# Monetary Economics

## Lecture 14: monetary/fiscal interactions in the new Keynesian model, part four

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# This class

- Optimal policy in a liquidity trap *with commitment*
- Reading:
  - Werning, "Managing a liquidity trap: Monetary and fiscal policy" MIT working paper 2012, sections 4–7

Available from the LMS

# This class

**1-** Monetary policy with commitment

- optimal path of interest rates, inflation and output
- importance of commitment to output boom after trap

**1-** Fiscal policy with commitment

- optimal pattern of government purchases
- decomposition into ‘opportunistic’ and ‘stimulus’ components

# Optimal monetary policy with commitment

- Monetary policy minimizes

$$L = \frac{1}{2}\int_0^\infty e^{-\rho t}\left(x(t)^2 + \lambda\pi(t)^2\right)dt$$

subject to the constraints

$$\begin{aligned}\dot{x}(t) &= \sigma^{-1}(i(t) - \pi(t) - r(t))\\ \dot{\pi}(t) &= \rho\pi(t) - \kappa x(t)\\ i(t) &\geq 0\end{aligned}$$

taking as given path $r(t)$

- Control $i$, state $x, \pi$ with free initial conditions $x(0), \pi(0)$

# Optimal monetary policy with commitment

- Hamiltonian for this problem

$$\mathcal{H} = \frac{1}{2}(x^2 + \lambda\pi^2) + \mu_x(\sigma^{-1}(i - \pi - r)) + \mu_\pi(\rho\pi - \kappa x) - \psi i$$

with costates $\mu_x, \mu_\pi$ and multiplier on ZLB constraint $\psi$

- Key optimality conditions

$$\mu_x(t)\sigma^{-1} = \psi(t), \qquad \psi(t)i(t) = 0 \qquad \text{with comp. slackness}$$

and

$$\rho\mu_x(t) - \dot{\mu}_x(t) = x(t) - \kappa\mu_\pi(t)$$
$$\rho\mu_\pi(t) - \dot{\mu}_\pi(t) = \lambda\pi(t) - \sigma^{-1}\mu_x(t) + \rho\mu_\pi(t)$$

# Optimal monetary policy with commitment

- Hence system can be written

$$\mu_x(t) \geq 0, \qquad \mu_x(t)i(t) = 0$$

with

$$\begin{aligned}
\dot{\mu}_x(t) &= \rho\mu_x(t) - x(t) + \kappa\mu_\pi(t) \\
\dot{\mu}_\pi(t) &= -\lambda\pi(t) + \sigma^{-1}\mu_x(t) \\
\dot{x}(t) &= \sigma^{-1}(i(t) - \pi(t) - r(t)) \\
\dot{\pi}(t) &= \rho\pi(t) - \kappa x(t)
\end{aligned}$$

taking as given path $r(t)$

- Boundary conditions: (i) $\mu_x(0) = 0$ and $\mu_\pi(0) = 0$, since both $x(0)$ and $\pi(0)$ are free, and (ii) two transversality conditions

# Preliminaries

- Suppose ZLB not binding, $\psi(t) = 0$ hence $\mu_x(t) = \dot{\mu}_x(t) = 0$ so that

  $$x(t) = \kappa\mu_\pi(t)$$

  hence

  $$\dot{x}(t) = \kappa\dot{\mu}_\pi(t) = \kappa\Big( -\lambda\pi(t) + \sigma^{-1}0\Big) = -\kappa\lambda\pi(t)$$

  but by the Euler equation

  $$\dot{x}(t) = \sigma^{-1}(i(t) - \pi(t) - r(t))$$

- Solving for $i(t)$ then gives

  $$i(t) = I(r(t), \pi(t)), \quad \text{where} \quad I(r, \pi) := r + (1 - \sigma\kappa\lambda)\pi$$

  This is the optimal nominal rate *whenever the ZLB is not binding*. $I(r, \pi) \geq 0$ is necessary for ZLB to not bind. But not sufficient.

# Approach

- Three phases

  **I.** During the liquidity trap, $t \in [0, T)$

  **II.** Just out of the trap, $t \in [T, \hat{T})$

  **III.** After the storm has passed, $t \in [\hat{T}, \infty)$

- Need to 'stitch together' three phase diagrams
- Key is whether $x(t), \pi(t)$ are free at critical dates $t = 0, T, \hat{T}$
- Solve backwards from terminal conditions

# Phase III. After the storm

- At beginning of Phase III $x(\hat{T}), \pi(\hat{T})$ are given (not free)
- ZLB is *not binding* so $i(t) = I(\bar{r}, \pi(t))$
- Under this control, motion of system given by

$$\begin{aligned}\dot{x}(t) &= -\kappa\lambda\pi(t)\\ \dot{\pi}(t) &= \rho\pi(t) - \kappa x(t)\end{aligned}$$

- Solve with method of undetermined coefficients. Guess $x(t) = \phi\pi(t)$ for some $\phi$. Then $\dot{x}(t) = \phi\dot{\pi}(t)$ so

$$-\kappa\lambda\pi(t) = \phi\Big(\rho\pi(t) - \kappa\phi\pi(t)\Big)$$

# Phase III. After the storm

- Since this must hold for all $\pi(t)$ we have the restriction

$$Q(\phi) = \kappa\phi^2 - \rho\phi - \kappa\lambda = 0$$

- Solving for the roots of this quadratic

$$\phi_1, \phi_2 = \frac{\rho \pm \sqrt{\rho^2 + 4\kappa^2\lambda}}{2\kappa}$$

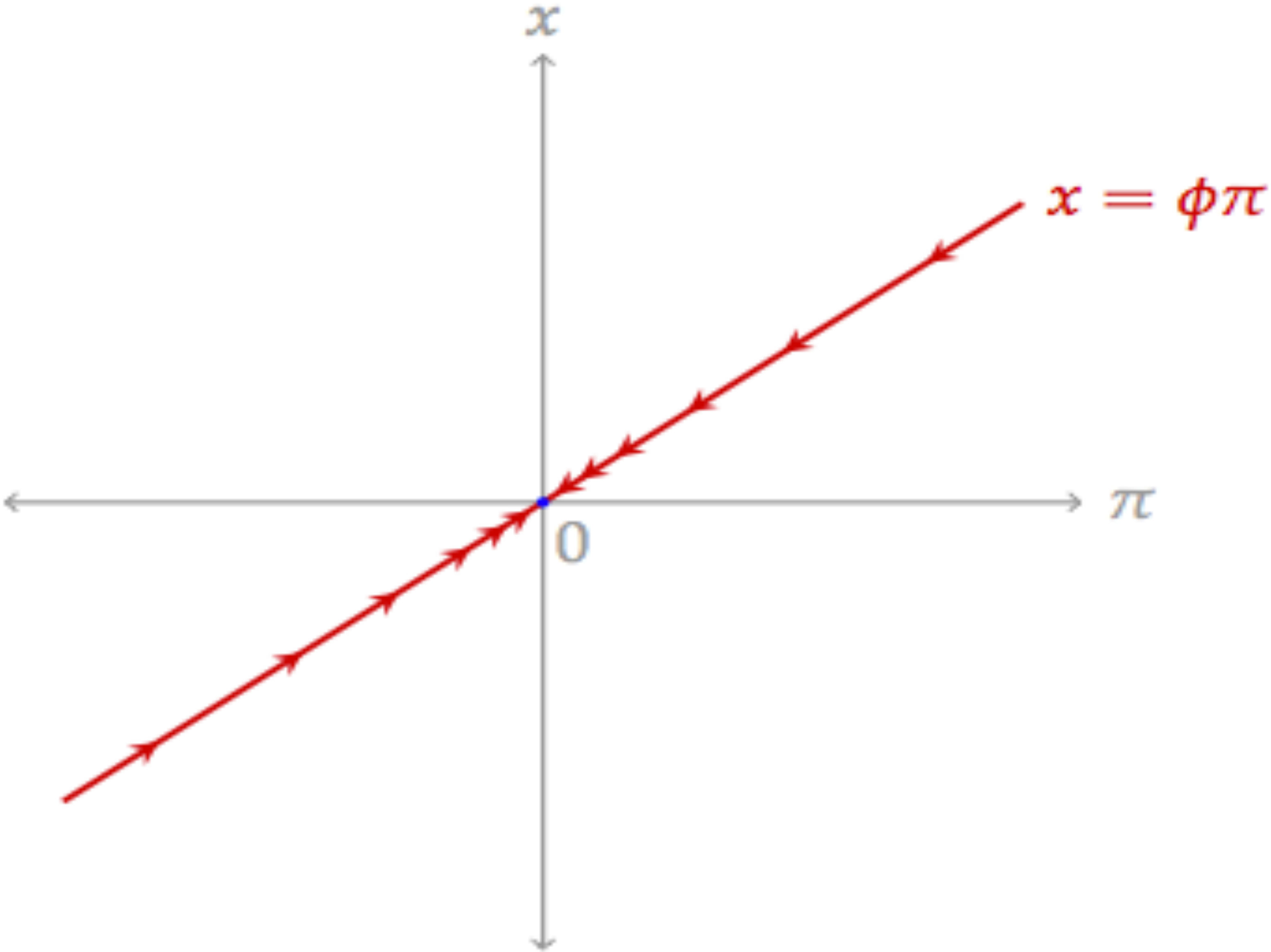
(one of which is positive, the other negative)

- We want these dynamics to take us *towards* $x(\infty) = \pi(\infty) = 0$, so we choose the positive solution

$$\phi = \frac{\rho + \sqrt{\rho^2 + 4\kappa^2\lambda}}{2\kappa} > \frac{\rho}{\kappa} > 0$$

# Phase III. After the storm

$\phi$ is slope of *saddle-path* through $(0,0)$ with $i(t) = I(\bar{r}, \pi(t))$ for $t \in [\hat{T}, \infty)$



# Phase II. Just out of the trap

- At beginning of Phase II $x(T), \pi(T)$ are given (not free)

- Liquidity trap is over but $i(t) = 0$ is *still optimal*. Policy commits to keeping $i(t) = 0$ even after trap is over

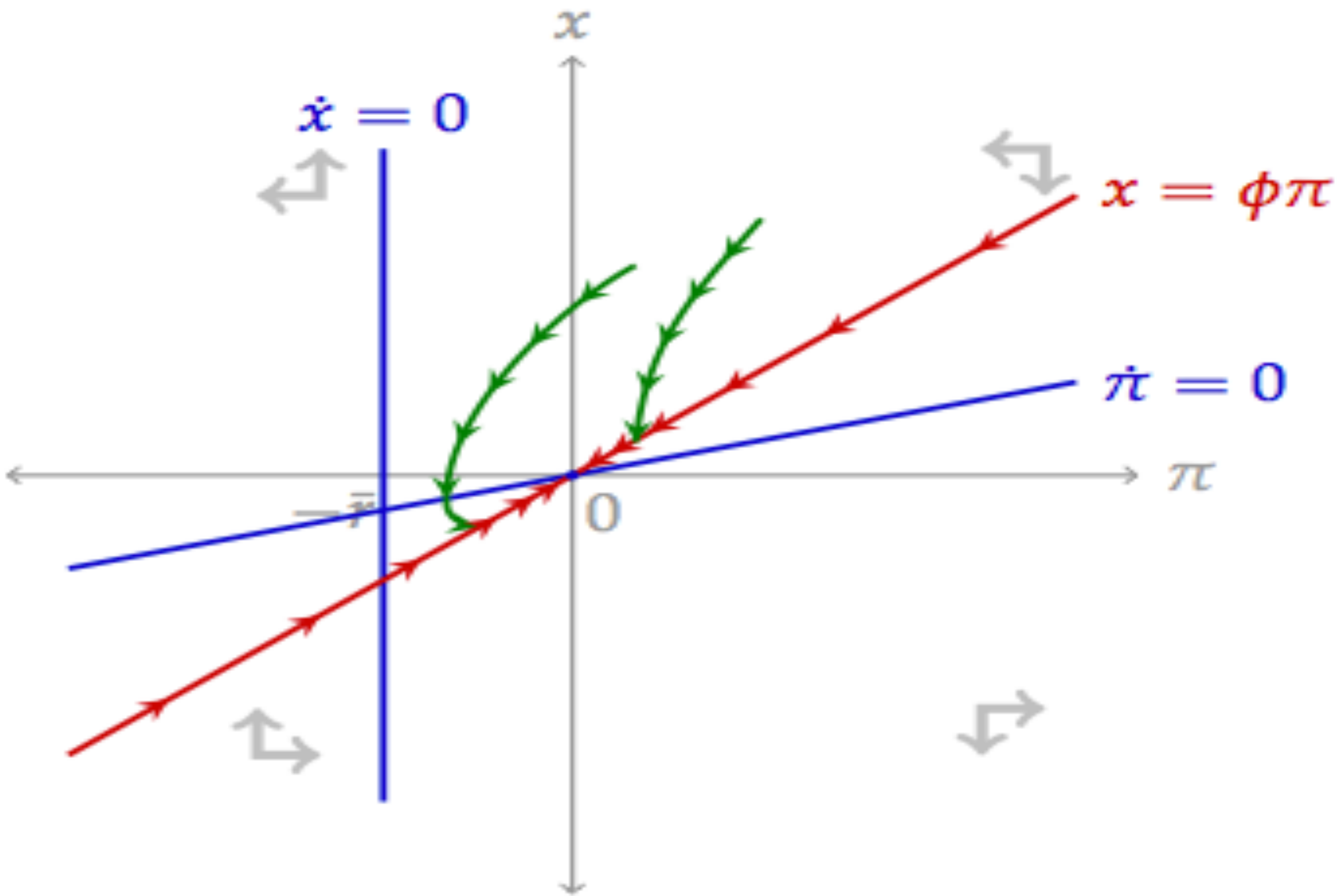
- Motion of system given by

$$\dot{x}(t) = -\sigma^{-1}(\pi(t) + \overline{r})$$
$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

- Same phase diagram as no-commitment case, except $\dot{x}(t) = 0$ locus at $\pi(t) = -\overline{r} < 0$ rather than at $-\underline{r} > 0$

# Phase II. Just out of the trap

Admissible dynamics *from* given $x(T), \pi(T)$ with $i(t) = 0$ for $t \in [T, \hat{T})$



# Phase I. During the liquidity trap

- At beginning of Phase I $x(0), \pi(0)$ free, but $x(T), \pi(T)$ given

- ZLB is *binding*, $i(t) = 0$

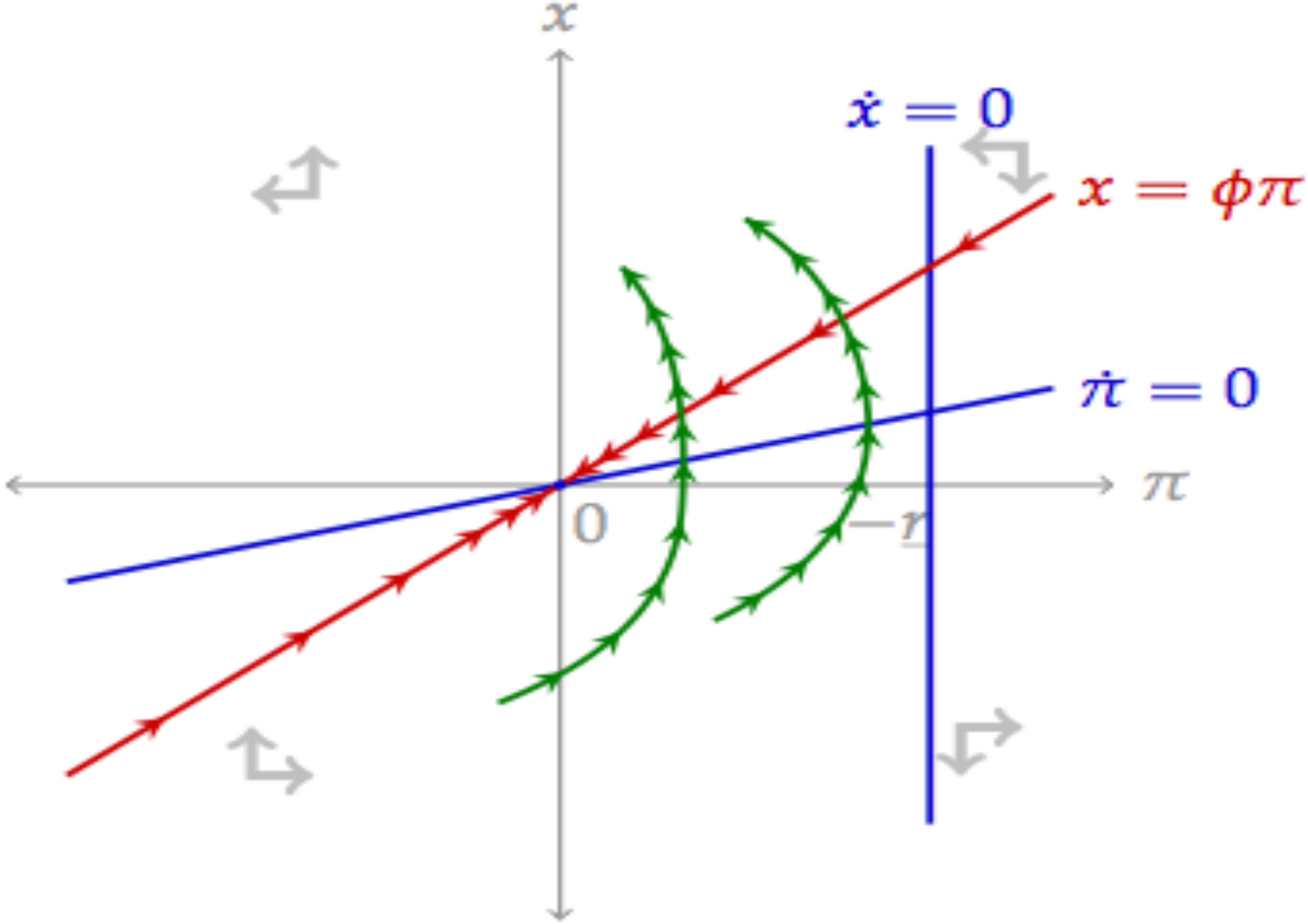
- Motion of system given by

$$\dot{x}(t) = -\sigma^{-1}(\pi(t) + \underline{r})$$
$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

- Same phase diagram as no-commitment case, $\dot{x}(t) = 0$ locus at $\pi(t) = -\underline{r} > 0$

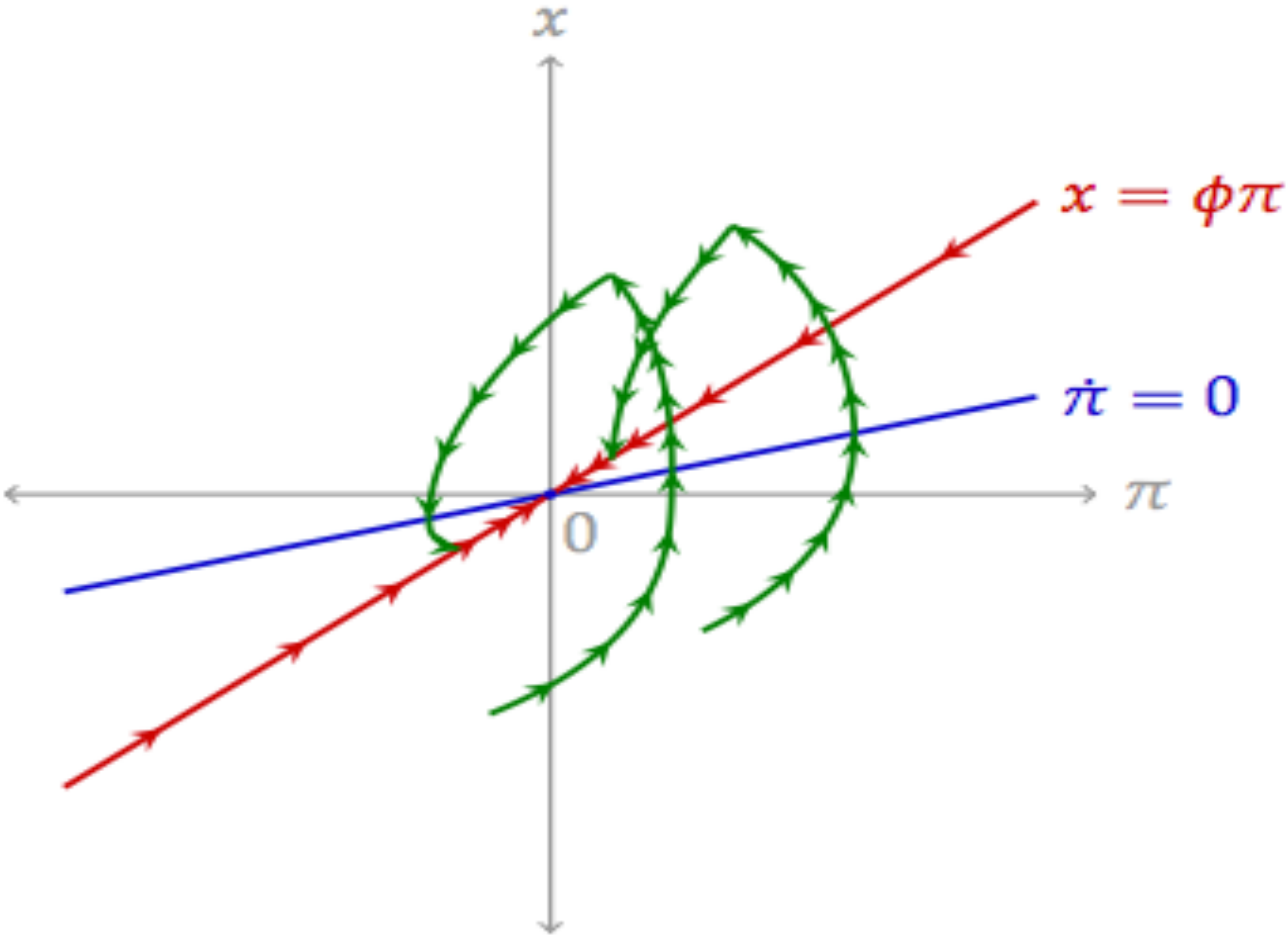
# Phase I. During the liquidity trap

Admissible dynamics *towards* $x(T), \pi(T)$ with $i(t) = 0$ for $t \in [0, T)$



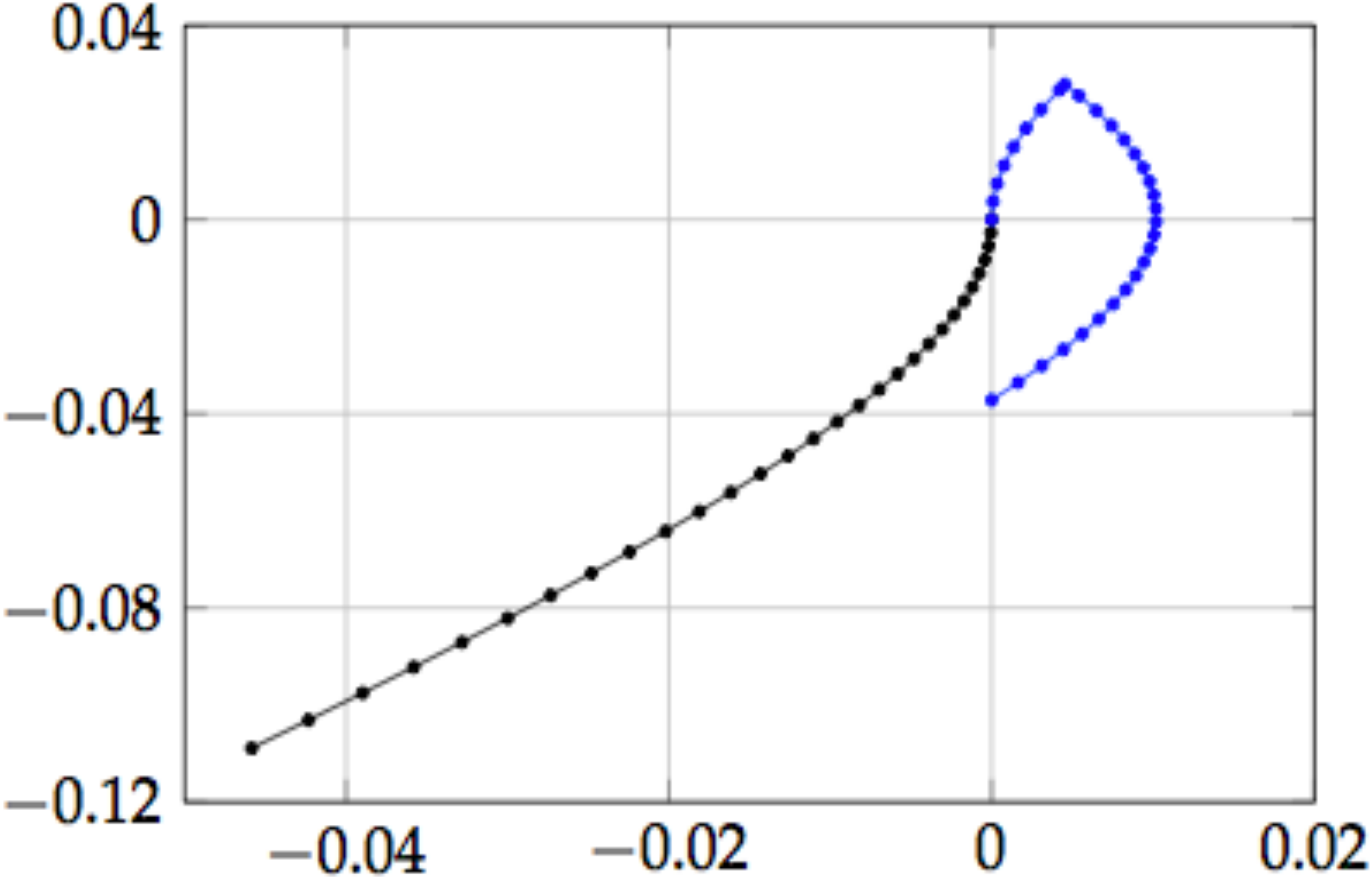
# Stitching it all together

Dynamics through the entire episode



# Commitment vs. No-commitment

Paths for $\pi(t), x(t)$. Commitment in blue, no-commitment in black



# Summary

**(1)** If ZLB is not binding, then $i(t) = I(r(t), \pi(t))$

**(2)** If $I(r(t), \pi(t)) < 0$ for $t \in [0, T)$ then $i(t) = 0$ for $t \in [0, \hat{T})$ for some $\hat{T} > T$

**(3)** Inflation must be positive at some point

**(4)** Output must be both positive and negative

**(5)** Depending on parameters, inflation may be positive throughout

# Communication

- Optimal policy requires commitment to $i(t) = 0$ for some $[T, \hat{T})$

- Two ways to summarize plan

  **(i)** $i(t) = 0$ for $t \in [0, T)$ and $x^*(T), \pi^*(T)$ satisfying

  $$x^*(T) > \phi\pi^*(T)$$

  (promised boom > promised boom implied by promised inflation)

  **(ii)** $i(t) = 0$ for $t \in [0, \hat{T})$ with $\hat{T} > T$ along with

  $$\pi(\hat{T})$$

  ($x(\hat{T}) = \phi\pi(\hat{T})$ is ex post optimal at $\hat{T}$ but not at $T$)

- Policy commitments for $t < T$ are irrelevant

# Commitment to inflation? Or boom?

- Krugman (1998) and older literature emphasizes importance of commitment to deliver *inflation*

- Werning argues that real goal is to deliver *boom* (though optimum generally features some positive inflation)

- Three devices to illustrate this point

  **(i)** completely rigid prices, $\kappa = 0$
  **(ii)** commitment to exit inflation $\pi(T)$ only
  **(iii)** exogenous constraint to avoid inflation

  In each case we obtain result that commitment to $i(t) = 0$ after trap is over is motivated by desire to deliver a boom

# Rigid prices

- Since $\kappa = 0$, inflation is $\pi(t) = 0$ always
- Suppose $i(t) = 0$ for $t \in [0, \hat{T})$ and $x(t) = \pi(t) = 0$ for $t > \hat{T}$
- Output gap then

$$x(t\,;\,\hat{T}) = \sigma^{-1} \int_t^{\hat{T}} r(s)\,ds$$

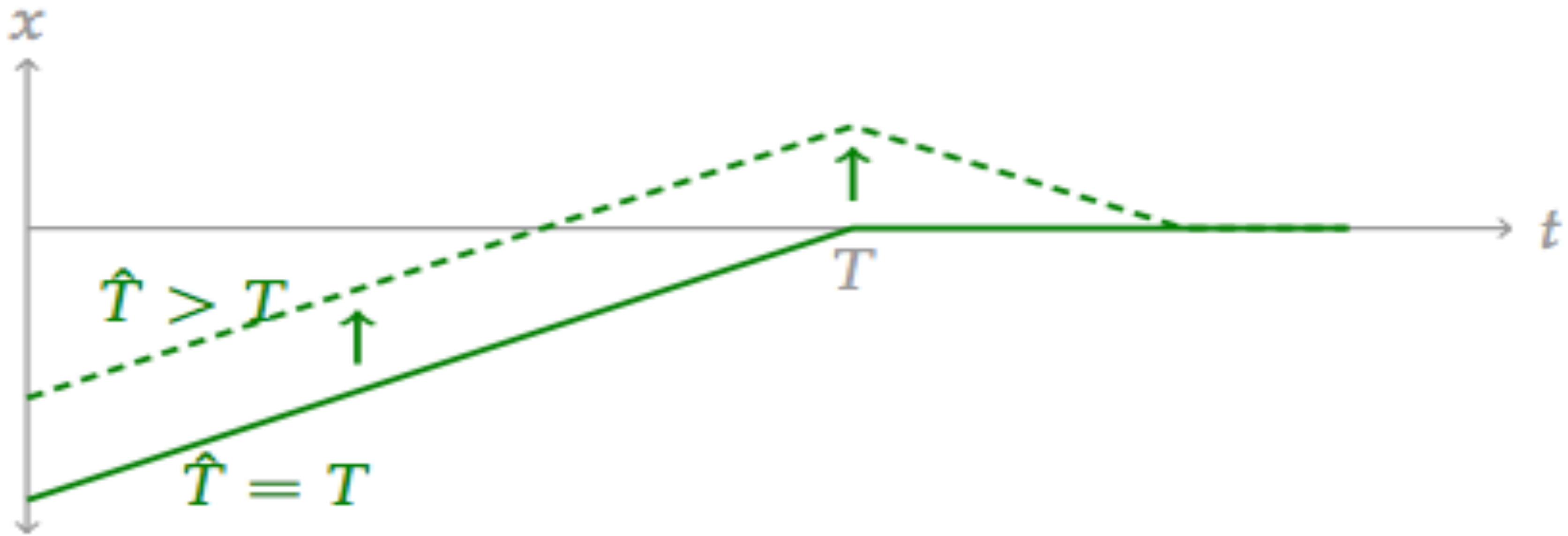
- Loss function is then

$$L(\hat{T}) = \frac{1}{2} \int_0^{\infty} e^{-\rho t} x(t\,;\,\hat{T})^2\,dt$$

(choose $\hat{T}$ to set PV of output gap to zero)

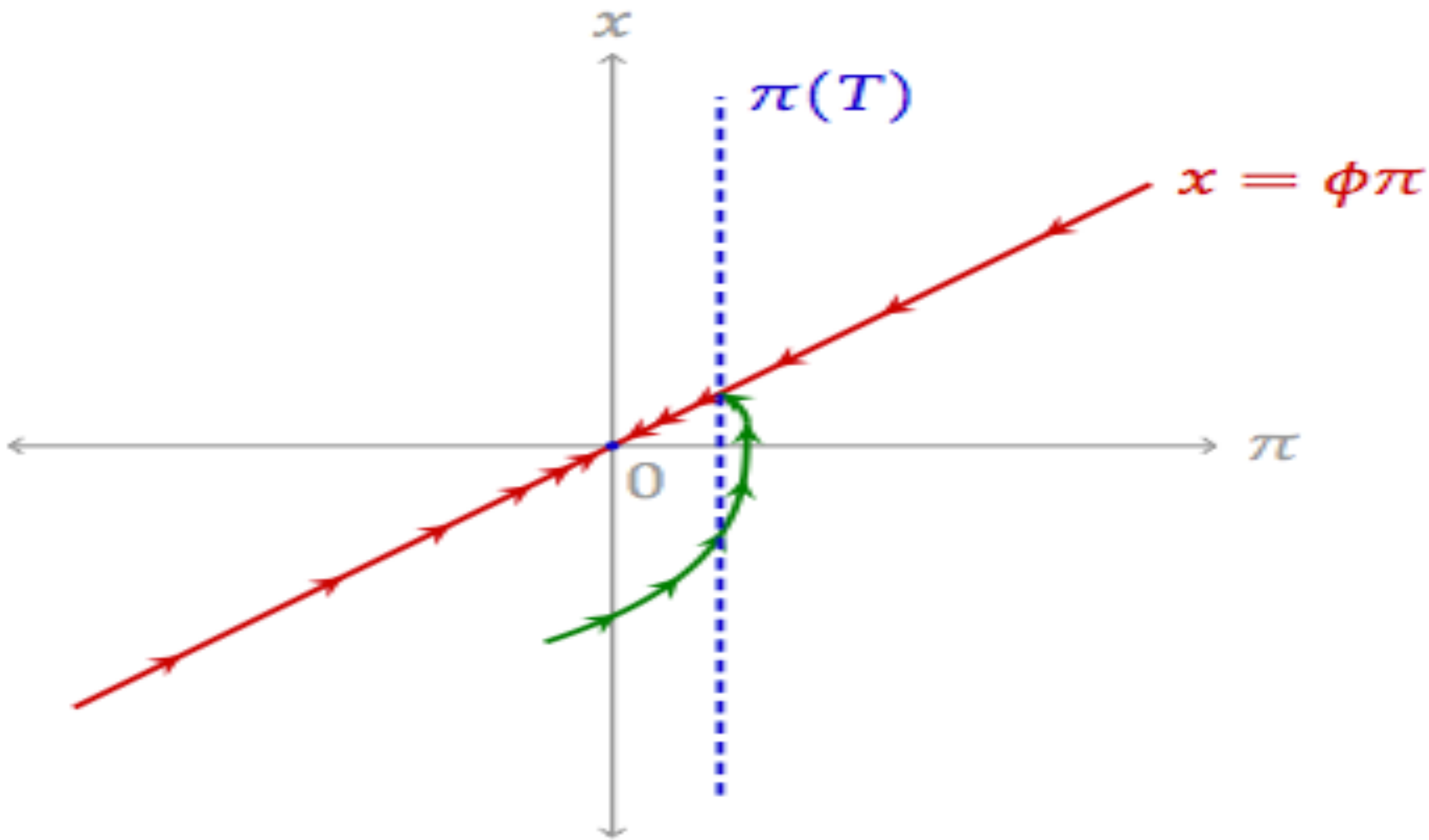
# Rigid prices

If $\hat{T} = T$, then $x(t) < 0$. If $\hat{T} > T$ then $x(t)$ higher and $x(T) > 0$




Since prices are fully rigid, creating inflation cannot be the purpose of monetary policy. Commitment to $i(t) = 0$ for $\hat{T} > T$ creates a boom to mitigate the welfare loss from the earlier recession. Current recession and future boom average out in PV. Creating inflation *not necessary* to rationalize commitment to $i(t) = 0$ after trap.

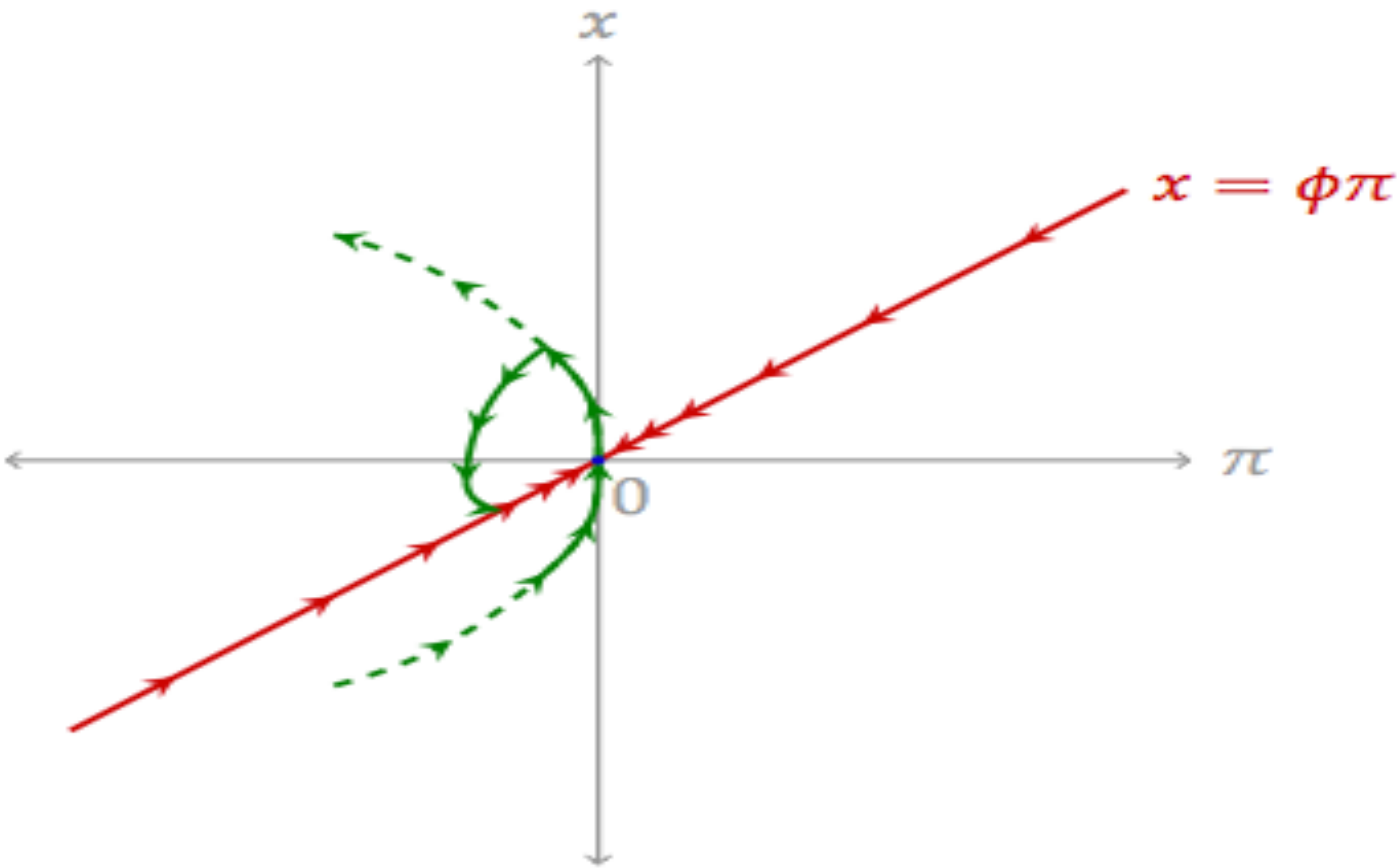
# Exit inflation




Commitment to exit inflation $\pi(T)$ without boom, $i(t) = I(r(t), \pi(t))$ for $t \geq T$.

Creating inflation *not sufficient* to rationalize commitment to $i(t) = 0$ after trap.

# Inflation constraint




Exogenous constraint $\pi(t) \leq 0$. Same arc as no-commitment, but go through origin earlier and deliver boom at $T$. Again, boom offsets earlier recession.

# Monetary policy summary

- Liquidity trap
  - if no-commitment, then deflation and recession
  - made worse by flexible prices
  - need to commit to polices after trap
- Optimal monetary policy
  - avoids deflation
  - features commitment to $i(t) = 0$ even after trap
  - commitment to $i(t) = 0$ even after trap to deliver boom

# Government purchases

- Representative consumer has preferences

$$U(C, N, G)$$

- Private consumption gap

$$c(t) = \frac{C(t) - C^*(t)}{C^*(t)}$$

- Government consumption gap

$$g(t) = \frac{G(t) - G^*(t)}{C^*(t)}$$

- Output gap

$$x(t) = c(t) - (1 - \Gamma)g(t), \quad \text{with multiplier } \Gamma \in (0, 1)$$

# Optimal policy with commitment

- Policy minimizes

$$L = \frac{1}{2}\int_0^\infty e^{-\rho t}\left(x(t)^2 + \lambda\pi(t)^2 + \eta g(t)^2\right) dt$$

subject to the constraints

$$\begin{aligned}\dot{x}(t) &= (1-\Gamma)\dot{g}(t) + \sigma^{-1}(i(t) - \pi(t) - r(t))\\ \dot{\pi}(t) &= \rho\pi(t) - \kappa x(t)\\ i(t) &\geq 0\end{aligned}$$

taking as given path $r(t)$

- Controls $i, \dot{g}$, states $x, \pi, g$, with free initial conditions $x(0), \pi(0), g(0)$

# Filling in the gap

- Consider (suboptimal) policy of setting $g(t)$ such that

$$c(t) + (1 - \Gamma)g(t) = \pi(t) = 0$$

- Requires

$$\dot{g}(t) = \frac{\sigma^{-1}}{1 - \Gamma}(r(t) - i(t))$$

which if $i(t) = 0$ for $t < T$ and $i(t) = r(t)$ for $t > T$ implies

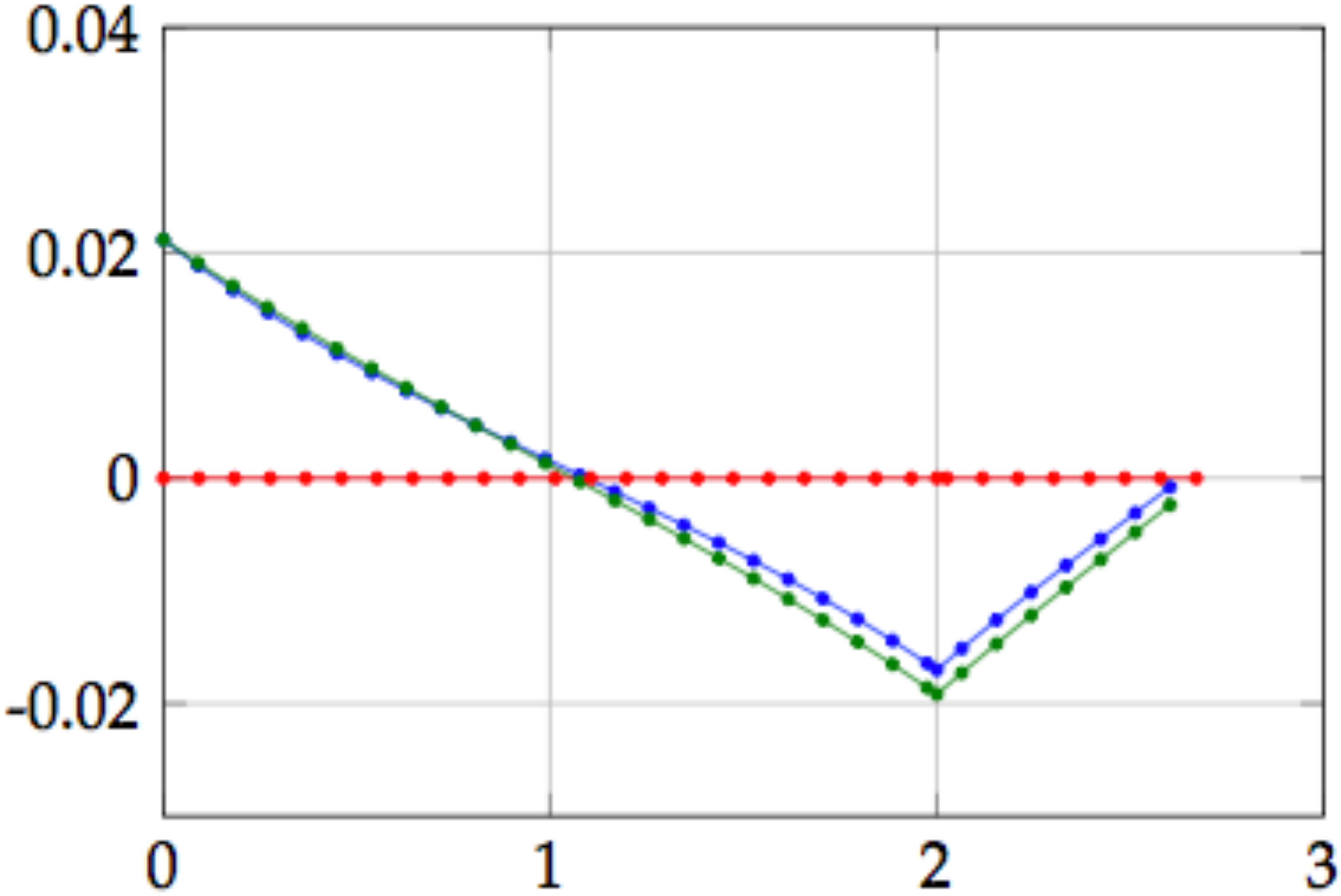
$$g(t) = \frac{\sigma^{-1}}{1 - \Gamma}\int_0^t r(s)\,ds + g(0), \qquad t < T$$

with $g(t) = g(T)$ for $t \geq T$. Now optimize over $g(0), g(T)$

- Solution features $g(0) > 0 > g(T)$. Suggests we should expect $g(t)$ policy to take on both signs

# Front-loading

Optimal $g(t)$ in blue. Initially positive, falling. Becomes negative.



# Decomposition

- Let $g^*(c)$ denote static 'opportunistic' government purchases, the $g$ that minimizes

$$(c - (1-\Gamma)g)^2 + \eta g^2$$

In recession (with low $c$) will get more $g$ just because opportunity cost is lower

- Let $\hat{g}(t)$ denote 'stimulus'

$$\hat{g}(t) = g(t) - g^*(c(t))$$

That part of $g(t)$ not accounted for by $g^*(c(t))$

- In previous figure, $g^*(c(t))$ in green and $\hat{g}(t)$ in red. In fact, exactly zero 'stimulus' if $\kappa = 0$ or $\sigma\kappa\lambda = 1$.

# Fiscal policy summary

- Stimulus component small $\hat{g}(t)$, most increase in $g(t)$ is opportunistic
- Optimal $g(t)$ is counter-cyclical, leans against the wind, but because it would be anyway
- That is, $g(t)$ is very close to what would be chosen by myopic policy-maker that completely ignored general equilibrium effects (e.g., ignores effects of $g(t)$ on inflation and hence $c(t)$)

# Next class

- Unemployment fluctuations in the new Keynesian model, part one

- Main reading:

  ◇ Gali, *Unemployment Fluctuations and Stabilization Policies: A New Keynesian Perspective*, MIT Press, 2011, chapter 1

- Alternate reading:

  ◇ Gali, "The return of the wage Phillips curve" *Journal of the European Economic Association*, 2011

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