Monetary Economics

Lecture 23a: inside and outside liquidity, part one

Chris Edmond

2nd Semester 2014
(not examinable)

This lecture

- Main reading:
  - ◇ Holmström and Tirole, *Inside and outside liquidity*, MIT Press. Chapter 1
- Further reading:
  - ◇ Holmström and Tirole "Private and public supply of liquidity" *Journal of Political Economy*, 1998
  - ◇ Tirole "Illiquidity and all its friends" *Journal of Economic Literature*, 2011

Available from the LMS

Holmström-Tirole overview

Liquidity: availability of assets for intertemporal smoothing

Q. Is the private supply of liquid assets socially optimal?

A. Private supply sufficient to achieve the socially optimal (second best) outcome if *no aggregate risk*

Implementation of this requires financial intermediaries that can *pool idiosyncratic risk*

Q. Is there a role for government intervention?

A. Yes, especially if there is aggregate risk or impediments to intermediation

Today: basic concepts

1- Credit rationing with fixed investment scale

2- Moral hazard and the wedge between value and pledgeable income

3- Variable investment scale

Credit rationing with fixed investment scale

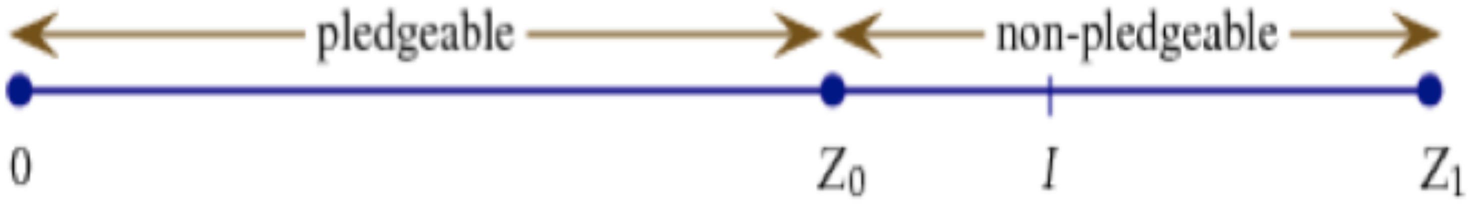
- Risk neutral entrepreneur with investment opportunity
- Opportunity worth Z_1 to entrepreneur but $Z_0 < Z_1$ to investors
- Initial investment I required to implement project

  $$Z_0 < I < Z_1$$

  Positive net present value $I < Z_1$, but *not self-financing*, $Z_0 < I$

- Shortfall $I - Z_0$ must be covered by entrepreneur
- *Entrepreneurial rent* $Z_1 - Z_0$ cannot be pledged to investors (e.g., because private benefits, different beliefs, non-transferability)

Limited pledgeability



Value of project to entrepreneurs is Z_1. Value to investors is Z_0. Entrepreneurial rent $Z_1 - Z_0$. Investment I required to implement project. Shortfall $I - Z_0$

Credit rationing with fixed investment scale

- Let $A > 0$ be entrepreneurial capital committed to project

- Project can proceed if and only if pledgeable income Z_0 exceeds financing need $I - A$, i.e.,

$$A \geq \bar{A} \equiv I - Z_0$$

- If $A < \bar{A}$, entrepreneur is *credit-rationed*

  - entrepreneurial rent $Z_1 - Z_0 > 0$ is necessary for credit-rationing (else all positive NPV projects are self-financing)

  - entrepreneur must also be *capital poor* $A < Z_1 - Z_0$ (else firm can pay ex ante for ex post rents)

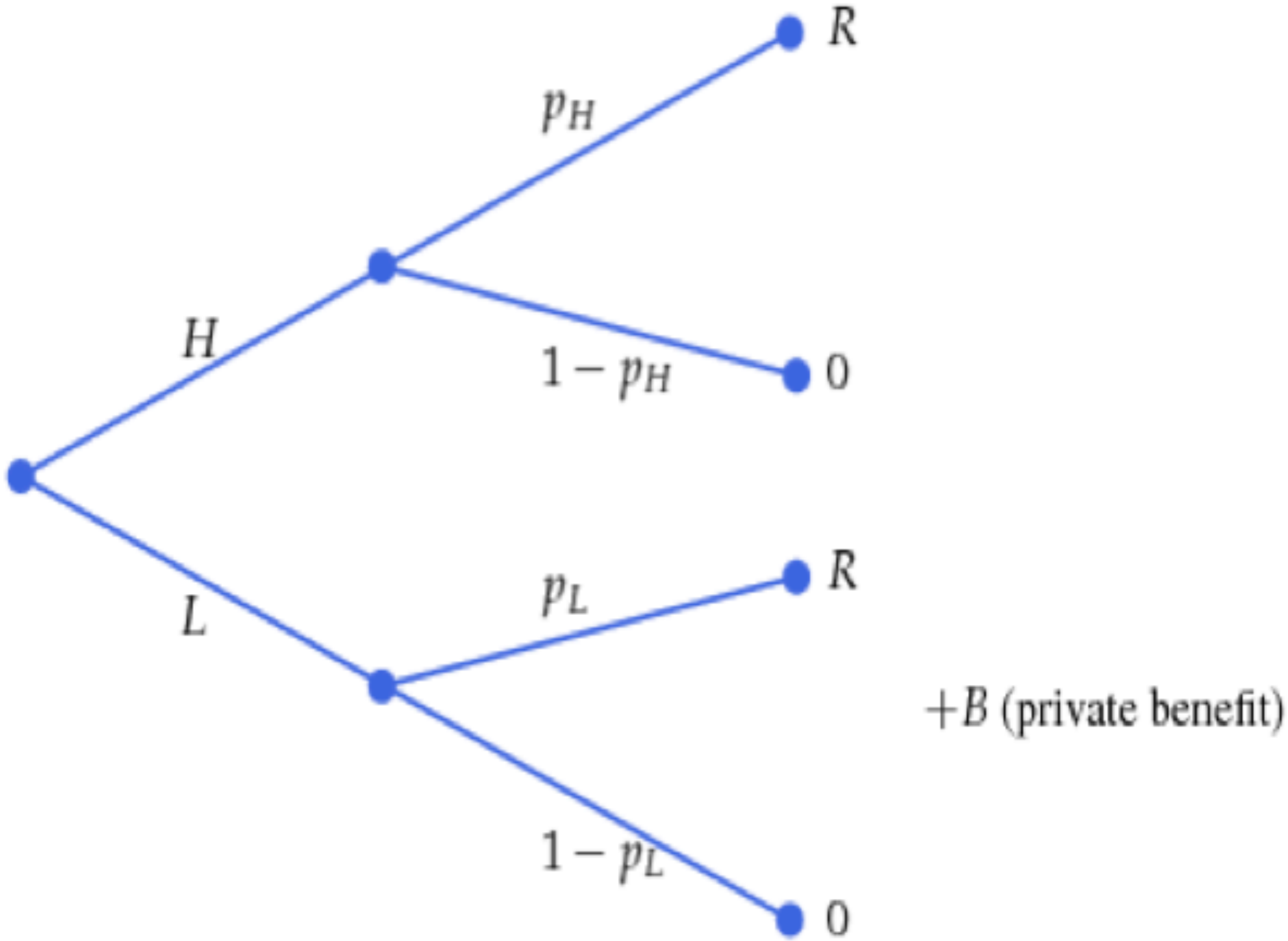
$$\text{NPV} = Z_1 - I \geq Z_1 - Z_0 - A = \text{net entrepreneurial rent}$$

- Positive NPV projects may go unfunded if capital poor

Moral hazard

- Model of wedge between project value and pledgeable income
- Two periods $t = \{0, 1\}$
- Project gross payoff R (success, s) or 0 (failure, f) at time $t = 1$
- Moral hazard problem: entrepreneur chooses probability of success
  - if diligent, probability of success is high p_H
  - if shirks, probability of success is low $p_L < p_H$, obtains private benefit B

Moral hazard timing



Moral hazard constraints

- Project returns shared between entrepreneur and investors

- Payments to entrepreneurs contingent on outcome, X_s or X_f

- *Individual rationality*: investors break even if

$$p_H(R - X_s) + (1 - p_H)(0 - X_f) \geq I - A(> 0)$$

- *Incentive compatibility*: entrepreneur diligent if

$$p_H X_s + (1 - p_H) X_f \geq p_L X_s + (1 - p_L) X_f + B$$

or

$$X_s - X_f \geq \frac{B}{\Delta p}, \qquad \Delta p \equiv p_H - p_L$$

- *Limited liability*: $X_f, X_s \geq 0$

Moral hazard and pledgeable income

- Limited liability and incentive compatibility together imply an entrepreneurial rent

- Entrepreneurial rent minimised by setting

$$X_s = \frac{B}{\Delta p}, \qquad X_f = 0$$

- *Pledgeable income* is maximum that can be promised to investors

$$Z_0 = p_H(R - X_s) = p_H\left(R - \frac{B}{\Delta p}\right)$$

Factors influencing pledgeable income

- Bias towards less risky projects
  (if entrepreneur has portfolio of projects to choose from)

- But diversification across projects increases pledgeable income from portfolio (if projects not perfectly correlated)

- Financial intermediation, loan covenants, costly monitoring etc

Variable investment scale

- Now I is scale of investment, not fixed amount

- Let ρ_1 denote expected return per unit investment, ρ_0 denote pledgeable return per unit investment

$$0 < \rho_0 < 1 < \rho_1$$

- Total project payoff $\rho_1 I$, with $\rho_0 I$ pledged to investors, entrepreneurial rent $(\rho_1 - \rho_0)I$

- Entrepreneur's endowed with capital A, $\rho_0 I$ raised from investors, remaining $(1 - \rho_0)I$ covered by own capital

$$(1 - \rho_0)I \leq A$$

Equity multiplier

- If this constraint is binding (maximum scale), I is a proportion of own funds

$$I = kA, \qquad k \equiv \frac{1}{1-\rho_0} > 1$$

- A measure of leverage

- Gross payoff to entrepreneur

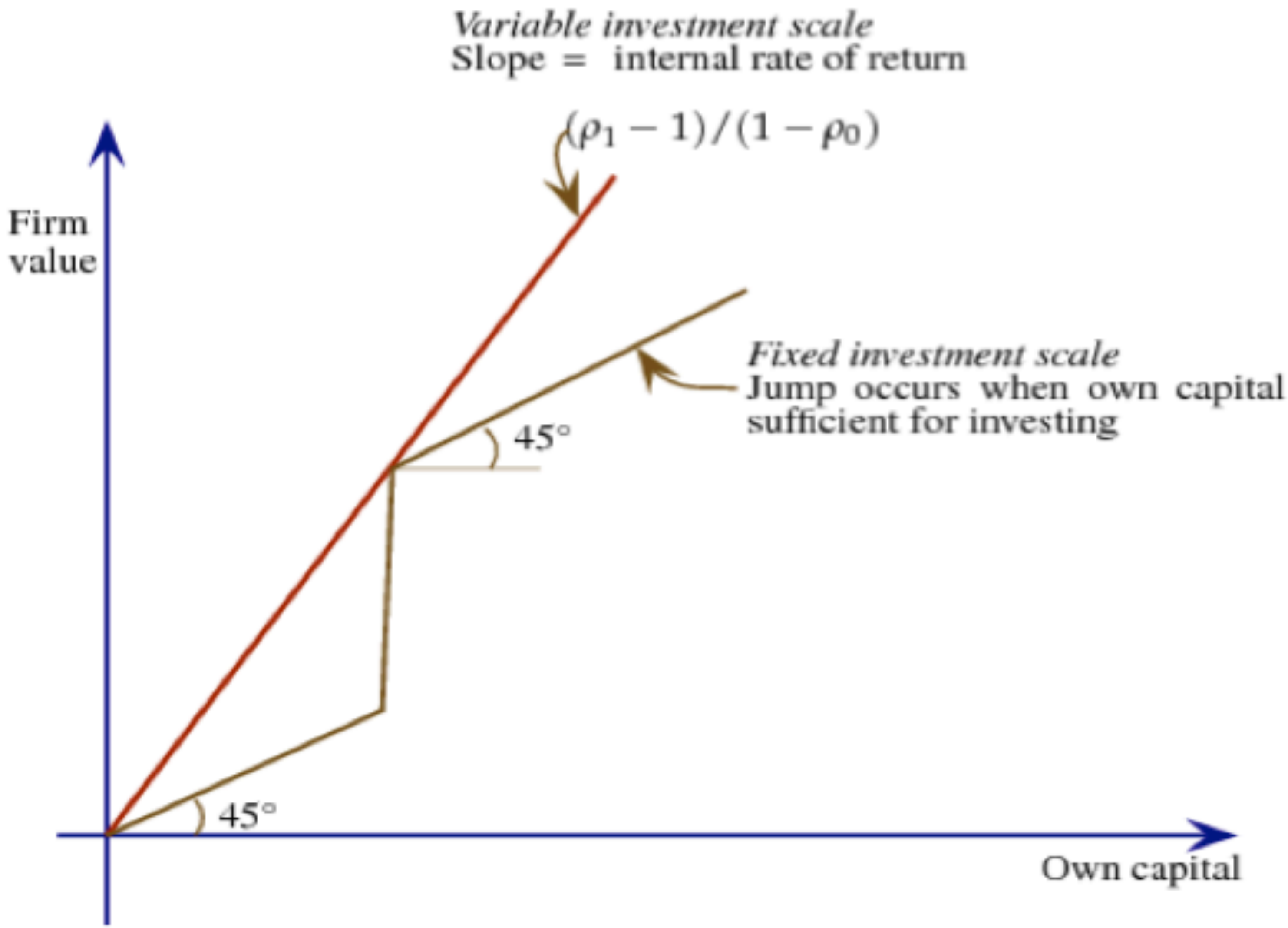
$$(\rho_1 - \rho_0)I = \frac{\rho_1 - \rho_0}{1-\rho_0} A \equiv \mu A, \qquad \mu > 1$$

where μ is gross rate of return on own capital (internal rate of return), greater than market return (=1)

- Net payoff to entrepreneur

$$U = (\mu - 1)A$$

Internal Rate of Return



NPV vs. pledgeable income

- Consider portfolio of projects distinguished by ρ_0, ρ_1
- Rate of return

$$\mu = \frac{\rho_1 - \rho_0}{1 - \rho_0}$$

- Holding μ fixed

$$\frac{d\rho_1}{d\rho_0} = 1 - \mu < 0$$

- Substitute NPV for more pledgeable income. Each ρ_0 is worth $\mu - 1$ units of ρ_1

Monetary Economics

Lecture 23b: inside and outside liquidity, part two

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This lecture

- Inside and outside liquidity, part two

  - Holmström and Tirole, *Inside and outside liquidity*, MIT Press. Chapter 2

- Further reading:

  - Holmström and Tirole “Private and public supply of liquidity” *Journal of Political Economy*, 1998

  - Tirole “Illiquidity and all its friends” *Journal of Economic Literature*, 2011

- Available from LMS

Today

1- Holmström-Tirole model

- binary shocks
- continuous shocks

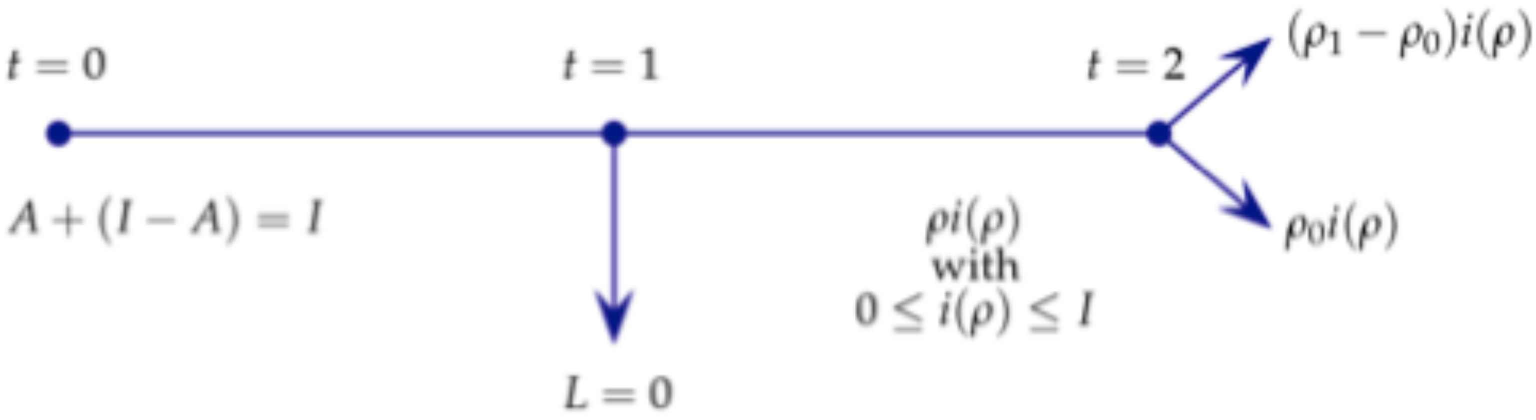
2- Implementing the optimal (second-best) contract

3- Idiosyncratic vs. aggregate risk

Holmström-Tirole setup

- Three dates $t = \{0, 1, 2\}$
- Firm has endowment A, chooses investment scale I at $t = 0$
- Liquidity shock $\rho \geq 0$ realised at $t = 1$
  - continuation scale $i(\rho) \leq I$
  - required reinvestment $\rho i(\rho)$, else project ceases
- Returns at $t = 2$
  - liquid (pledgeable) return $\rho_0 i(\rho)$
  - illiquid (private) return $(\rho_1 - \rho_0)i(\rho)$ to entrepreneur

Timing



Investment scale I. Outside investment $I - A$. Liquidity shock $\rho \geq 0$. Required reinvestment $\rho i(\rho)$ with $i(\rho) \leq I$. Liquid (pledgeable) return $\rho_0 i(\rho)$. Illiquid (private) return $(\rho_1 - \rho_0)i(\rho)$ to entrepreneur.

Binary liquidity shocks

- Two possible values

  $$\rho \in \{\rho_L, \rho_H\}$$

  with probabilities f_L, f_H respectively

- To focus on interesting cases, suppose

  $$0 \leq \rho_L < \rho_0 < \rho_H < \rho_1$$

- Low shock ρ_L does not require pre-arranged financing, but high shock ρ_H does

- Also assumed that project is (i) socially desirable, and (ii) not self-financing

Second-best contract

- Specifies three terms

$$I, \qquad i_L \equiv i(\rho_L), \qquad i_H \equiv i(\rho_H)$$

and payments to outside investors and entrepreneurs

- These maximise expected social return

$$\max_{I,\, i_L,\, i_H} \left[f_L(\rho_1 - \rho_L) i_L + f_H(\rho_1 - \rho_H) i_H - I\right] \tag{SBC}$$

subject to the entrepreneur's budget constraint

$$f_L(\rho_0 - \rho_L) i_L + f_H(\rho_0 - \rho_H) i_H \geq I - A$$

and feasibility

$$0 \leq i_L,\ i_H \leq I$$

- When low shock, firm pays investors $\rho_0 - \rho_L > 0$. When high shock, investors pay firm $\rho_H - \rho_0$

- Contract trades off ex ante scale vs. ex post liquidity

Entrepreneurial rent

- Using budget constraint to eliminate I, we get an equivalent optimisation problem that involves maximising net entrepreneurial rent

$$U = \max_{i_L,\, i_H} \left[f_L(\rho_1 - \rho_0) i_L + f_H(\rho_1 - \rho_0) i_H - A \right]$$

subject to

$$0 \le i_L,\ i_H \le I$$

- Full social surplus goes to the entrepreneur (investors get their outside option)

Solving the contract

- If low liquidity shock, no tension. Since

$$\rho_1 - \rho_L > 0 \quad \text{and} \quad \rho_0 - \rho_L > 0$$

it is in everyone's interest to continue at full scale. Hence

$$i_L = I$$

for some I to be determined

- Tension between I and i_H, both involve outlays by investors

- Fraction of project continued if high shock

$$x \equiv \frac{i_H}{I}$$

- Expected unit cost of continuing project

$$\bar{\rho}(x) \equiv f_L \rho_L + f_H \rho_H x$$

Solving the contract, cont.

- Implies entrepreneurial rent (net social surplus)

$$U(x) = (\mu(x) - 1)A$$

where $\mu(x)$ is gross value of extra unit of entrepreneurial capital A

$$\mu(x) \equiv \frac{(\rho_1 - \rho_0)(f_L + f_H x)}{(1 + \bar{\rho}(x)) - \rho_0(f_L + f_H x)}$$

- Original problem (SBC) is a *linear program*, hence solution is at one of the *extreme points*

- These correspond to $x = 0$ (continue project only if low shock) or $x = 1$ (always continue)

Summary of solution

- If $\rho = \rho_L$, project continues and $i_L = I$

- If $\rho = \rho_H$, project continues and $i_H = I$ if and only if

$$\rho_H < c \equiv \min\left\{1 + \bar{\rho}(1), \frac{1 + f_L \rho_L}{\rho_L}\right\}$$

  i.e., the unit cost of the liquidity shock is less than c, the *unit cost of effective investment*

- Project is continued in both states if and only if

$$f_L(\rho_H - \rho_L) < 1$$

  Both a larger ρ_H and smaller ρ_L serve to increase ex ante scale I at cost of reducing ex post liquidity

Ex ante scale

- From budget constraint

$$I = A + f_L(\rho_0 - \rho_L)i_L + f_H(\rho_0 - \rho_H)i_H$$

- Two cases:

(i) $\rho_H < c$ so that $i_L = i_H = I$. Then

$$I = \frac{1}{1 + \bar{\rho}(1) - \rho_0} A$$

(ii) $\rho_H > c$ so that $i_L = I$ but $i_H = 0$. Then

$$I = \frac{1}{1 + (\bar{\rho}(0) - \rho_0) f_L} A$$

Continuous liquidity shocks

- Continuous distribution of liquidity shocks $\rho \geq 0$

- Probability density function (PDF)

$$f(\rho) \geq 0, \qquad \int_0^\infty f(\rho)\, d\rho = 1$$

- Cumulative distribution function (CDF)

$$F(\rho) = \int_0^\rho f(r)\, dr = \Pr[r \leq \rho]$$

Second best contract, continuous case

- Maximises entrepreneur's expected rent

$$U = \max_{I,\, i(\rho)} \int (\rho_1 - \rho_0) i(\rho) f(\rho)\, d\rho$$

subject to the budget constraint

$$\int (\rho_0 - \rho) i(\rho) f(\rho)\, d\rho \geq I - A$$

and feasibility

$$0 \leq i(\rho) \leq I$$

Continuation policy

- Linearity of the optimisation problem implies continuation policy is a *cutoff rule*

$$i(\rho) = I \qquad \text{for } \rho < \hat{\rho}$$

and

$$i(\rho) = 0 \qquad \text{for } \rho > \hat{\rho}$$

- Critical value $\hat{\rho}$ to be determined

Ex ante scale, continuous case

- Binding budget constraint implies

$$
\begin{aligned}
A &= I - \int (\rho_0 - \rho) i(\rho) f(\rho)\, d\rho = I - \int_0^{\hat{\rho}} (\rho_0 - \rho) I f(\rho)\, d\rho \\
&= \left(1 - \rho_0 F(\hat{\rho}) + \int_0^{\hat{\rho}} \rho f(\rho)\, d\rho\right) I
\end{aligned}
$$

or simply

$$I = k(\hat{\rho}) A$$

- *Investment multiplier*

$$k(\hat{\rho}) = \frac{1}{1 - \rho_0 F(\hat{\rho}) + \int_0^{\hat{\rho}} \rho f(\rho)\, d\rho}$$

- This is maximised at $\hat{\rho} = \rho_0$ with $k(\rho_0) > 1$ (continuing at full scale when $\rho_0 \geq \rho$), and is decreasing in $\hat{\rho}$ at ρ_1

Entrepreneurial rent

- Plugging back into objective

$$U(\hat{\rho}) = m(\hat{\rho})I = m(\hat{\rho})k(\hat{\rho})A$$

- Total expected return per unit investment (marginal return)

$$m(\hat{\rho}) = F(\hat{\rho})\rho_1 - 1 - \int_0^{\hat{\rho}} \rho f(\rho)\, d\rho$$

- This is maximised at $\hat{\rho} = \rho_1$ (continuing at full scale whenever $\rho_1 \geq \rho$), and is increasing in $\hat{\rho}$ at ρ_0

Fundamental tradeoff

- Tension between investing in initial scale vs. saving funds to meet anticipated liquidity shocks

  (i) lower $\hat{\rho}$ towards ρ_0 to increase size of investment $I = k(\hat{\rho})A$, or

  (ii) increase $\hat{\rho}$ towards ρ_1 to increase ability to withstand liquidity shock ρ, this raises marginal return $m(\hat{\rho})$ on initial investment I

  (not both, binding IR constraint places limit on firm's investment)

- Solution is a ρ^* that balances $k(\hat{\rho})$ and $m(\hat{\rho})$ effects

$$\rho_0 < \rho^* < \rho_1$$

- Compromise between credit rationing initial scale and credit rationing reinvestment to meet liquidity shock

Solving for optimal cutoff ρ^*

- Can write entrepreneurial rent

$$U(\hat{\rho}) = \frac{\rho_1 - c(\hat{\rho})}{c(\hat{\rho}) - \rho_0} A$$

- Expected unit cost of effective investment

$$c(\hat{\rho}) = \frac{1 + \int_0^{\hat{\rho}} \rho f(\rho)\, d\rho}{F(\hat{\rho})}$$

- Maximising $U(\hat{\rho})$ is achieved by minimising $c(\hat{\rho})$, first order condition for this can be written

$$1 = \int_0^{\rho^*} F(\rho) d\rho$$

- Interior solutions depend only on $F(\rho)$, not ρ_0, ρ_1, A etc

Overview of second best contract solution

- Firm with capital A invests $I = k(\rho^*)A$
- Project continued if and only if $\rho < \rho^*$ where $\rho^* \in (\rho_0, \rho_1)$
- If project continued, then
  - firm paid $(\rho_1 - \rho_0)I$ for all ρ
  - outside investors paid $\rho_0 I$

Implementing the optimal contract

1- *Credit line*

- outside investors lend $I - A$ at $t = 0$
- credit line $\rho^* I$, can be used by firms at $t = 1$
- such funds cannot be consumed, firm prefers to continue if possible

[twist: credit line of $(\rho^* - \rho)I$ but allow investors claims to be diluted to cover shock]

2- *Liquidity ratio*

- outside investors lend $(1 + \rho^*)I - A$ at $t = 0$
- covenant that minimum $\rho^* I$ be kept in liquid assets, liquidity ratio

$$\frac{\rho^*}{1 + \rho^*}$$

These are equivalent in this partial equilibrium scenario. But not in general equilibrium ($\because$ then liquid assets at a premium)

Endogenous liquidity, no aggregate risk

- No storage technology, only assets created by firms can be used to store value

- Ex ante identical firms. *Idiosyncratic liquidity shocks* $\rho \sim$ IID $f(\rho)$ make firms heterogeneous ex post

- Risk neutral firms and consumers. Consumers have endowments large enough to finance any taxes and to finance all required investments. Cannot issue their own assets

- To implement the second-best, additional funds needed at $t = 1$ are

$$D = I \int_0^{\rho^*} \rho f(\rho)\, d\rho$$

(since firms are identical ex ante, I is the same for all firms)

- Credit line and liquidity ratio implementations of second best relied on *exogenous* supply of the liquid asset

- Can financial market generate *endogenously* the needed supply of liquid assets? Possible instruments

  - additional claims issued at date $t = 1$
  - holding shares in other firms

Distribution of liquidity

- Can show that without aggregate risk, total liquidity needs can be met endogenously

- Main problem is possible *inefficient distribution* of liquidity

  – firms with $\rho < \rho_0$ have liquid assets they do not need

  – firms with $\rho > \rho^*$ will shut down, release liquid assets

  – firms with $\rho \in (\rho_0, \rho^*]$ want liquidity

- Need a way to transfer from excess liquidity firms to shortfall firms

Liquidity supply from financial intermediaries

- Financial intermediation can *pool the idiosyncratic risk* of all firms thereby cross-subsidising unlucky firms

- With *no aggregate uncertainty*, financial intermediary can pool risk and second best can be implemented

- No particular role for government intervention

Endogenous liquidity, pure aggregate risk

- All firms receive *the same* ρ shock, perfectly correlated
- Firms cannot generally be self sufficient. For $\rho_0 < \rho < \rho^*$, firms need ρI but can only raise $\rho_0 I$
- Intermediaries cannot pool aggregate risk
- Role for government supplied liquid assets
  - issue $(\rho^* - \rho_0)I$ bonds at $t = 0$, provides "storage facility" for cash
  - firms invest $(1 + \rho^*)I - A$ at $t = 0$, spend $(\rho^* - \rho_0)I$ of this amount on bonds
- Government bonds "crowd-out" initial investment I at $t = 0$ but increase reinvestment at $t = 1$

Next lecture

- Leverage cycles, part one
- Leverage and balance sheet effects
  - Adrian and Shin "Liquidity and leverage" *Journal of Financial Intermediation*, 2010

Available from the LMS