Monetary Economics

Lecture 13: monetary/fiscal interactions in the new Keynesian model, part three

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2nd Semester 2014

This class

- *Optimal policy in a liquidity trap*
- Reading:
    - Werning "Managing a liquidity trap: Monetary and fiscal policy" MIT working paper 2012, sections 2–3

Available from the LMS

This class

1- Basic new Keynesian model in continuous time

- focus on price setting under sticky prices

2- Liquidity trap *without commitment*

- harmful effects of price flexibility
- value of commitment

Appendix: brief review of continuous time tools

- Hamiltonians, phase diagrams etc

New Keynesian model in continuous time

- Taking prices and interest rates as given, household maximizes

$$\int_0^\infty e^{-\rho t}\, U(C(t), N(t))\, dt, \qquad \rho > 0$$

subject to flow budget constraint

$$P(t)C(t) + \dot{B}(t) = i(t)B(t) + W(t)N(t) - T(t)$$

- Here controls are consumption $C(t)$ and labor supply $N(t)$, state variable is bonds $B(t)$ with $B(0) = B_0$ given

- Hamiltonian for this problem

$$\mathcal{H}(C, N, B, \lambda\,;\, t) = U(C, N) + \lambda(i(t)B + W(t)N - T(t) - P(t)C)$$

- Key optimality conditions for the household

$$\mathcal{H}_C: \quad U_C(C(t), N(t)) = \lambda(t)P(t)$$

$$\mathcal{H}_N: \quad -U_N(C(t), N(t)) = \lambda(t)W(t)$$

$$\mathcal{H}_B: \quad \lambda(t)i(t) = \rho\lambda(t) - \dot{\lambda}(t)$$

- Suppose utility function

$$U(C, N) = \log C - \frac{N^{1+\varphi}}{1+\varphi}$$

- Then familiar conditions

$$N(t)^{\varphi}C(t) = \frac{W(t)}{P(t)}, \qquad \frac{\dot{C}(t)}{C(t)} = i(t) - \frac{\dot{P}(t)}{P(t)} - \rho$$

Intermediate producers

- Usual CES demand system

$$y(p) = \left(\frac{p}{P}\right)^{-\varepsilon} Y$$

- Constant nominal marginal cost $W(t)/A(t)$, flow profits

$$\Pi(p\,;\,t) = \left(p - \frac{W(t)}{A(t)}\right)\left(\frac{p}{P(t)}\right)^{-\varepsilon} Y(t)$$

- In symmetric flexible price equilibrium

$$p(t) = P(t) = \frac{\varepsilon}{\varepsilon - 1}\frac{W(t)}{A(t)}$$

Sticky prices

- Rather than Calvo setup, firms face price *adjustment costs*

$$\Theta\left(\frac{\dot{p}}{p}\,;\,t\right) = \frac{\theta}{2}\left(\frac{\dot{p}}{p}\right)^2 P(t)Y(t)$$

where $\theta > 0$ captures costliness of price adjustment. If $\theta = 0$, then fully flexible prices

- Flow profits net of adjustment costs

$$\Pi(p\,;\,t) - \Theta\left(\frac{\dot{p}}{p}\,;\,t\right)$$

Sticky prices

- Taking $i(t), P(t), Y(t)$ as given, firm problem is to maximize

$$\int_0^\infty e^{-\int_0^t i(s)\,ds}\left[\Pi(p(t)\,;\,t) - \Theta\left(\frac{\dot{p}(t)}{p(t)}\,;\,t\right)\right]dt$$

Here control is $\dot{p}$ and state is p

- Hamiltonian for this problem

$$\mathcal{H}(\dot{p}, p, \eta\,;\ t) = \Pi(p\,;\,t) - \Theta\left(\frac{\dot{p}}{p}\,;\,t\right) + \eta\dot{p}$$

- Key optimality conditions for the firm

$$\mathcal{H}_{\dot{p}}(\dot{p}(t), p(t), \eta(t)\,;\ t) = 0$$
$$\mathcal{H}_{p}(\dot{p}(t), p(t), \eta(t)\,;\ t) = i(t)\eta(t) - \dot{\eta}(t)$$
$$\mathcal{H}_{\eta}(\dot{p}(t), p(t), \eta(t)\,;\ t) = \dot{p}(t)$$

Sticky prices

- Calculating the derivatives of the Hamiltonian

$$\mathcal{H}_{\dot{p}} = -\theta\left(\frac{\dot{p}(t)}{p(t)}\right)\frac{P(t)}{p(t)}Y(t) + \eta(t)$$

$$\mathcal{H}_{p} = \left\{\left[(1-\varepsilon) + \varepsilon\frac{W(t)}{p(t)A(t)}\right]\left(\frac{p(t)}{P(t)}\right)^{-\varepsilon} + \theta\left(\frac{\dot{p}}{p}\right)^{2}\frac{P(t)}{p(t)}\right\}Y(t)$$

Symmetric equilibrium

- In symmetric equilibrium $p(t) = P(t)$ so these simplify to

$$\mathcal{H}_{\dot{p}} = -\theta\pi(t)Y(t) + \eta(t)$$

$$\mathcal{H}_{p} = \left\{\left[(1-\varepsilon) + \varepsilon\frac{W(t)}{P(t)A(t)}\right] + \theta\pi(t)^2\right\}Y(t)$$

where $\pi(t) \equiv \dot{P}(t)/P(t)$ is the inflation rate

- So we have the equilibrium conditions

$$\theta\pi(t)Y(t) = \eta(t)$$

$$\left\{\left[(1-\varepsilon) + \varepsilon\frac{W(t)}{P(t)A(t)}\right] + \theta\pi(t)^2\right\}Y(t) = i(t)\eta(t) - \dot{\eta}(t)$$

Symmetric equilibrium

- Differentiating the former of these with respect to t

$$\theta\dot{\pi}(t)Y(t) + \theta\pi(t)\dot{Y}(t) = \dot{\eta}(t)$$

- Which we can use to eliminate $\dot{\eta}(t)$ from the latter to get

$$\left(i(t) - \pi(t) - \frac{\dot{Y}(t)}{Y(t)}\right)\pi(t) = \dot{\pi}(t) + \frac{\varepsilon - 1}{\theta}\left(\frac{\varepsilon}{\varepsilon - 1}\frac{W(t)}{P(t)}\frac{1}{A(t)} - 1\right)$$

- Since in equilibrium $Y(t) = C(t)$ we can use the household consumption Euler equation to write this as

$$\rho\pi(t) = \dot{\pi}(t) + \frac{\varepsilon - 1}{\theta}\left(\frac{\varepsilon}{\varepsilon - 1}\frac{W(t)}{P(t)}\frac{1}{A(t)} - 1\right)$$

  This is the continuous time version of our familiar relationship between inflation and real marginal cost

Flexible price equilibrium

- Turn this into a new Keynesian Phillips curve by rewriting real marginal cost in terms of output gap

- In flexible price equilibrium

$$\frac{W^n(t)}{P^n(t)} = \frac{\varepsilon - 1}{\varepsilon} A(t)$$

and whether prices are sticky or not, household labor supply and market clearing implies

$$\frac{W(t)}{P(t)} = N(t)^{\varphi} C(t) = Y(t)^{\varphi+1} A(t)^{-\varphi}$$

- Hence natural output is, here,

$$Y^n(t) = \left(\frac{\varepsilon - 1}{\varepsilon}\right)^{\frac{1}{1+\varphi}} A(t)$$

- And the natural real wage can then be written

$$\frac{W^n(t)}{P^n(t)} = \left(\frac{\varepsilon - 1}{\varepsilon}\right)^{\frac{\varphi}{1+\varphi}} Y^n(t)$$

- While with sticky prices the real wage is

$$\frac{W(t)}{P(t)} = Y(t)^{\varphi+1} A(t)^{-\varphi} = \left(\frac{Y(t)}{Y^n(t)}\right)^{\varphi} \left(\frac{\varepsilon - 1}{\varepsilon}\right)^{\frac{\varphi}{1+\varphi}} Y(t)$$

- So we can write

$$\frac{\varepsilon}{\varepsilon - 1} \frac{W(t)}{P(t)} \frac{1}{A(t)} = \frac{W(t)/P(t)}{W^n(t)/P^n(t)} = \left(\frac{Y(t)}{Y^n(t)}\right)^{\varphi+1}$$

Output gap and Phillips curve

- Then define the output gap

$$X(t) \equiv \frac{Y(t)}{Y^n(t)}$$

- Inflation dynamics therefore given by

$$\rho\pi(t) = \dot{\pi}(t) + \frac{\varepsilon - 1}{\theta}\Big(X(t)^{\varphi+1} - 1\Big)$$

- Let $x \equiv \log X$. Then $X^{\varphi+1} - 1 \approx (\varphi + 1)x$ so

$$\rho\pi(t) = \dot{\pi}(t) + \kappa x(t), \qquad \kappa \equiv \frac{\varepsilon - 1}{\theta}(\varphi + 1)$$

- This is the new Keynesian Phillips curve in continuous time

Output gap and dynamic IS curve

- Household Euler equation and market clearing implies

$$\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{C}(t)}{C(t)} = i(t) - \pi(t) - \rho$$

- Define natural real rate $r(t)$ from natural output growth

$$\frac{\dot{Y}^n(t)}{Y^n(t)} = r(t) - \rho$$

- Log output gap $x(t) = \log X(t)$ satisfies

$$\dot{x}(t) = \frac{\dot{X}(t)}{X(t)} = \frac{\dot{Y}(t)}{Y(t)} - \frac{\dot{Y}^n(t)}{Y^n(t)}$$

so

$$\dot{x}(t) = i(t) - \pi(t) - r(t)$$

- This is the dynamic IS curve in continuous time

Three-equation model

- Dynamic IS curve

$$\dot{x}(t) = i(t) - \pi(t) - r(t)$$

- New Keynesian Phillips curve

$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

- Interest rate rule, say

$$i(t) = r(t) + \phi_\pi \pi(t),$$

- Unique equilibrium if $\phi_\pi > 1$ but multiple equilibria if $\phi_\pi < 1$

Three-equation model

- Qualitative dynamics, for phase diagram

$$\dot{x}(t) = (\phi_\pi - 1)\pi(t) > 0 \quad \Leftrightarrow \quad \begin{cases} \pi(t) > 0 & \text{if} \quad \phi_\pi > 1 \\ \pi(t) < 0 & \text{if} \quad \phi_\pi < 1 \end{cases}$$

$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t) > 0 \quad \Leftrightarrow \quad x(t) < (\rho/\kappa)\pi(t)$$

- Magnitude of ϕ_π determines *direction of $x(t)$ flow* and hence stability properties of system

- Initial values $x(0), \pi(0)$ both free — both output gap and inflation are jump variables

- Jump to $(x(0), \pi(0)) = (0, 0)$ if $\phi_\pi > 1$

Werning (2012)

- What is *optimal policy* if economy in liquidity trap?

- Dynamic IS curve, slightly generalized

$$\dot{x}(t) = \sigma^{-1}(i(t) - \pi(t) - r(t)), \qquad \sigma > 0$$

  where σ^{-1} is the intertemporal elasticity of substitution

- New Keynesian Phillips curve

$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

- Nominal interest rate $i(t)$ must satisfy the ZLB constraint

$$i(t) \geq 0$$

Werning (2012)

- Integrating the new Keynesian Phillips curve forward

$$\pi(t) = \kappa \int_0^\infty e^{-\rho s} x(t+s)\, ds$$

(positive gaps increase inflation, negative gaps decrease it)

- Welfare evaluated according to quadratic loss function

$$L = \frac{1}{2} \int_0^\infty e^{-\rho t} \left(x(t)^2 + \lambda \pi(t)^2 \right) dt, \qquad \lambda \equiv \bar{\lambda}/\kappa \geq 0$$

Weight on inflation $\lambda = \bar{\lambda}/\kappa \to 0$ as prices become fully flexible, $\kappa \to \infty$ (recall Lecture 10 derivation of loss function)

Liquidity trap

- Suppose natural real rate takes form

$$r(t) = \begin{cases} \underline{r} & t \in [0, T) \\ \overline{r} & t \in [T, \infty) \end{cases} \qquad \text{with} \quad \underline{r} < 0 < \overline{r}$$

  for some given horizon $T > 0$

- If $r(t) > 0$ for all t then can achieve $(x(t), \pi(t)) = (0, 0)$ with a sufficiently reactive interest rate rule

- But if $r(t) < 0$ for some t, economy stuck in a liquidity trap

No commitment scenario

- Monetary authority cannot credibly commit to plans for future
  - in other words, a *time-consistency* problem
- From $t \geq T$ monetary authority will try to do what is optimal from $t \geq T$ on irrespective of past announcements
- Hence from $t \geq T$ on, monetary authority will implement

$$(x(t), \pi(t)) = (0, 0) \qquad \text{for} \qquad t \geq T$$

(e.g., by sufficiently reactive interest rate rule)

- How does this affect dynamics during liquidity trap $t < T$?

No commitment scenario

- System of differential equations with $i(t) = 0$ during trap $t < T$

$$\dot{x}(t) = -\sigma^{-1}(\pi(t) + \underline{r})$$
$$\dot{\pi}(t) = \rho\pi(t) - \kappa x(t)$$

with terminal condition

$$(x(T), \pi(T)) = (0, 0)$$

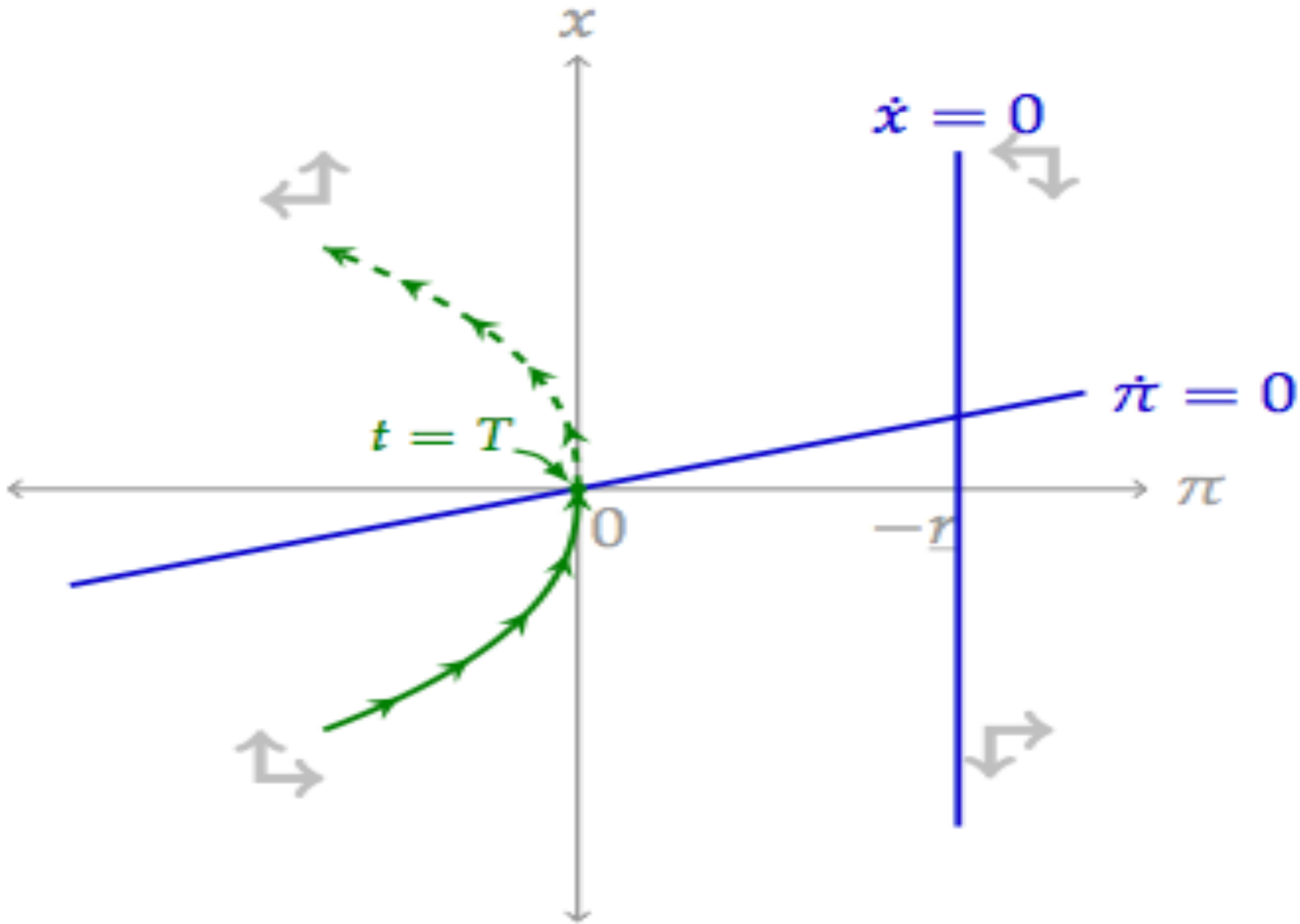
- Initial values $x(0), \pi(0)$ both free — both output gap and inflation are jump variables

- Qualitative dynamics, for phase diagram

$$\dot{x}(t) > 0 \quad \Leftrightarrow \quad \pi(t) < -\underline{r} > 0$$
$$\dot{\pi}(t) > 0 \quad \Leftrightarrow \quad x(t) < (\rho/\kappa)\pi(t)$$

No commitment scenario

Before T, binding ZLB $i(t) = 0$. Obtains $x(T) = \pi(T) = 0$ at end of liquidity trap.



Deflation and recession

- Solution features *deflation,* $\pi(0) < 0$ and *recession,* $x(0) < 0$. Both gradually alleviated as $t \to T$

- Extent of initial recessionn is increasing in length of trap T

$$x(0), \pi(0) \to -\infty \qquad \text{as} \qquad T \to \infty$$

  (larger T means starting further from origin)

- Intuition: real interest rate $i(t) - \pi(t) = -\pi(t) > 0$ is *too high* during liquidity trap. Suppresses consumption and output, makes forward-looking inflation even lower, exacerbates problem

- Problem is inability to commit to policies *after* the liquidity trap, in particular inability to commit to other than $x(T) = \pi(T) = 0$

Harmful effects of price flexibility

- Surprisingly, outcomes worse if prices more flexible (high κ)

- High κ means a given $x(t) < 0$ creates more deflation $\pi(t) < 0$, making real rates even higher

- Euler equation then implies higher growth $\dot{x}(t)$ to reach $x(T) = 0$. But this means $x(0)$ must be even lower

- Perfectly rigid prices $\kappa = 0$ deliver a better outcome

- Benefits of price flexibility only obtained if monetary policy permits $\pi(t) > 0$ under some circumstances (which here it does not)

Elbow room

- Consider suboptimal policy that delivers the steady-state

$$\pi(t) = \bar{\pi} \equiv -\underline{r} > 0, \quad \text{and} \quad x(t) = \bar{x} \equiv -\frac{\rho}{\kappa}\underline{r} > 0$$

(positive inflation, positive output gap for all $t \geq 0$)

- Commitment to higher inflation *after* the trap improves welfare

- Permanent sacrifice to solve a temporary problem. Large sacrifice if $\bar{\pi}$ high or T short etc, small sacrifice if prices flexible

- This is not optimal but approaches optimality as prices become fully flexible, $\kappa \to \infty$, since $\lambda = \bar{\lambda}/\kappa$ implies $L \to 0$

Next class

- Optimal policy in a liquidity trap *with commitment*
- Reading:
  - ◇ Werning, "Managing a liquidity trap: Monetary and fiscal policy" MIT working paper 2012, sections 4–7

Monetary Economics

Appendix to Lecture 13

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2nd Semester 2014

A standard optimal control problem

- Consider the problem of maximizing

$$\int_0^\infty e^{-\rho t}\, h(u(t), x(t))\, dt, \qquad \rho > 0$$

with state variable x, control u, subject to the law of motion

$$\dot{x}(t) = g(u(t), x(t)), \qquad x(0) = x_0 \text{ given}$$

and feasible controls $u(t) \in \mathcal{U}$

- Characterize solution of this problem using *Hamiltonian*

Hamiltonian

- Hamiltonian for this problem (in *current-value* representation)

$$\mathcal{H}(u,x,\lambda) := h(u,x) + \lambda g(u,x)$$

- Key optimality conditions, for all $t \geq 0$,

$$\mathcal{H}_u(u(t),x(t),\lambda(t)) = 0$$

$$\mathcal{H}_x(u(t),x(t),\lambda(t)) = \rho\lambda(t) - \dot{\lambda}(t)$$

$$\mathcal{H}_\lambda(u(t),x(t),\lambda(t)) = \dot{x}(t)$$

Along with initial condition $x(0) = x_0$ and *transversality condition*

$$\lim_{t\to\infty} e^{-\rho t}\lambda(t)x(t) = 0$$

Optimal growth example

- State variable capital k, control variable consumption c

- Planner's problem is to maximize

$$\int_0^\infty e^{-\rho t}\, u(c(t))\, dt, \qquad \rho > 0$$

subject to resource constraint

$$\dot{k}(t) = f(k(t)) - \delta k(t) - c(t), \qquad k(0) = k_0 \text{ given}$$

and feasible consumption $c(t) \geq 0$

- Hamiltonian for this problem

$$\mathcal{H}(c, k, \lambda) := u(c) + \lambda(f(k) - \delta k - c)$$

$$\mathcal{H}(c, k, \lambda) := u(c) + \lambda(f(k) - \delta k - c)$$

- Key optimality conditions, for all $t \geq 0$,

$$\mathcal{H}_c(c(t), k(t), \lambda(t)) = 0$$
$$\mathcal{H}_k(c(t), k(t), \lambda(t)) = \rho\lambda(t) - \dot{\lambda}(t)$$
$$\mathcal{H}_\lambda(c(t), k(t), \lambda(t)) = \dot{k}(t)$$

along with initial condition and transversality condition

- Calculating the derivatives of the Hamiltonian

$$\mathcal{H}_c(c, k, \lambda) = u'(c) - \lambda$$
$$\mathcal{H}_k(c, k, \lambda) = \lambda(f'(k) - \delta)$$
$$\mathcal{H}_\lambda(c, k, \lambda) = f(k) - \delta k - c$$

Optimal growth example

- Hence system of optimality conditions can be written

$$\begin{aligned} u'(c(t)) &= \lambda(t) \\ \dot{\lambda}(t) &= (\rho - (f'(k(t)) - \delta))\lambda(t) \\ \dot{k}(t) &= f(k(t)) - \delta k(t) - c(t) \end{aligned}$$

- Differentiating the first condition with respect to t gives

$$u''(c(t))\dot{c}(t) = \dot{\lambda}(t)$$

- So the first two conditions can be combined to eliminate $\lambda(t)$, giving the familiar *consumption Euler equation*

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \delta - \rho}{\sigma(c(t))}, \qquad \sigma(c) := -\frac{u''(c)c}{u'(c)}$$

where $\sigma(c)$ is the Arrow/Pratt coefficient of relative risk aversion

Optimal growth example

- System of differential equations in $c(t), k(t)$

$$\frac{\dot{c}(t)}{c(t)} = \frac{f'(k(t)) - \delta - \rho}{\sigma(c(t))}$$
$$\dot{k}(t) = f(k(t)) - \delta k(t) - c(t)$$

given initial condition k_0 and transversality condition

$$\lim_{t \to \infty} e^{-\rho t} u'(c(t)) k(t) = 0$$

- One given initial condition k_0, initial consumption c_0 can jump

- Unique solution if dynamical system has one stable and one unstable root

Steady state

- Steady state c^*, k^* where $\dot{c}(t) = 0$ and $\dot{k}(t) = 0$, implied by

  $$f'(k^*) = \rho + \delta$$

  and

  $$c^* = f(k^*) - \delta k^*$$

  note c^* independent of $u(c)$ function (from time separability)

- If production function is

  $$f(k) = k^\alpha, \qquad 0 < \alpha < 1$$

  then steady state evaluates to

  $$k^* = \left(\frac{\alpha}{\rho + \delta}\right)^{\frac{1}{1-\alpha}}$$

  $$c^* = \left(\frac{\rho + (1-\alpha)\delta}{\rho + \delta}\right)\left(\frac{\alpha}{\rho + \delta}\right)^{\frac{\alpha}{1-\alpha}}$$

Qualitative dynamics

- From consumption Euler equation

$$\dot{c}(t) > 0 \quad \Leftrightarrow \quad f'(k(t)) > \rho + \delta \quad \Leftrightarrow \quad k(t) < k^*$$

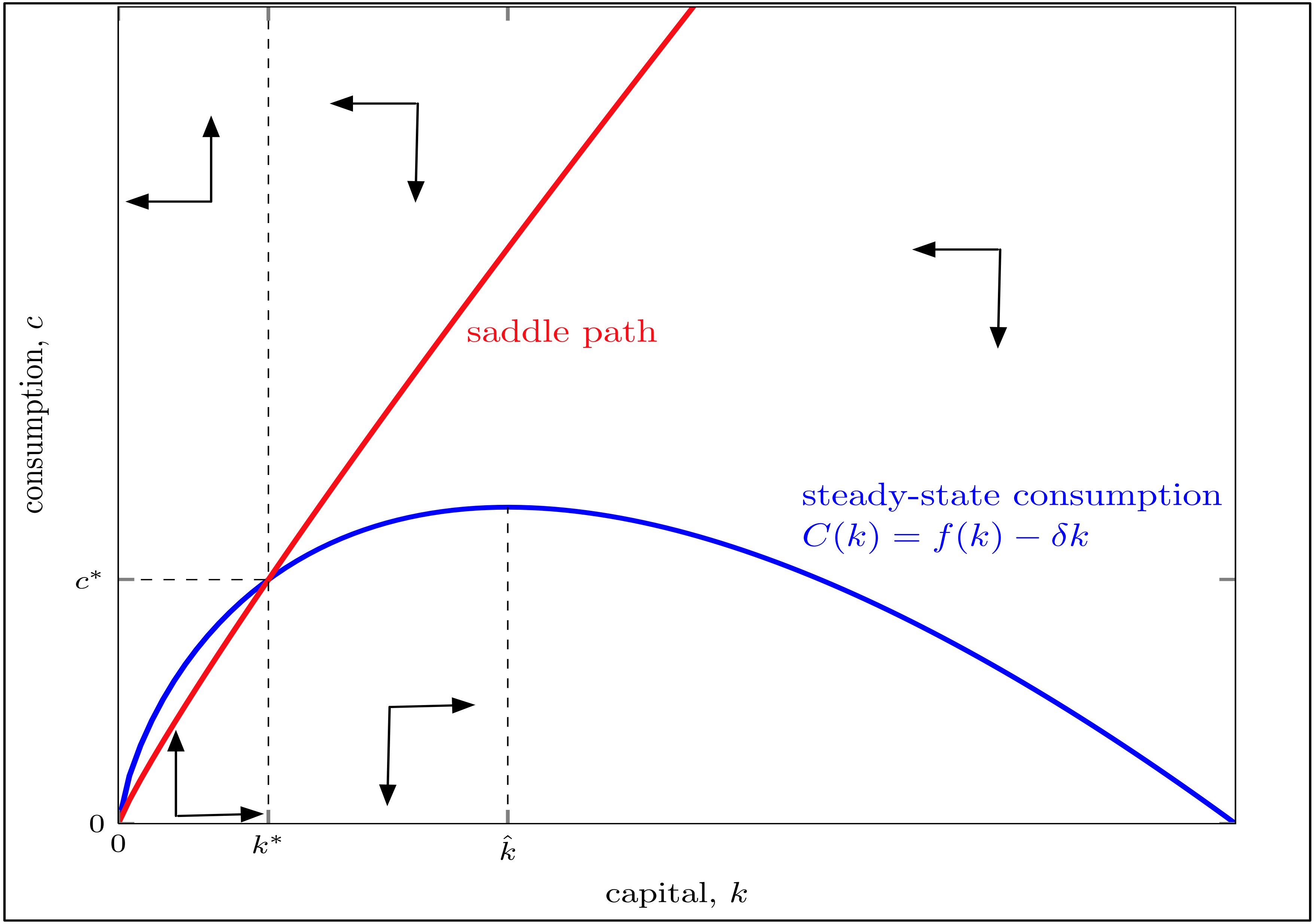
- Let $C(k)$ denote steady state consumption implied by capital k

$$C(k) := f(k) - \delta k$$

- Then from resource constraint

$$\dot{k}(t) > 0 \quad \Leftrightarrow \quad f(k(t)) - \delta k(t) > c(t) \quad \Leftrightarrow \quad C(k(t)) > c(t)$$

- Analyze these qualitative dynamics in a *phase diagram*


saddle path
steady-state consumption
$C(k) = f(k) - \delta k$
consumption, c
c^*
0
0
k^*
$\hat{k}$
capital, k