Monetary Economics

Lecture 21: financial market frictions, part four

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This lecture

- Macroeconomics with financial market frictions, part four
- Leverage cycles
  - Brunnermeier, Eisenbach and Sannikov “Macroeconomics with financial frictions: a survey,” NBER working paper, 2012

    section 3.3
  - Geanakoplos “Leverage cycles,” *NBER Macro Annual*, 2009
  - Adrian and Shin “Liquidity and leverage,” *Journal of Financial Intermediation*, 2010

Readings available from the LMS

This lecture

1- Geanakoplos (1997, 2009) model of leverage cycles

- heterogeneous beliefs
- ‘leveraging optimism’
- endogenous collateral constraints (sketch)

2- Adrian and Shin (2010) cross-sectional facts on leverage cyclicality

- balance sheet management
- households, countercylical leverage ratios
- investment banks, procyclical leverage ratios

Geanakoplos

- Asset market determines *both* interest rates and leverage ratios

- Fluctuations in leverage ratios more important than interest rates

- Key elements of theory:

  - natural buyers for assets
  - '*scary*' bad news lowers expectations, increases volatility
  - increased volatility tightens margins, reduces leverage
  - price falls, amplified by lower leverage, wealth redistributed from buyers, prices fall further ...

- Today: simple examples showing how leverage boosts asset prices and how leverage is determined in equilibrium

Two-period example

- Two dates $t \in \{0, 1\}$
- Two states $s \in \{U, D\}$ at date $t = 1$
- Two commodities

  (i) consumption good, durable
  (costlessly storable $\rightarrow$ also risk-free asset)

  (ii) risky asset, not consumable but state-contingent payoffs, $x_U > x_D$
  in units of consumption

- Continuum $h \in [0, 1]$ of agents with *heterogeneous beliefs*
  - agents differ in optimism about $s = U$

Heterogeneous beliefs

- Continuum $h \in [0,1]$ of agents with heterogeneous beliefs

$$\text{Prob}[s = U \mid h] = h$$

$$\text{Prob}[s = D \mid h] = 1 - h$$

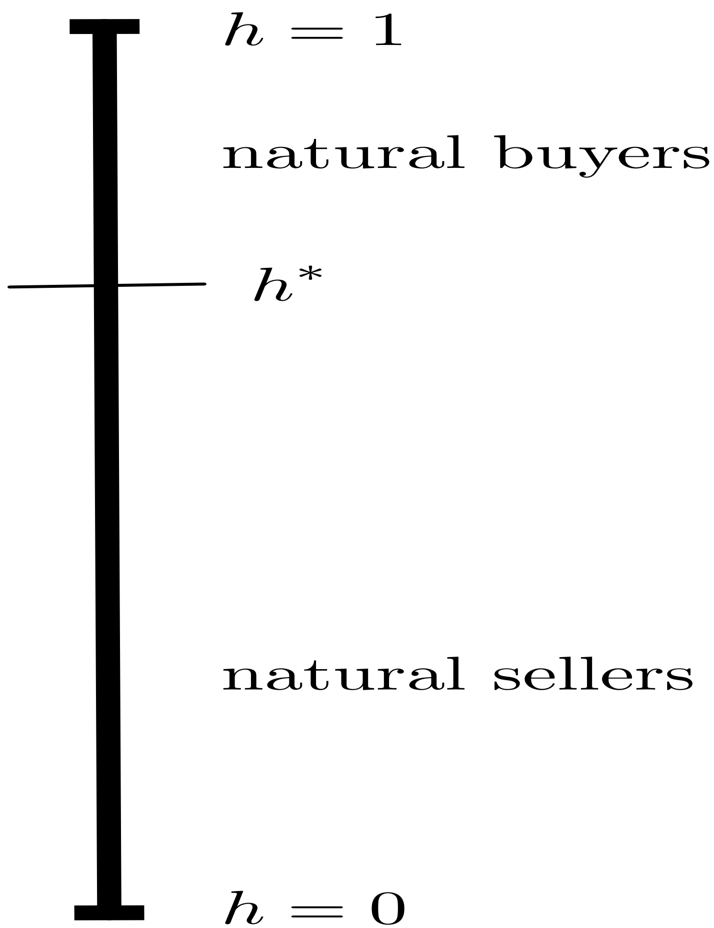
- Agent $h = 1$ is most optimistic about $s = U$, agent $h = 0$ is most pessimistic about $s = U$

- Agents with sufficiently high h are *natural buyers* of the asset

- Agents otherwise identical

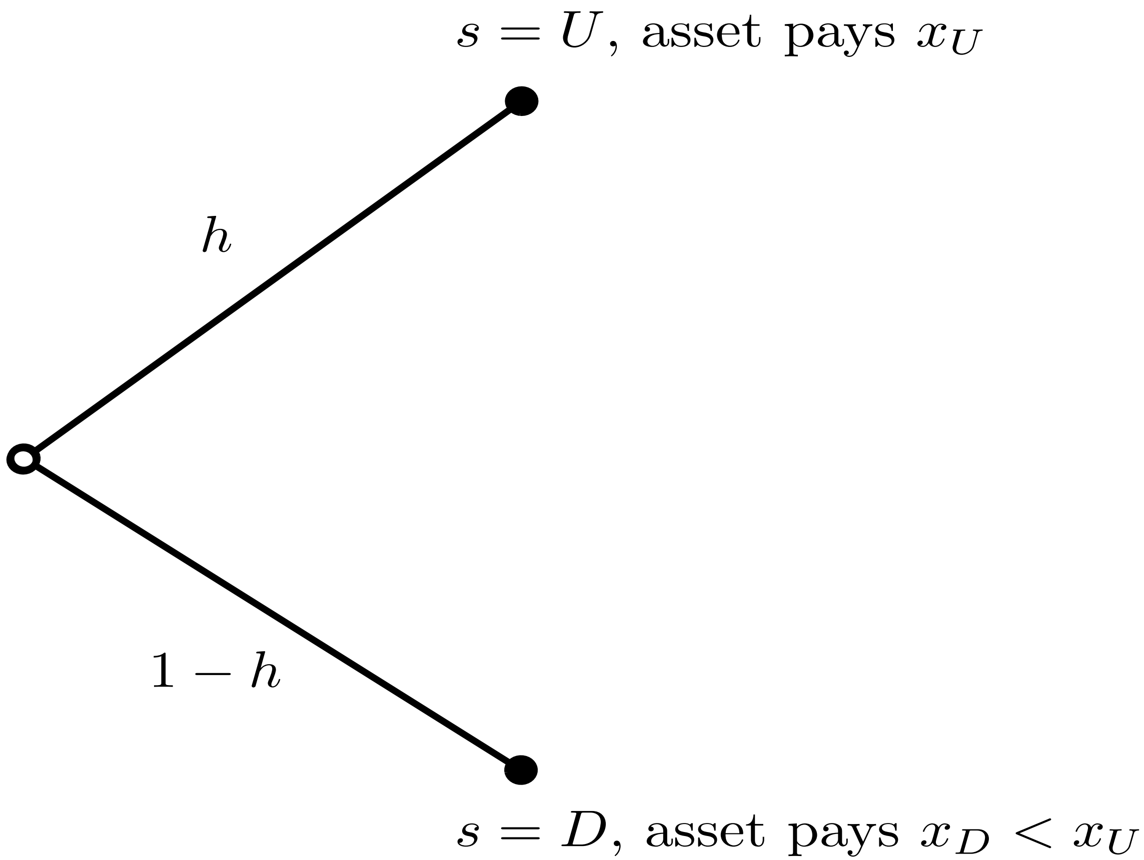
  - risk neutral expected utility, indifferent to timing of consumption
  - identical initial endowments, each have one unit of each commodity

Heterogeneous beliefs



Agent $h = 1$ is most optimistic about $s = U$. Agents with sufficiently high h are *natural buyers* of the asset. Cutoff h^* determined endogenously in equilibrium.

Two-period example



Two states $s \in \{U, D\}$ possible at date $t = 1$. Asset pays x_U in good state but only $x_D < x_U$ in bad state. Agent $h \in [0, 1]$ assigns subjective probability h to x_U and probability $1 - h$ to x_D

No borrowing benchmark: optimisation

- Expected utility

$$u_h = c_0 + h\, c_U + (1-h)\, c_D$$

- Budget constraints if no borrowing

$$c_0 + w_0 + p\, y_0 = 1 + p$$

$$c_U = w_0 + x_U\, y_0$$

$$c_D = w_0 + x_D\, y_0$$

- Consumption good is numeraire, p and y_0 are relative price of and quantity of risky asset held at date $t = 0$, w_0 is real storage of consumption good held at date $t = 0$. All agents have initial endowment of one unit of each commodity

- *No short selling*, sales of asset limited by endowment (i.e., $y_0 \geq 0$)

No borrowing benchmark: market clearing

- Markets for risky asset at initial date and consumption at each date and state

$$\int_0^1 y_0^h \, dh = 1$$

$$\int_0^1 (c_0^h + w_0^h) \, dh = 1$$

$$\int_0^1 c_U^h \, dh = x_U + \int_0^1 w_0^h \, dh$$

$$\int_0^1 c_D^h \, dh = x_D + \int_0^1 w_0^h \, dh$$

Cutoff agent h^*

- Linear objective and constraints, solution typically at a corner.
  Any agent h such that

  $$x_U\, h + x_D\, (1 - h) > p$$

  expects payoff greater than price, buys as much as possible

- Any agent h such that

  $$x_U\, h + x_D\, (1 - h) < p$$

  expects payoff less than price, sells as much as possible

- *Cutoff agent* has belief $h = h^*$ such that just indifferent

  $$x_U\, h^* + x_D\, (1 - h^*) = p \qquad \Rightarrow \qquad h^* = \frac{p - x_D}{x_U - x_D}$$

Solving simultaneously for p and h^*

- Demands for asset

$$y_0^h = \begin{cases} 0 & h \in [0, h^*) \\ \frac{1+p}{p} & h \in [h^*, 1] \end{cases} \qquad \text{where } h^* = \frac{p - x_D}{x_U - x_D}$$

- Market clearing for asset

$$1 = \int_0^1 y_0^h \, dh = \int_0^{h^*} 0 \, dh + \int_{h^*}^1 \frac{1+p}{p} \, dh = \frac{1+p}{p}(1 - h^*)$$

Two equations to solve for p, h^*

Numerical example

- Suppose $x_U = 1, x_D = 0.2$. Then cutoff belief h^*

$$h^* = \frac{p - 0.2}{1 - 0.2} = 1.25p - 0.25$$

- Market clearing

$$1 = \frac{1+p}{p}(1 - h^*) = \frac{1+p}{p}(1.25 - 1.25p)$$

- Rearrange to get quadratic in p

$$p^2 + 0.8p - 1 = 0$$

Only positive solution is $p = 0.68$ which then implies $h^* = 0.60$

Borrowing at exogenous collateral rates

- Suppose borrowing, but constrained by *exogenous collateral rates*

- Loan *promises* φ are noncontingent, same in every state

- Loan collateral is the asset, which can be seized if default. A promise of φ gives lender

$$\min[\varphi, x_U] \qquad \text{if } s = U \text{, good news}$$

$$\min[\varphi, x_D] \qquad \text{if } s = D \text{, bad news}$$

- Motivates simple exogenous *collateral constraint*

$$\varphi_0 \leq x_D y_0$$

Biggest promise that is *sure* to be covered by collateral

Borrowing at exogenous collateral rates

- Expected utility

$$u^h = c_0 + h\, c_U + (1-h)\, c_D$$

- Constraints if borrowing at exogenous collateral rate

$$c_0 + w_0 + p\, y_0 = 1 + p + \frac{1}{1+r}\varphi_0$$

$$\varphi_0 \leq x_D y_0$$

$$c_U = w_0 + x_U\, y_0 - \varphi_0$$

$$c_D = w_0 + x_D\, y_0 - \varphi_0$$

- Borrowing if $\varphi_0 > 0$, lending if $\varphi_0 < 0$, r is interest rate. No borrowing is special case with collateral constraint $\varphi_0 \leq 0 y_0$.

Market clearing with exogenous collateral rates

- Risky asset and loans at initial date and consumption at each date and state

$$\int_0^1 y_0^h \, dh = 1$$

$$\int_0^1 \varphi_0^h \, dh = 0$$

$$\int_0^1 (c_0^h + w_0^h) \, dh = 1$$

$$\int_0^1 c_U^h \, dh = x_U + \int_0^1 w_0^h \, dh$$

$$\int_0^1 c_D^h \, dh = x_D + \int_0^1 w_0^h \, dh$$

Analysis

- Guess interest rate $r = 0$ (linear utility, endowments large enough)

- As before, cutoff agent h^* just indifferent

$$x_U\, h^* + x_D\,(1 - h^*) = p \qquad \Rightarrow \qquad h^* = \frac{p - x_D}{x_U - x_D}$$

- Agents $h < h^*$ sell as much as possible, $y_0^h = 0$ for all $h < h^*$

- Agents $h > h^*$ buy as much as possible. To do this, *borrow the maximum*

$$\varphi_0^h = x_D y_0^h$$

and so for these agents

$$y_0^h = \frac{1 + p + \varphi_0^h}{p} = \frac{1 + p + x_D y_0^h}{p} \quad \Leftrightarrow \quad y_0^h = \frac{1 + p}{p - x_D}$$

Solve simultaneously for p, h^* as before

Numerical example

- Suppose again $x_U = 1, x_D = 0.2$. Then market clearing for asset is

$$1 = \int_0^1 y_0^h \, dh = \int_{h^*}^1 \frac{1+p}{p-0.2} \, dh = \frac{1+p}{p-0.2}(1-h^*)$$

- Eliminating h^* using the indifference condition for the cutoff agent now gives quadratic

$$p^2 + 0.8p - 1.16 = 0$$

Only positive solution is $p = 0.75$ which then implies $h^* = 0.69$. *Asset prices higher, marginal buyer is more optimistic* than without borrowing

Numerical example

- At price $p = 0.75$, buyers $h > h^* = 0.69$ have

$$\text{risky asset} = y_0^h = 3.2, \qquad \text{promise} = \varphi_0^h = 0.64$$

- Sellers $h < h^*$ have zero asset purchases and lend (from loans market clearing, about $\varphi_0^h = -(1-h^*)0.64/h^* = -0.3$ each)

- The leverage ratio is, in this example,

$$\text{leverage} = \frac{\text{asset value}}{\text{asset value} - \text{debt value}} = \frac{p}{p - 0.2} = \frac{0.75}{0.55} \approx 1.4$$

(of course, all $h < h^*$ are not levered). Equivalently, loan/value ratio is $0.2/0.75 = 27\%$ and margin or haircut is $0.55/0.75 = 73\%$

Discussion

- Ability to borrow allows most optimistic agents to ‘leverage’ their beliefs, borrowing to spend more on asset

- Fewer optimistic agents required to buy asset stock, marginal buyer h^* is more optimistic, asset prices higher

- Asset prices don’t just depend on payoff fundamentals, but also on lending standards. Loose lending standards $\Rightarrow$ higher asset prices

- Why? Because asset prices depend on beliefs and beliefs of marginal buyer change as lending standards change (because *who the marginal buyer is* changes)

Endogenous leverage (sketch)

- Collection $\mathcal{L}$ of *loan types*, indexed by collateral requirements

  $$\text{loan contract} = (\text{promise, collateral})$$

- Homogenous of degree one, so can normalize by collateral

- Loan contract $l \in \mathcal{L}$ promises *l units in both states* backed by one unit of asset as collateral

- Each contract l has its own price π_l

Constraints with many loan contracts

$$c_0 + w_0 + p\,y_0 = 1 + p + \sum_l \pi_l \varphi_l$$

$$\sum_l \max[\varphi_l, 0] \le y_0 \qquad \text{(total collateral requirement)}$$

$$c_U = w_0 + x_U\,y_0 - \sum_l \varphi_l \min[x_U, l]$$

$$c_D = w_0 + x_D\,y_0 - \sum_l \varphi_l \min[x_D, l]$$

Implied interest rates $1 + r_l = l/\pi_l$. Borrowing $\varphi_l > 0$ requires collateral. Lending $\varphi_l < 0$ requires no collateral. No-recourse lending: deliver promise or collateral, whichever is less

Equilibrium

- Only *traded loan contract* is $l = x_D$, as before. Other loan contracts priced but not traded

- Loan contracts values by cutoff agent h^*, equilibrium price

  $$\pi_l = h^* l + (1 - h^*) x_D$$

  Hence $l = x_D$ promise has price $\pi_l = x_D$ and interest rate $1 + r_l = l/\pi_l = 1$

- Bigger promises have bigger (shadow) interest rates

  In other words, agents can implicitly borrow more for same collateral by paying higher interest rate

Numerical example

- Consider example with $x_D = 0.2$ and $h^* = 0.69$ in equilibrium
- Price of $l = x_D = 0.2$ loan contract

$$\pi_{0.2} = h^* 0.2 + (1 - h^*) 0.2 = 0.2, \qquad 1 + r_l = \frac{l}{\pi_l}$$

- Price of bigger $l' = 0.3$ loan contract

$$\pi_{0.3} = h^* 0.3 + (1 - h^*) 0.2 = 0.269, \qquad 1 + r_{0.3} = \frac{0.3}{0.269} = 1.12$$

(since here $h^* = 0.69$)

Intuition

- Why is only the $l = 0.2$ contract traded in equilibrium?

- Optimistic agents (say $h = 1$) believe for every $p = 0.75$ paid, get $x_U = 1$ for sure. Wouldn't they borrow more? No.

  – to get bigger loan, $l = 0.4$ say, have to promise to *pay more in good state* and same in bad state

  – but these are the optimistic agents who believe good state will happen for sure, so this is not rational

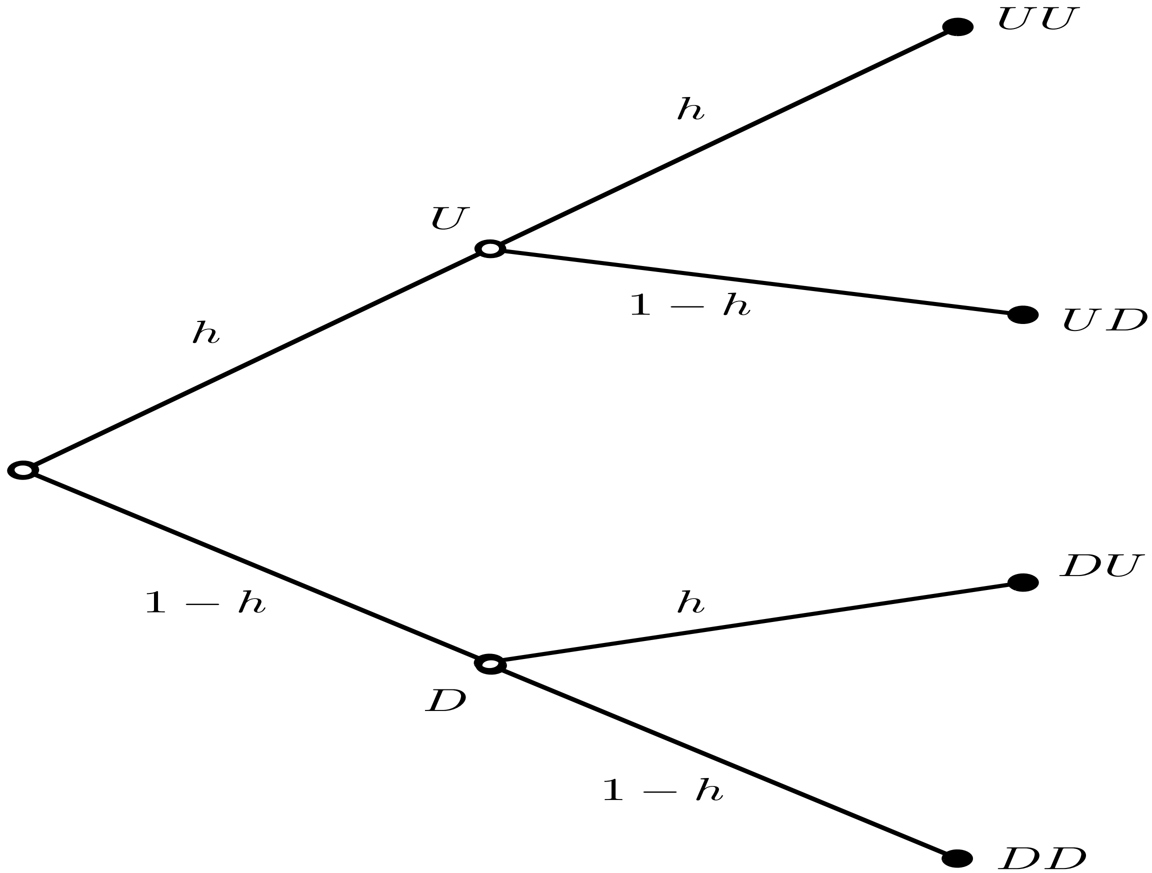
- Pessimistic agents (say $h = 0$) won't give up more at $t = 0$ to get bigger payout in state U they think won't happen

- Only traded loans are those with margins *just tight enough* to rule out default (though, special to this 2×2 example)

Leverage cycle: three-period example

- Three dates $t \in \{0, 1, 2\}$
- Binomial tree
  - two states $s_1 \in \{U, D\}$ at date $t = 1$
  - so four states $s_2 \in \{UU, UD, DU, DD\}$ at date $t = 2$
- Agent $h \in [0, 1]$ believes upticks occur with probability h
- Risky asset pays off at terminal date $t = 2$, nothing at $t = 1$
- But is traded based on interim information s_1 at $t = 1$

Leverage cycle: three-period example



Two states $s_1 \in \{U, D\}$ at date $t = 1$. Four states $s_2 \in \{UU, UD, DU, DD\}$ possible at date $t = 2$. State-contingent payoff at terminal date, nothing at date $t = 1$. But traded at date $t = 1$ based on interim information s_1.

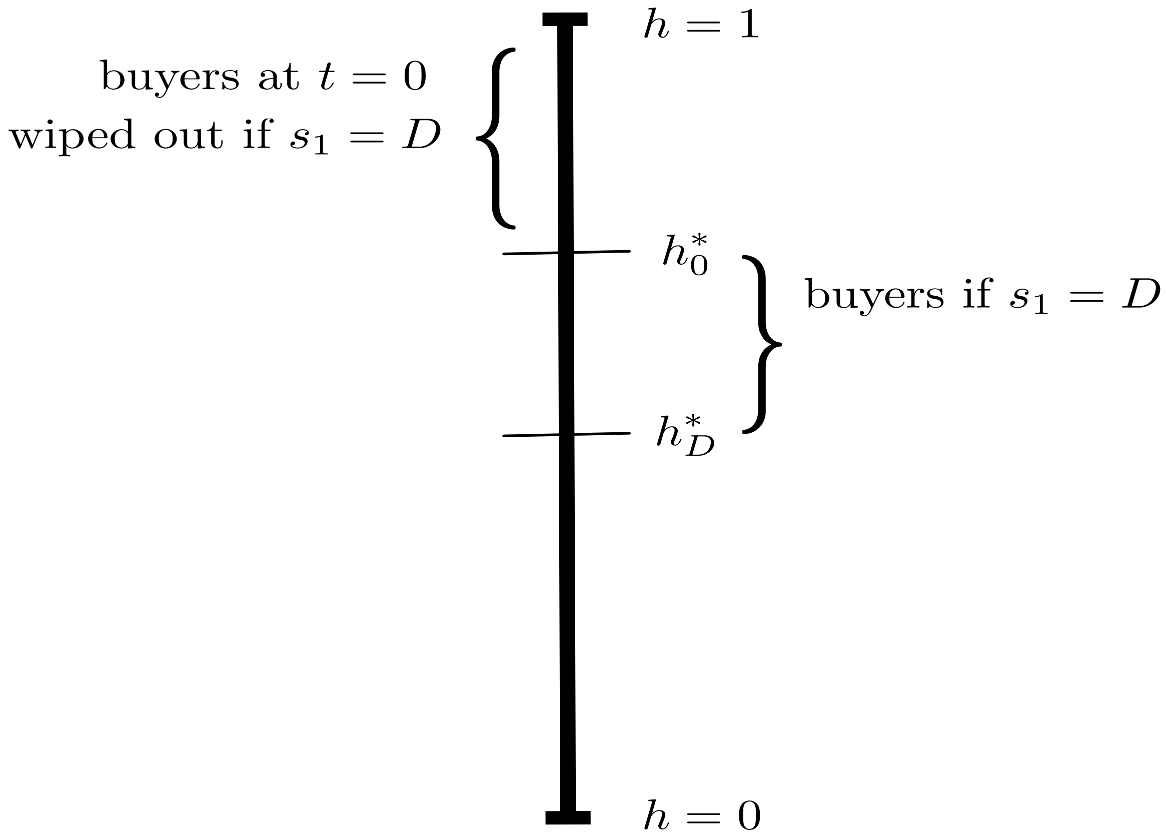
Leverage cycle: three-period example

- Suppose $x_{UU} = x_{UD} = x_{DU} = 1$ but $x_{DD} < 1$
- Then if $s_1 = U$, all uncertainty has been resolved
- Focus on $s_1 = D$, for which (i) there has been bad news, and (ii) there is remaining uncertainty
- Equilibrium characterized in terms of four numbers

$$p_0\,,\ p_D\,,\ h_0^*\,,\ h_D^*$$

asset prices p_0, p_D and cutoff beliefs h_0^*, h_D^* at $t = 0$ and $s_1 = D$

Cutoff beliefs h_0^*, h_D^*



Initial buyers wiped out if bad news, $s_1 = D$. Risky asset then bought by agents $h \in [h_D^*, h_0^*]$ with less optimistic beliefs.

Equilibrium conditions

Four conditions in four unknowns. Solve backwards

- Indifference condition for cutoff belief in state $s_1 = D$

$$h_D^* \, 1 + (1 - h_D^*) x_{DD} = p_D$$

- Market clearing for asset in state $s_1 = D$

$$1 = \int_{h_D^*}^{h_0^*} y_D^h \, dh, \qquad y_D^h = \left(\frac{1 + p_D}{p_D - x_{DD}}\right) \frac{1}{h_0^*}$$

(initial buyers have sold all assets and paid off all loans, giving $1/h_0^*$ each to the remaining agents; new buyers can borrow with collateral rate x_{DD})

Equilibrium conditions

- Market clearing for asset at $t = 0$

$$1 = \int_{h_0^*}^{1} y_0^h \, dh, \qquad y_0^h = (\frac{1 + p_0}{p_0 - p_D})$$

(initial buyers can borrow with collateral rate p_D)

- Cutoff agent at $t = 0$ must be indifferent between buying asset at $t = 0$ or waiting and buying at $s_2 = D$

$$h_0^* \, (\frac{1 - p_D}{p_0 - p_D}) = h_0^* \, 1 + (1 - h_0^*) \left[h_0^* \, (\frac{1 - x_{DD}}{p_D - x_{DD}}) \right]$$

(LHS is expected return from buying at $t = 0$, RHS is expected return from waiting and buying at $s_2 = D$)

- Solve these four equations in four unknowns

Numerical example

- Suppose $x_{DD} = 0.2$ as in previous examples. Then solution is

$$p_0 = 0.95, \quad p_D = 0.69, \quad h_0^* = 0.87, \quad h_D^* = 0.61$$

- Asset price crashes from $p_0 = 0.95$ to $p_D = 0.69$ on bad news

- But bad news alone only explains part of the fall in asset prices

- In addition, marginal buyer is an agent with less optimistic beliefs, initial buyers (most optimistic) wiped out

- Moreover, it becomes harder to borrow (collateral rate falls from $p_D = 0.69$ to $x_{DD} = 0.2$)

Leverage ratios

- Initial leverage

$$\frac{p_0}{p_0 - p_D} = \frac{0.95}{0.95 - 0.69} = 3.65$$

- Falls to

$$\frac{p_D}{p_D - x_{DD}} = \frac{0.69}{0.69 - 0.20} = 1.41$$

- Or equivalently, initial margins (haircuts) rise from $1/3.65 = 27\%$ to $1/1.41 = 71\%$

- In short, the bad news dramatically tightens borrowing constraints, which amplifies the fall in asset prices

Scary news

- The bad news is *scary*

- One-period variance of asset price from $t = 0$ to $t = 1$

$$\mathrm{Var}[p \,|\, h] = h(1-h)(1-p_D)^2 = 0.096\, h(1-h)$$

- One-period variance of asset price from $t = 1$ to $t = 2$ conditional on state $s_1 = D$

$$\mathrm{Var}[p \,|\, h\,,\ D] = h(1-h)(1-x_{DD})^2 = 0.64\, h(1-h)$$

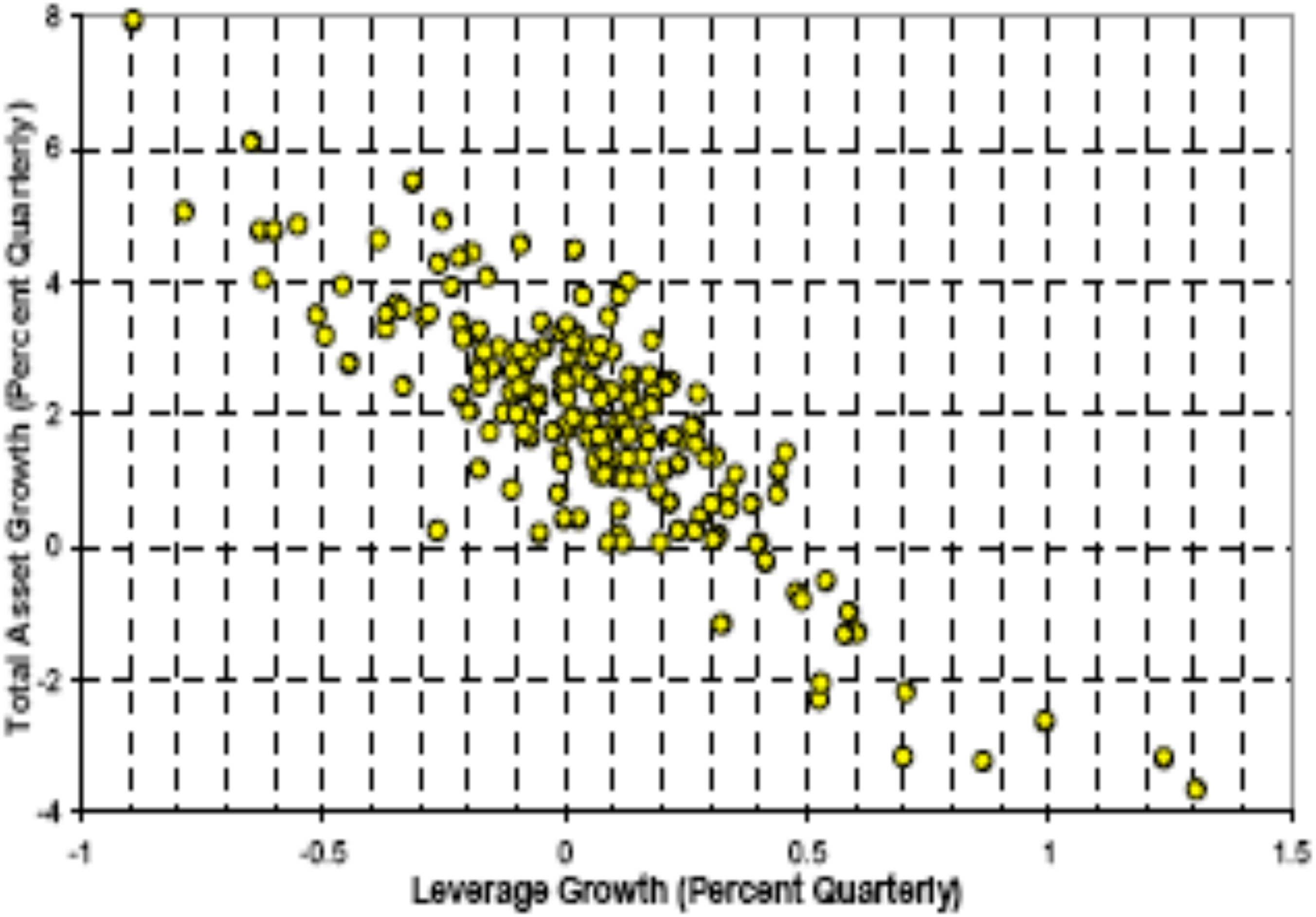
- That is, arrival of bad news increases variance by a factor of

$$(\frac{1-x_{DD}}{1-p_D})^2 = \frac{0.64}{0.096} \approx 6.67$$

Adrian/Shin

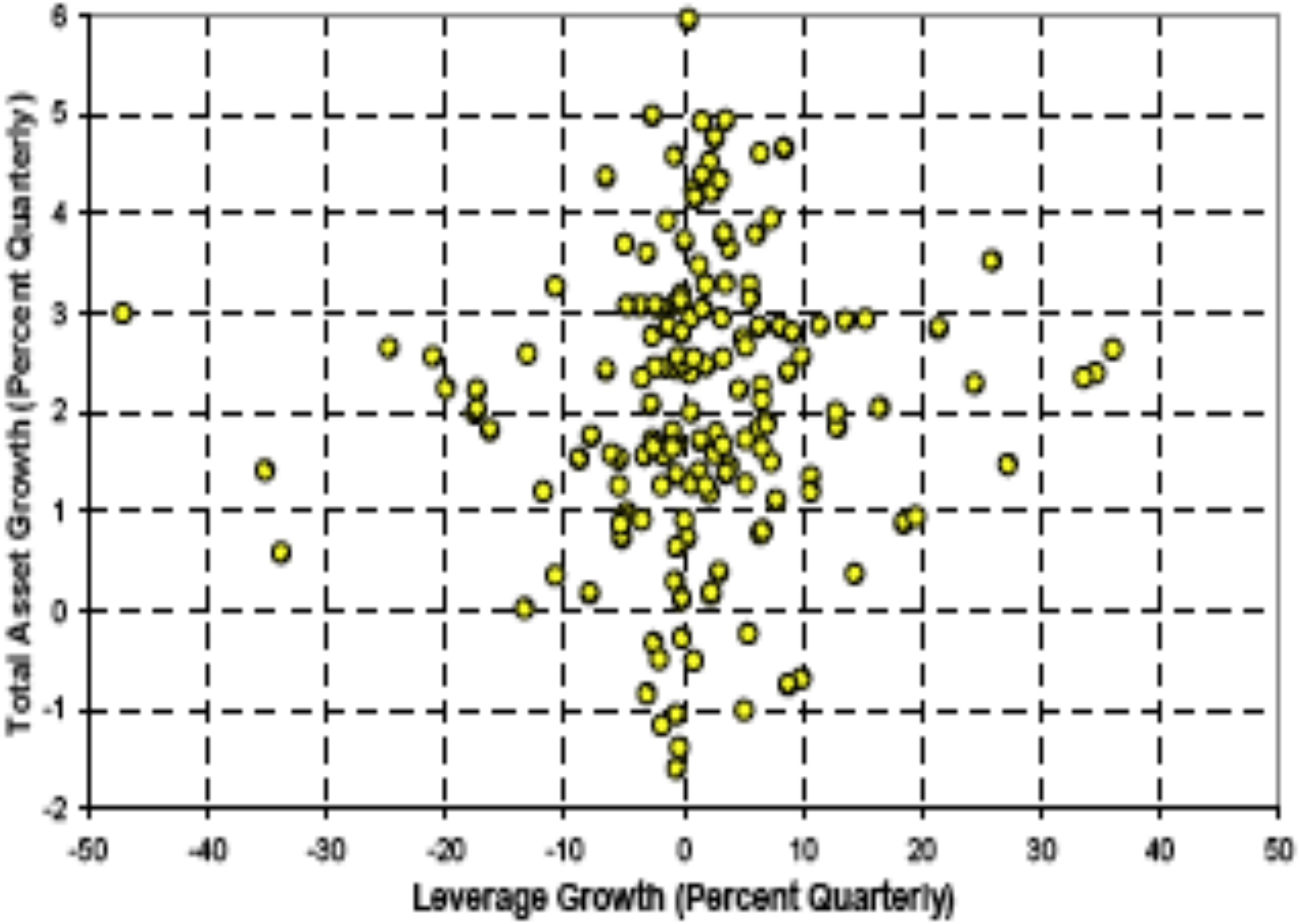
- Households
  - *passive* balance sheet management
    increase in asset value not matched by increase in debt
  - countercyclical leverage ratio
- Financial intermediaries
  - *active* balance sheet management
    increase in asset value matched by increase in debt
  - commercial banks, constant leverage ratio
  - investment banks, procyclical leverage ratio

Households: *passive balance sheets*



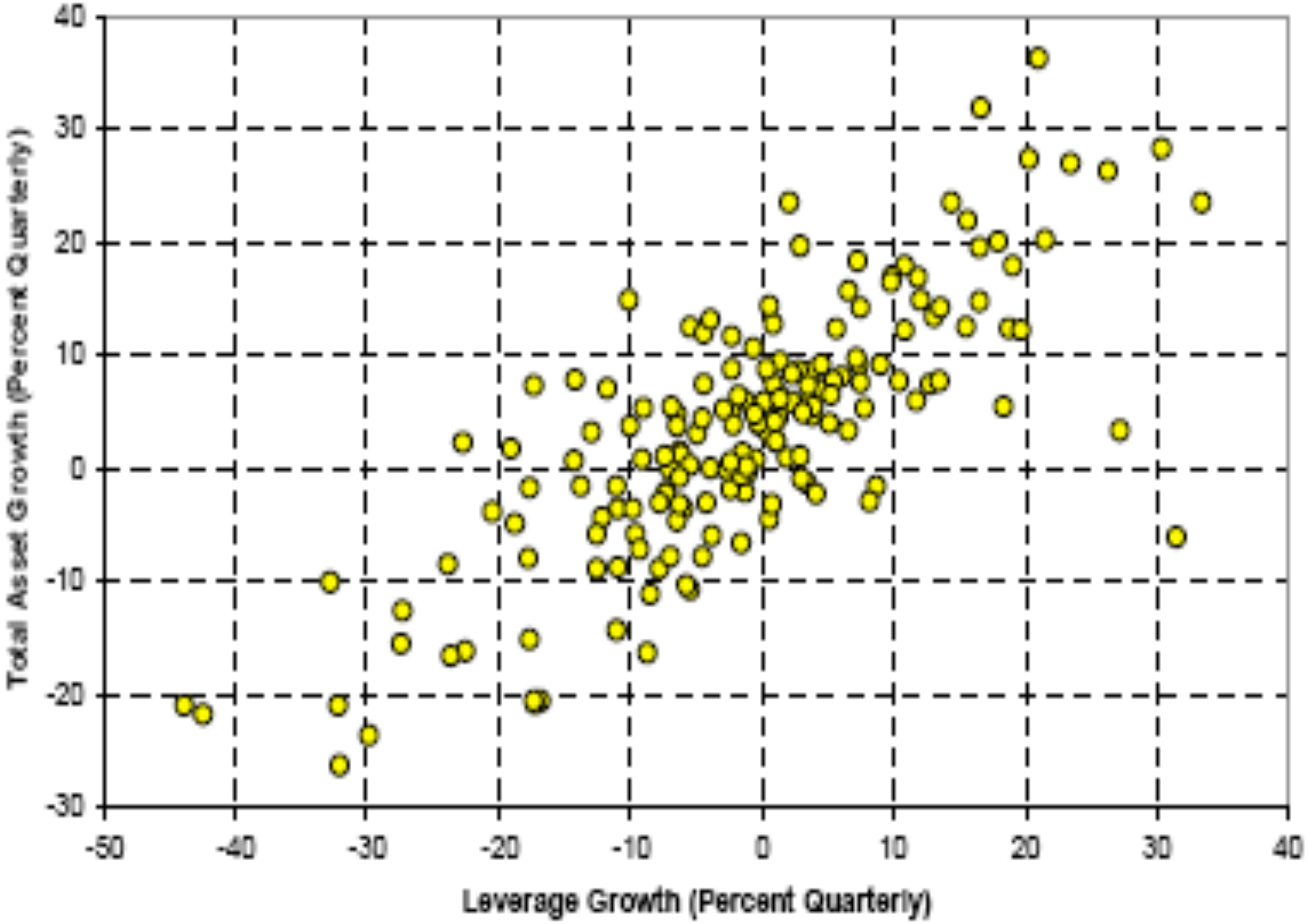
Increase in asset value not matched by increase in debt. Leverage falls. Source: Adrian and Shin (2010).

Commercial banks: constant leverage



Increase in asset value matched by increase in debt. Leverage approximately constant. Source: Adrian and Shin (2010).

Investment banks: *active balance sheets*



For security brokers and dealers (including investment banks), increase in asset value more than matched by increase in debt. Leverage rises. Source: Adrian and Shin (2010).