Monetary Economics

Lecture 15: unemployment in the new Keynesian model, part one

Chris Edmond

2nd Semester 2014

This class

- Unemployment fluctuations in the new Keynesian model, part one

- Main reading:

  ◇ Gali, *Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective*, MIT Press, 2011, sections 1.1–1.3

- Alternate reading:

  ◇ Gali, "The return of the wage Phillips curve" *Journal of the European Economic Association*, 2011

Available from the LMS

This class

1- Static model

- unemployment due to *real* wage rigidity

2- Dynamic model

- fluctuations in unemployment due to *nominal* wage rigidity
- parallel treatment of sticky wages and sticky prices

Static model: overview

- Monopolistically competitive firms
- Representative household which supplies differentiated labor inputs (perfect consumption insurance)
- Union, acting on behalf of household, has wage-setting power (gap between labor demand and supply $\Rightarrow$ unemployment)

Firms

- Many differentiated products $z \in [0, 1]$

- Many differentiated types of labor $i \in [0, 1]$

- Firm z produces with CES bundle of labor inputs

$$Y(z) = AN(z)^{1-\alpha}$$

where

$$N(z) = \left(\int_0^1 N(i, z)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} \, di \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \qquad \varepsilon_w > 1$$

- Final consumption good is CES bundle of differentiated products

$$C = \left(\int_0^1 C(z)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} \, dz \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}}, \qquad \varepsilon_p > 1$$

Representative household

- Continuum of members indexed by $(i, j) \in [0, 1] \times [0, 1]$

  i type of labor
  j amount of disutility from working is χj^{φ}

  indivisible labor: either work or not, $l(i, j) \in \{0, 1\}$

- Utility function

$$U = \log C - \chi \int_0^1 \int_0^1 j^{\varphi} l(i, j)\, dj\, di$$

where C is the CES index of differentiated products

Firms' problem

- Choose price $P(z)$ and labor $\{N(i,z)\}$ to maximize profits

$$P(z)Y(z) - \int_0^1 W(i)N(i,z)\,di$$

subject to the usual product demand curve

$$Y(z) = C(z) = \left(\frac{P(z)}{P}\right)^{-\varepsilon_p} C$$

and the technological constraints

$$Y(z) = AN(z)^{1-\alpha}$$

$$N(z) = \left(\int_0^1 N(i,z)^{\frac{\varepsilon_w - 1}{\varepsilon_w}}\,di\right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \qquad \varepsilon_w > 1$$

Firms: labor demand

- *Labor demand*: demand curve for differentiated labor inputs

$$N(i,z) = \left(\frac{W(i)}{W}\right)^{-\varepsilon_w} N(z)$$

- *Wage index*: plugging labor demands back into wage bill

$$W = \left(\int_0^1 W(i)^{1-\varepsilon_w}\, di\right)^{\frac{1}{1-\varepsilon_w}}$$

so that the wage bill can be written

$$WN(z) \equiv \int_0^1 W(i)N(i,z)\, di$$

Firms: price-setting

- Total cost of $Y(z)$ output

$$W\left(\frac{Y(z)}{A}\right)^{\frac{1}{1-\alpha}}$$

- Marginal cost of $Y(z)$ is then

$$\frac{1}{1-\alpha}\frac{W}{A}\left(\frac{Y(z)}{A}\right)^{\frac{\alpha}{1-\alpha}}$$

- *Price setting*: price is constant markup over marginal cost

$$P(z)=\frac{\varepsilon_p}{\varepsilon_p-1}\left(\frac{1}{1-\alpha}\frac{W}{A}\left(\frac{Y(z)}{A}\right)^{\frac{\alpha}{1-\alpha}}\right)$$

or in terms of the *marginal product of labor* (MPN)

$$(1-\alpha)AN(z)^{-\alpha}=\frac{\varepsilon_p}{\varepsilon_p-1}\frac{W}{P(z)}$$

Household problem

- Choose aggregate C and participation $l(i,j) \in \{0,1\}$ to maximize

$$\log C - \chi \int_0^1 \int_0^1 j^{\varphi} l(i,j)\, dj\, di$$

subject to the budget constraint

$$PC \leq \int_0^1 \int_0^1 W(i) l(i,j)\, dj\, di + \Pi$$

- *Lagrangian* can be written

$$\mathcal{L} = \log C + \lambda(\Pi - PC) + \int_0^1 \int_0^1 \left(\lambda W(i) - \chi j^{\varphi}\right) l(i,j)\, dj\, di$$

Household participation decisions

- *Increment* to the Lagrangian from choosing $l(i,j)=1$

$$(\lambda W(i) - \chi j^{\varphi})$$

  Utility increases for every (i,j) such that this increment is positive

- For each i there is a $L(i)$ such that all $j \in [0, L(i)]$ *participate.* Marginal participant solves

$$\lambda W(i) = \chi L(i)^{\varphi}$$

- And from the first order condition for aggregate consumption

$$\frac{1}{C} = \lambda P$$

  so in terms of the household's *marginal rate of substitution* (MRS)

$$\frac{W(i)}{P} = \chi C L(i)^{\varphi}$$

- Participation margin set by neoclassical labor supply condition

Wage setting

- A *union* chooses $W(i)$ for each labor type to maximize utility

$$\log C - \chi \int_0^1 \int_0^{N(i)} j^{\varphi} \, dj \, di = \log C - \chi \int_0^1 \frac{N(i)^{1+\varphi}}{1+\varphi} \, di$$

subject to labor demand curve

$$N(i) = \left(\frac{W(i)}{W} \right)^{-\varepsilon_w} N$$

and the household budget constraint

$$PC \leq \int_0^1 W(i) N(i) \, di + \Pi$$

- Employment $N(i)$ is then *demand-determined*

Wage setting

- Lagrangian for union wage-setting can be written

$$\mathcal{L} = \log C + \lambda(\Pi - PC)$$
$$+ \int_0^1 \left[\lambda W(i)^{1-\varepsilon_w} W^{\varepsilon_w} N - \frac{\chi}{1+\varphi} \left(\left(\frac{W(i)}{W} \right)^{-\varepsilon_w} N \right)^{1+\varphi} \right] di$$

Wage markup

- First order condition for this problem can be written

$$W(i) = \frac{\varepsilon_w}{\varepsilon_w - 1}\frac{1}{\lambda}\chi N(i)^{\varphi}$$

where $\lambda = 1/PC$ is the multiplier on the budget constraint

- Thus real wage is a markup over the "marginal rate of substitution"

$$\frac{W(i)}{P} = \frac{\varepsilon_w}{\varepsilon_w - 1}\chi C N(i)^{\varphi}$$

But note this is in terms of $N(i)$ whereas the household's MRS is in terms of $L(i)$

- The wage markup puts a *wedge* between labor demand and labor supply, hence unemployment

Symmetric equilibrium

- Symmetry: $P(z) = P$, $N(z) = N$, $W(i) = W$, $L(i) = L$ etc

- Goods market clearing

$$C = Y = AN^{1-\alpha}$$

- Labor demand / price-markup condition

$$(1-\alpha)AN^{-\alpha} = \frac{\varepsilon_p}{\varepsilon_p - 1}\frac{W}{P}$$

- Wage-markup condition

$$\frac{W}{P} = \frac{\varepsilon_w}{\varepsilon_w - 1}\chi C N^{\varphi}$$

- Participation condition

$$\frac{W}{P} = \chi C L^{\varphi}$$

Log-linear equilibrium

- Notation: $p = \log P$, $n = \log N$, $w = \log W$, etc

- Log marginal product of labor

$$mpn \equiv a - \alpha n + \log(1 - \alpha)$$

- Log marginal rate of substitution (in terms of n)

$$mrs \equiv c + \varphi n + \log \chi$$

- Unemployment

$$u \equiv l - n$$

Labor market

- Log labor demand / price-markup condition

$$w - p = mpn - \mu_p$$

- Log wage-markup condition

$$w - p = c + \varphi n + \log \chi + \mu_w$$

where μ_p and μ_w are log markups

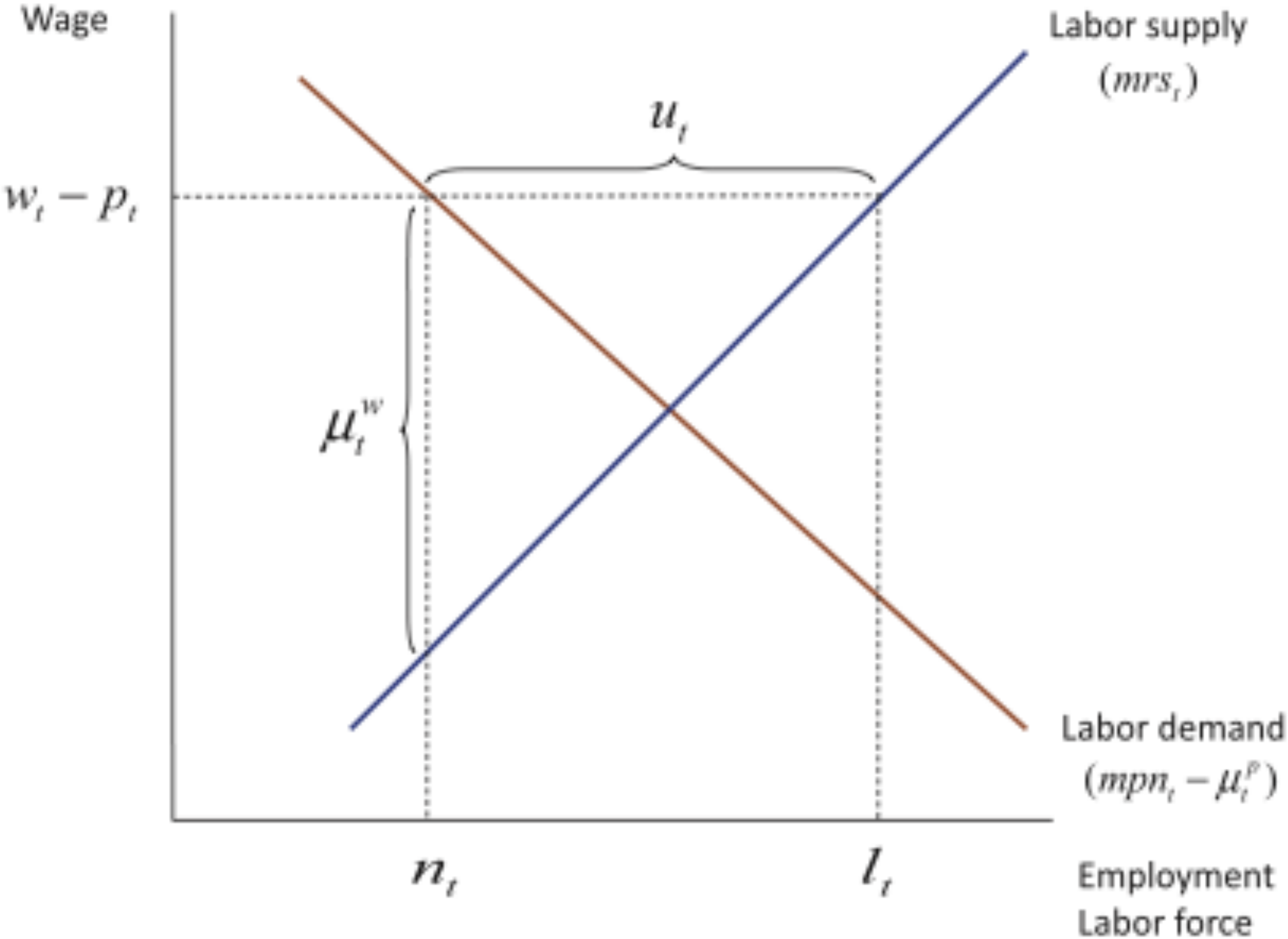
- Log participation decision and definition of unemployment

$$w - p = c + \varphi(n + u) + \log \chi$$

- Hence

$$u = \frac{\mu_w}{\varphi}$$

Unemployment and wage markup



Summary

- Level of unemployment determined by wage markup μ_w, a *real rigidity* that puts a wedge between labor demand and supply

- *Nominal wage rigidities* then give rise to fluctuations in unemployment around this level

- In the absence of nominal wage rigidities, unemployment would be constant (the *natural* level of unemployment)

- Now turn to dynamic model with nominal wage rigidities

Calvo stickiness

- Firms have IID probability θ_p of being stuck with same price
- Unions have IID probability θ_w of being stuck with same wage
- Law of motion for log price level

$$p_t = \theta_p p_{t-1} + (1 - \theta_p) p_t^*$$

- Law of motion for log wage level

$$w_t = \theta_w w_{t-1} + (1 - \theta_w) w_t^*$$

where p_t^* and w_t^* are reset price and wage

Reset price and wage conditions

- As in basic model, reset price satisfies the condition

$$p_t^* = \mu^p + (1 - \theta_p \beta)\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta_p \beta)^k [mc_{t,t+k} + p_{t+k}] \right\}$$

  where $mc_{t,t+k}$ is real marginal cost in period $t + k$ of firm that last set price in t

- Likewise reset wage satisfies the condition

$$w_t^* = \mu^w + (1 - \theta_w \beta)\mathbb{E}_t \left\{ \sum_{k=0}^{\infty} (\theta_w \beta)^k [mrs_{t,t+k} + p_{t+k}] \right\}$$

  where $mrs_{t,t+k}$ is real marginal rate of substitution in period $t + k$ given wage last set in t

- In absence of stickiness ($\theta_p = 0$ and/or $\theta_w = 0$) have static markups

$$p_t^* = \mu^p + mc_t + p_t \quad \text{and} \quad w_t^* = \mu^w + mrs_t + p_t$$

- As in basic model, real marginal cost satisfies

$$
\begin{aligned}
mc_{t,t+k} &= mc_{t+k} + \frac{\alpha}{1-\alpha}(y_{t,t+k} - y_{t+k}) \\
&= mc_{t+k} - \frac{\alpha \varepsilon_p}{1-\alpha}(p_t^* - p_{t+k})
\end{aligned}
$$

- Likewise, real marginal rate of substitution satisfies

$$
\begin{aligned}
mrs_{t,t+k} &= mrs_{t+k} + \varphi(n_{t,t+k} - n_{t+k}) \\
&= mrs_{t+k} - \varphi \varepsilon_w (w_t^* - w_{t+k})
\end{aligned}
$$

Aggregate price and wage markups

- Define the aggregate (economy wide) price markup

$$\mu_t^p \equiv p_t - \psi_t = -mc_t$$

- Define the aggregate (economy wide) wage markup

$$\mu_t^w \equiv w_t - p_t - mrs_t$$

Solutions for p_t^* and w_t^*

- Plug these into the reset price condition and rearrange to get

$$p_t^* = (1-\theta_p\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta_p\beta)^k\left[p_{t+k} - \Theta_p(\mu_t^p - \mu^p)\right]\right\}$$

- Likewise for the reset wage condition

$$w_t^* = (1-\theta_w\beta)\mathbb{E}_t\left\{\sum_{k=0}^{\infty}(\theta_w\beta)^k\left[w_{t+k} - \Theta_w(\mu_t^w - \mu^w)\right]\right\}$$

where

$$\Theta_p \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p} \quad \text{and} \quad \Theta_w \equiv \frac{1}{1+\varphi\varepsilon_w}$$

- Write the sum as a difference equation

$$\begin{aligned} p_t^* &= (1-\theta_p\beta)[p_t - \Theta_p(\mu_t^p - \mu^p)] + \theta_p\beta\mathbb{E}_t\left\{p_{t+1}^*\right\} \\ &= -(1-\theta_p\beta)\Theta_p(\mu_t^p - \mu^p) + p_t + \theta_p\beta\mathbb{E}_t\left\{p_{t+1}^* - p_t\right\} \end{aligned}$$

- Subtract p_{t-1} from both sides

$$p_t^* - p_{t-1} = -(1-\theta_p\beta)\Theta_p(\mu_t^p - \mu^p) + (p_t - p_{t-1}) + \theta_p\beta\mathbb{E}_t\left\{p_{t+1}^* - p_t\right\}$$

- Use the law of motion for the price level

$$\pi_t^p \equiv p_t - p_{t-1} = (1-\theta_p)(p_t^* - p_{t-1})$$

Price inflation

- Gives *price-inflation* equation

$$\pi_t^p = -\lambda_p(\mu_t^p - \mu^p) + \beta\mathbb{E}_t\left\{\pi_{t+1}^p\right\}$$

where

$$\lambda_p \equiv \frac{(1-\theta_p)(1-\theta_p\beta)}{\theta_p}\Theta_p$$

Wage inflation

- Same derivation gives *wage-inflation* equation

$$\pi_t^w = -\lambda_w(\mu_t^w - \mu^w) + \beta \mathbb{E}_t \left\{ \pi_{t+1}^w \right\}$$

where

$$\pi_t^w \equiv w_t - w_{t-1}$$

and

$$\lambda_w \equiv \frac{(1-\theta_w)(1-\theta_w \beta)}{\theta_w} \Theta_w$$

Wage inflation

- Recall that unemployment is proportional to wage-markup

$$u_t = \frac{\mu_t^w}{\varphi}$$

Hence

$$(\mu_t^w - \mu^w) = \varphi(u_t - u^n)$$

where u^n is the natural rate of unemployment, the rate that would prevail if no wage stickiness

- Gives wage inflation in terms of unemployment

$$\pi_t^w = -\lambda_w \varphi(u_t - u^n) + \beta \mathbb{E}_t \left\{ \pi_{t+1}^w \right\}$$

Next class

- Unemployment fluctuations in the new Keynesian model, part two
- Responses to shocks, volatility and persistence of unemployment
- Main reading:
  - Gali, *Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective*, MIT Press, 2011, sections 1.3–1.4