Monetary Economics

Lecture 2: classical building blocks, part one

Chris Edmond

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This class

- Solving the classical monetary model

  1- real variables independent of nominal variables

  2- fluctuations in real variables driven by fluctuations in productivity

- Reading: Gali (2008), chapter 2 section 2.3 and appendix 2.1

Equilibrium in classical model

- A competitive equilibrium involves
  - households optimising taking prices as given
  - firms optimising taking prices as given
  - prices such that markets clear

- Optimality conditions for labor supply and demand give

$$-\frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = \frac{W_t}{P_t} = A_t F'(N_t)$$

- Goods market clearing

$$Y_t = C_t$$

- Bond market clears if goods market clears

Standard functional forms

- Utility function

$$U(C,N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}$$

σ is (constant) coefficient of relative risk aversion (CRRA)

$$\sigma = -\frac{U_{cc}(\cdot)\,C}{U_c(\cdot)}$$

$1/\sigma$ is (constant) intertemporal elasticity of substitution (IES)

$$\frac{C^{1-\sigma}-1}{1-\sigma} \to \log C \qquad \text{as } \sigma \to 1$$

$1/\varphi$ is (λ-constant) Frisch elasticity of labor supply

- Typical numbers in macro, $\sigma = 1$ or 2 and $\varphi = 0.4$

Standard functional forms

- Production function

$$Y = A\,F(N) = A\,N^{1-\alpha}$$

$1-\alpha$ is labor's (constant) share in final output

$$1-\alpha = \frac{W\,N}{P\,Y}$$

- Implicitly α share is paid to capital and other fixed factors

- Typical number in macro, $\alpha = 1/3$ so that labor's share is $2/3$

Solving the model

- With these functional forms

$$N^{\varphi} C^{\sigma} = \frac{W}{P} = (1-\alpha) A N^{1-\alpha}$$

- Goods market clearing

$$C = Y = A N^{1-\alpha}$$

- Two equations in two unknowns C, N given exogenous A

- Solve for real variables independently of all nominal variables

Notation

- Little letters denote logs (or other proportional variables)

$$c \equiv \log C, \qquad n \equiv \log N, \qquad a \equiv \log A$$

- Bars denote non-stochastic steady state

$$\bar{c} \equiv \log \bar{C}, \qquad \bar{n} \equiv \log \bar{N}, \qquad \bar{a} \equiv \log \bar{A}$$

where

$$\bar{A} \equiv \mathbb{E}\{A\}$$

- Hats denote log deviations

$$\hat{c} \equiv c - \bar{c}, \qquad \hat{n} \equiv n - \bar{n}, \qquad \hat{a} \equiv a - \bar{a}$$

(approximate percentage deviation from steady state)

Solution

- Equilibrium labor

$$\hat{n} = \psi_{na}\,\hat{a}, \qquad \psi_{na} \equiv \frac{1-\sigma}{(1-\alpha)\sigma + \alpha + \varphi}$$

- Equilibrium output and consumption

$$\hat{y} = \psi_{ya}\,\hat{a}, \qquad \psi_{ya} \equiv \frac{1+\varphi}{(1-\alpha)\sigma + \alpha + \varphi}, \qquad \hat{c} = \hat{y}$$

Interpreting the solution

- Coefficients are *elasticities*

$$\frac{dn}{da} = \psi_{na} = \frac{1-\sigma}{(1-\alpha)\sigma + \alpha + \varphi}$$

one percent change in exogenous productivity a gives ψ_{na} percent change in equilibrium labor n

- Easy to calculate volatilities

$$\text{std}\{n\} = |\psi_{na}|\,\text{std}\{a\}$$

and

$$\text{std}\{y\} = |\psi_{ya}|\,\text{std}\{a\}$$

Interpreting the solution

- Relative volatilities

$$\frac{\text{std}\{n\}}{\text{std}\{y\}} = \frac{|\psi_{na}|}{|\psi_{ya}|} = \frac{|1-\sigma|}{1+\varphi}$$

- Correlations

$$\text{corr}\{n,y\} \equiv \frac{\text{cov}\{n,y\}}{\text{std}\{n\}\,\text{std}\{y\}} = \frac{\psi_{na}\,\psi_{ya}}{|\psi_{na}|\,|\psi_{ya}|}$$

- Endogenous real variables $n, c, y, w-p$ inherit time series properties of exogenous productivity a (autocorrelation, impulse responses, etc)

Euler equation

- Recall nominal bond prices Q_t given by

$$Q_t = \mathbb{E}_t \left\{ \beta \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

- With our standard separable preferences

$$Q_t = \mathbb{E}_t \left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}$$

- Write in terms of nominal interest rate i_t, inflation π_t etc

$$i_t \equiv -\log Q_t, \quad \rho \equiv -\log \beta, \quad p_t \equiv \log P_t, \quad \pi_{t+1} \equiv p_{t+1} - p_t$$

- So Euler equation can be written

$$1 = \mathbb{E}_t \left\{ \exp(-\rho - \sigma \Delta c_{t+1} - \pi_{t+1} + i_t) \right\}$$

Log-linearised Euler equation

- Euler equation

$$1 = \mathbb{E}_t \left\{ \exp(z_{t+1}) \right\}$$

where

$$z_{t+1} \equiv -\rho - \sigma \Delta c_{t+1} - \pi_{t+1} + i_t$$

- First order approximation of $\exp(z)$ around $z \approx 0$ is

$$\exp(z) \approx \exp(0) + \exp(0)(z - 0) = 1 + z$$

- Treat approximation as exact and simplify

$$i_t = \rho + \sigma \mathbb{E}_t\{\Delta c_{t+1}\} + \mathbb{E}_t\{\pi_{t+1}\}$$

Euler equation and Fisher equation

- Define ex ante real interest rate r_t by Fisher equation

$$r_t \equiv i_t - \mathbb{E}_t\{\pi_{t+1}\}$$

- And since log linear Euler equation is

$$i_t = \rho + \sigma\mathbb{E}_t\{\Delta c_{t+1}\} + \mathbb{E}_t\{\pi_{t+1}\}$$

- Can eliminate nominal interest rate i_t to get

$$r_t = \rho + \sigma\mathbb{E}_t\{\Delta c_{t+1}\}$$

- Familiar version of Euler equation

$$\mathbb{E}_t\{\Delta c_{t+1}\} = \frac{r_t - \rho}{\sigma}$$

(recall, $1/\sigma$ is intertemporal elasticity of substitution)

Equilibrium real interest rates

- Up to log linear approximation, real interest rates r_t given by

$$r_t = \rho + \sigma \mathbb{E}_t\{\Delta c_{t+1}\}$$

- Aside: in new Keynesian literature, this often re-written as

$$y_t = -\frac{r_t - \rho}{\sigma} + \mathbb{E}_t\{y_{t+1}\}$$

and called the *dynamic IS curve* (why?)

- Equilibrium real interest rates therefore

$$r_t = \rho + \sigma \psi_{ya} \mathbb{E}_t\{\Delta a_{t+1}\}$$

Equilibrium real interest rates

- Conditional expectation determined by productivity process
- AR(1) example

$$a_{t+1} = \phi_a \, a_t + \varepsilon_{a,t+1} \qquad \varepsilon_{a,t+1} \sim \text{IID and } N(0, \sigma_\varepsilon^2)$$

  with persistence $0 < \phi_a < 1$

  Then

$$r_t = \rho - \sigma \psi_{ya}(1 - \phi_a) a_t$$

  moves in opposite direction to productivity, since $\phi_a < 1$

- All interesting real variables now determined, all independent of nominal variables

Next class

- Money demand and price level determination
- Reading: Gali (2008), chapter 2 sections 2.4–2.5