rho)$, else project ceases
- Returns at $t = 2$
  - liquid (pledgeable) return $\rho_0 i(\rho)$
  - illiquid (private) return $(\rho_1 - \rho_0)i(\rho)$ to entrepreneur
Timing

Investment scale $I$. Outside investment $I - A$. Liquidity shock $\rho \geq 0$. Required reinvestment $\rho i(\rho)$ with $i(\rho) \leq I$. Liquid (pledgeable) return $\rho_0 i(\rho)$. Illiquid (private) return $(\rho_1 - \rho_0) i(\rho)$ to entrepreneur.
Binary liquidity shocks

- Two possible values

\[ \rho \in \{\rho_L, \rho_H\} \]

with probabilities \( f_L, f_H \) respectively

- To focus on interesting cases, suppose

\[ 0 \leq \rho_L < \rho_0 < \rho_H < \rho_1 \]

- Low shock \( \rho_L \) does not require pre-arranged financing, but high shock \( \rho_H \) does

- Also assumed that project is (i) socially desirable, and (ii) not self-financing
Second-best contract

• Specifies three terms

\[ I, \quad i_L \equiv i(\rho_L), \quad i_H \equiv i(\rho_H) \]

and payments to outside investors and entrepreneurs

• These maximise expected social return

\[ \max_{I, i_L, i_H} [f_L(\rho_1 - \rho_L)i_L + f_H(\rho_1 - \rho_H)i_H - I] \quad (\text{SBC}) \]

subject to the entrepreneur’s budget constraint

\[ f_L(\rho_0 - \rho_L)i_L + f_H(\rho_0 - \rho_H)i_H \geq I - A \]

and feasibility

\[ 0 \leq i_L, i_H \leq I \]

• When low shock, firm pays investors \( \rho_0 - \rho_L > 0 \). When high shock, investors pay firm \( \rho_H - \rho_0 \)

• Contract trades off ex ante scale vs. ex post liquidity
Entrepreneurial rent

- Using budget constraint to eliminate $I$, we get an equivalent optimisation problem that involves maximising net entrepreneurial rent

$$U = \max_{i_L, i_H} \left[ f_L(\rho_1 - \rho_0)i_L + f_H(\rho_1 - \rho_0)i_H - A \right]$$

subject to

$$0 \leq i_L, i_H \leq I$$

- Full social surplus goes to the entrepreneur (investors get their outside option)
Solving the contract

• If low liquidity shock, no tension. Since
  \[ \rho_1 - \rho_L > 0 \quad \text{and} \quad \rho_0 - \rho_L > 0 \]
  it is in everyone’s interest to continue at full scale. Hence
  \[ i_L = I \]
  for some \( I \) to be determined

• Tension between \( I \) and \( i_H \), both involve outlays by investors

• Fraction of project continued if high shock
  \[ x \equiv \frac{i_H}{I} \]

• Expected unit cost of continuing project
  \[ \bar{\rho}(x) \equiv f_L \rho_L + f_H \rho_H x \]
Solving the contract, cont.

• Implies entrepreneurial rent (net social surplus)

\[ U(x) = (\mu(x) - 1)A \]

where \( \mu(x) \) is gross value of extra unit of entrepreneurial capital \( A \)

\[ \mu(x) \equiv \frac{(\rho_1 - \rho_0)(f_L + f_H x)}{(1 + \bar{\rho}(x)) - \rho_0(f_L + f_H x)} \]

• Original problem (SBC) is a *linear program*, hence solution is at one of the *extreme points*

• These correspond to \( x = 0 \) (continue project only if low shock) or \( x = 1 \) (always continue)
Summary of solution

• If $\rho = \rho_L$, project continues and $i_L = I$

• If $\rho = \rho_H$, project continues and $i_H = I$ if and only if

$$\rho_H < c \equiv \min \left\{ 1 + \bar{\rho}(1), \frac{1 + f_L \rho_L}{\rho_L} \right\}$$

i.e., the unit cost of the liquidity shock is less than $c$, the unit cost of effective investment

• Project is continued in both states if and only if

$$f_L(\rho_H - \rho_L) < 1$$

Both a larger $\rho_H$ and smaller $\rho_L$ serve to increase ex ante scale $I$ at cost of reducing ex post liquidity
Ex ante scale

- From budget constraint

\[ I = A + f_L(\rho_0 - \rho_L)i_L + f_H(\rho_0 - \rho_H)i_H \]

- Two cases:

  (i) \( \rho_H < c \) so that \( i_L = i_H = I \). Then

  \[ I = \frac{1}{1 + \bar{\rho}(1) - \rho_0} A \]

  (ii) \( \rho_H > c \) so that \( i_L = I \) but \( i_H = 0 \). Then

  \[ I = \frac{1}{1 + (\bar{\rho}(0) - \rho_0)f_L} A \]
Continuous liquidity shocks

- Continuous distribution of liquidity shocks $\rho \geq 0$

- Probability density function (PDF)

  $$f(\rho) \geq 0, \quad \int_{0}^{\infty} f(\rho) \, d\rho = 1$$

- Cumulative distribution function (CDF)

  $$F(\rho) = \int_{0}^{\rho} f(r) \, dr = \Pr[r \leq \rho]$$
Second best contract, continuous case

- Maximises entrepreneur’s expected rent

\[ U = \max_{I, i(\rho)} \int (\rho_1 - \rho_0)i(\rho)f(\rho) d\rho \]

subject to the budget constraint

\[ \int (\rho_0 - \rho)i(\rho)f(\rho) d\rho \geq I - A \]

and feasibility

\[ 0 \leq i(\rho) \leq I \]
Continuation policy

- Linearity of the optimisation problem implies continuation policy is a *cutoff rule*
  \[ i(\rho) = I \quad \text{for} \quad \rho < \hat{\rho} \]

and

\[ i(\rho) = 0 \quad \text{for} \quad \rho > \hat{\rho} \]

- Critical value \( \hat{\rho} \) to be determined
Ex ante scale, continuous case

- Binding budget constraint implies

\[ A = I - \int (\rho_0 - \rho) i(\rho) f(\rho) \, d\rho = I - \int_{0}^{\hat{\rho}} (\rho_0 - \rho) I f(\rho) \, d\rho \]

\[ = \left( 1 - \rho_0 F(\hat{\rho}) + \int_{0}^{\hat{\rho}} \rho f(\rho) \, d\rho \right) I \]

or simply

\[ I = k(\hat{\rho}) A \]

- Investment multiplier

\[ k(\hat{\rho}) = \frac{1}{1 - \rho_0 F(\hat{\rho}) + \int_{0}^{\hat{\rho}} \rho f(\rho) \, d\rho} \]

- This is maximised at \( \hat{\rho} = \rho_0 \) with \( k(\rho_0) > 1 \) (continuing at full scale when \( \rho_0 \geq \rho \)), and is decreasing in \( \hat{\rho} \) at \( \rho_1 \)
Entrepreneurial rent

- Plugging back into objective

\[ U(\hat{\rho}) = m(\hat{\rho})I = m(\hat{\rho})k(\hat{\rho})A \]

- Total expected return per unit investment (marginal return)

\[ m(\hat{\rho}) = F(\hat{\rho})\rho_1 - 1 - \int_0^{\hat{\rho}} \rho f(\rho) \, d\rho \]

- This is maximised at \( \hat{\rho} = \rho_1 \) (continuing at full scale whenever \( \rho_1 \geq \rho \)), and is increasing in \( \hat{\rho} \) at \( \rho_0 \)
Fundamental tradeoff

• Tension between investing in initial scale vs. saving funds to meet anticipated liquidity shocks

(i) lower $\hat{\rho}$ towards $\rho_0$ to increase size of investment $I = k(\hat{\rho})A$, or

(ii) increase $\hat{\rho}$ towards $\rho_1$ to increase ability to withstand liquidity shock $\rho$, this raises marginal return $m(\hat{\rho})$ on initial investment $I$

(not both, binding IR constraint places limit on firm’s investment)

• Solution is a $\rho^*$ that balances $k(\hat{\rho})$ and $m(\hat{\rho})$ effects

$\rho_0 < \rho^* < \rho_1$

• Compromise between credit rationing initial scale and credit rationing reinvestment to meet liquidity shock
Solving for optimal cutoff $\rho^*$

- Can write entrepreneurial rent

\[ U(\hat{\rho}) = \frac{\rho_1 - c(\hat{\rho})}{c(\hat{\rho}) - \rho_0} A \]

- Expected unit cost of effective investment

\[ c(\hat{\rho}) = \frac{1 + \int_0^{\hat{\rho}} \rho f(\rho) d\rho}{F(\hat{\rho})} \]

- Maximising $U(\hat{\rho})$ is achieved by minimising $c(\hat{\rho})$, first order condition for this can be written

\[ 1 = \int_0^{\rho^*} F(\rho) d\rho \]

- Interior solutions depend only on $F(\rho)$, not $\rho_0, \rho_1, A$ etc
Overview of second best contract solution

- Firm with capital $A$ invests $I = k(\rho^*)A$

- Project continued if and only if $\rho < \rho^*$ where $\rho^* \in (\rho_0, \rho_1)$

- If project continued, then
  - firm paid $(\rho_1 - \rho_0)I$ for all $\rho$
  - outside investors paid $\rho_0I$
Implementing the optimal contract

1- Credit line

- outside investors lend $I - A$ at $t = 0$
- credit line $\rho^* I$, can be used by firms at $t = 1$
- such funds cannot be consumed, firm prefers to continue if possible

[twist: credit line of $(\rho^* - \rho)I$ but allow investors claims to be diluted to cover shock]

2- Liquidity ratio

- outside investors lend $(1 + \rho^*)I - A$ at $t = 0$
- covenant that minimum $\rho^* I$ be kept in liquid assets, liquidity ratio

$$\frac{\rho^*}{1 + \rho^*}$$

These are equivalent in this partial equilibrium scenario. But not in general equilibrium (\therefore then liquid assets at a premium)
Endogenous liquidity, no aggregate risk

- No storage technology, only assets created by firms can be used to store value

- Ex ante identical firms. *Idiosyncratic liquidity shocks* \( \rho \sim \text{IID } f(\rho) \) make firms heterogeneous ex post

- Risk neutral firms and consumers. Consumers have endowments large enough to finance any taxes and to finance all required investments. Cannot issue their own assets
• To implement the second-best, additional funds needed at $t = 1$ are

$$D = I \int_0^{\rho^*} \rho f(\rho) \, d\rho$$

(since firms are identical ex ante, $I$ is the same for all firms)

• Credit line and liquidity ratio implementations of second best relied on *exogenous* supply of the liquid asset

• Can financial market generate *endogenously* the needed supply of liquid assets? Possible instruments
  
  – additional claims issued at date $t = 1$
  – holding shares in other firms
Distribution of liquidity

- Can show that without aggregate risk, total liquidity needs can be met endogenously

- Main problem is possible *inefficient distribution* of liquidity
  - firms with $\rho < \rho_0$ have liquid assets they do not need
  - firms with $\rho > \rho^*$ will shut down, release liquid assets
  - firms with $\rho \in (\rho_0, \rho^*)$ want liquidity

- Need a way to transfer from excess liquidity firms to shortfall firms
Liquidity supply from financial intermediaries

- Financial intermediation can *pool the idiosyncratic risk* of all firms thereby cross-subsidising unlucky firms

- With *no aggregate uncertainty*, financial intermediary can pool risk and second best can be implemented

- No particular role for government intervention
Endogenous liquidity, pure aggregate risk

• All firms receive the same $\rho$ shock, perfectly correlated

• Firms cannot generally be self sufficient. For $\rho_0 < \rho < \rho^*$, firms need $\rho I$ but can only raise $\rho_0 I$

• Intermediaries cannot pool aggregate risk

• Role for government supplied liquid assets
  
  − issue $(\rho^* - \rho_0)I$ bonds at $t = 0$, provides “storage facility” for cash
  − firms invest $(1 + \rho^*)I - A$ at $t = 0$, spend $(\rho^* - \rho_0)I$ of this amount on bonds

• Government bonds “crowd-out” initial investment $I$ at $t = 0$ but increase reinvestment at $t = 1$
Next lecture

- Leverage cycles, part one
- Leverage and balance sheet effects

- Adrian and Shin “Liquidity and leverage” *Journal of Financial Intermediation*, 2010

Available from the LMS