Monetary Economics

Lecture 10: monetary policy in the new Keynesian model, part three

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2nd Semester 2014

This class

- Monetary policy in the new Keynesian model, part three
  - welfare function, evaluating policy rules
- Reading: Gali, chapter 4 sections 4.3–4.4 and appendix

This class

1- Shortcomings of optimal policy rules

2- Evaluation of simple policy rules

- (tedious) approximate welfare function

3- Overview of policy in the new Keynesian model

Practical shortcomings of optimal policy rules

- Require knowledge of true "structure" of economy (to compute natural output y_t^n, natural real rate r_t^n etc) including:

  - all functional forms
  - all parameter values
  - values of realised shocks in *real-time*

- Alternative is to look at "simple rules" that make policy instrument (say, i_t) a function of observable variables only

- Goal then is to find rules that are "robust" across many models, since no model is true (though some may be useful)

- Evaluate proposed rules according to a *welfare function*

Approximate welfare function

- Representative household has utility

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \right\}$$

with $U(C_t, N_t)$ separable between C_t and N_t, evaluated at equilibrium for given policy rule

- MAIN IDEA: rewrite this as (approximate) welfare *loss* function over output gap and inflation *variances*

Approximate welfare function

- **Step 1**: *second order* approximation of $U_t \equiv U(C_t, N_t)$ around *first-best* (efficient) steady state $\bar{U} \equiv U(\bar{C}, \bar{N})$, specifically

$$U_t - \bar{U} \approx \bar{U}_c \bar{C} \left(\frac{C_t - \bar{C}}{\bar{C}} \right) + \bar{U}_n \bar{N} \left(\frac{N_t - \bar{N}}{\bar{N}} \right)$$

$$+ \frac{1}{2} \bar{U}_{cc} \bar{C}^2 \left(\frac{C_t - \bar{C}}{\bar{C}} \right)^2 + \frac{1}{2} \bar{U}_{nn} \bar{N}^2 \left(\frac{N_t - \bar{N}}{\bar{N}} \right)^2$$

(note: separability $\Rightarrow U_{cn} = 0$)

- **Step 2**: rewrite in terms of *log-deviations*

$$\frac{Z_t - \bar{Z}}{\bar{Z}} \approx \hat{z}_t - \frac{1}{2} \hat{z}_t^2$$

So up to second order

$$U_t - \bar{U} \approx \bar{U}_c \bar{C} \left(\hat{c}_t + \frac{1 - \sigma}{2} \hat{c}_t^2 \right) + \bar{U}_n \bar{N} \left(\hat{n}_t + \frac{1 + \varphi}{2} \hat{n}_t^2 \right)$$

where $\sigma \equiv -\bar{U}_{cc} \bar{C} / \bar{U}_c$ and $\varphi \equiv \bar{U}_{nn} \bar{N} / \bar{U}_n$

Approximate welfare function

- **Step 3**: Use equilibrium relationships. Market clearing implies

$$C_t = Y_t \qquad \Leftrightarrow \qquad \hat{c}_t = \hat{y}_t$$

and from "aggregate production function"

$$N_t = \int_0^1 \left[\left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon} \frac{Y_t}{A_t} \right]^{\frac{1}{1-\alpha}} dj$$

which implies

$$(1-\alpha)\hat{n}_t = \hat{y}_t - \hat{a}_t + \hat{d}_t$$

where the relative price dispersion term is defined by

$$\hat{d}_t \equiv (1-\alpha) \log \left[\int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} dj \right], \qquad \bar{d} = 0$$

Approximate welfare function

- **Step 4**: Plug these into utility approximation to get

$$U_t - \bar{U} \approx \bar{U}_c \bar{C} \left(\hat{y}_t + \frac{1-\sigma}{2} \hat{y}_t^2 \right)$$

$$+ \frac{\bar{U}_n \bar{N}}{1-\alpha} \left(\hat{y}_t - \hat{a}_t + \hat{d}_t + \frac{1+\varphi}{2(1-\alpha)} (\hat{y}_t - \hat{a}_t + \hat{d}_t)^2 \right)$$

and then use $\bar{C} = \bar{Y}$ and $-\bar{U}_n / \bar{U}_c = (1-\alpha)\bar{Y}/\bar{N}$ to simplify

$$\frac{U_t - \bar{U}}{\bar{U}_c \bar{C}} \approx -\frac{1}{2} \left(-(1-\sigma)\hat{y}_t^2 - 2\hat{a}_t + 2\hat{d}_t + \frac{1+\varphi}{1-\alpha} (\hat{y}_t - \hat{a}_t + \hat{d}_t)^2 \right)$$

Approximate welfare function

- Cleaning things up a bit

$$\frac{U_t - \bar{U}}{\bar{U}_c \bar{C}} \approx -\frac{1}{2}\left(-(1-\sigma)\hat{y}_t^2 + 2\hat{d}_t + \frac{1+\varphi}{1-\alpha}(\hat{y}_t^2 - 2\hat{a}_t\hat{y}_t)\right) + \text{“junk”}$$

where “junk” means terms higher than 2nd order or that are independent of policy

- **Step 5**: Natural output in terms of productivity

$$\hat{a}_t = \frac{\sigma(1-\alpha)+\varphi+\alpha}{1+\varphi}\hat{y}_t^n$$

And plug in and collect terms

$$\frac{U_t - \bar{U}}{\bar{U}_c \bar{C}} \approx -\frac{1}{2}\left(2\hat{d}_t + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(\hat{y}_t{}^2 - 2\hat{y}_t\hat{y}_t^n)\right) + \text{“more junk”}$$

And in terms of the *output gap* $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^n$ we then have

$$\frac{U_t - \bar{U}}{\bar{U}_c \bar{C}} \approx -\frac{1}{2}\left(2\hat{d}_t + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)\tilde{y}_t{}^2\right) + \text{“yet more junk”}$$

Approximate welfare function

- **Step 6a**: Approximate relative price dispersion term around symmetric steady state with $\bar{P}(j) = \bar{P}$ for all j to get

$$\hat{d}_t \approx \frac{\varepsilon}{2\Theta}\text{Var}\{\hat{p}_t(j)\}$$

where, as earlier, $\Theta \equiv (1-\alpha)/(1-\alpha+\alpha\varepsilon)$ and where $\text{Var}\{\hat{p}_t(j)\}$ is the *cross-sectional* variance of the price distribution at date t

- **Step 6b**: Relate cross-sectional variance to inflation, in fact up to a second order approximation it is true that

$$\sum_{t=0}^{\infty}\beta^t\text{Var}\{\hat{p}_t(j)\} \approx \frac{\theta}{(1-\beta\theta)(1-\theta)}\sum_{t=0}^{\infty}\beta^t\hat{\pi}_t^2$$

Approximate welfare function

- **Step 7**: Construct approximate welfare (loss) function

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{U_t - \bar{U}}{\bar{U}_c \bar{C}} \right\} = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t L_t$$

where (expected) period loss is

$$\begin{aligned} L_t &= \mathbb{E}_0 \left\{ \frac{\varepsilon}{\lambda} \hat{\pi}_t^2 + \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t{}^2 \right\} \\ &= \frac{\varepsilon}{\lambda} \mathrm{Var}\{\hat{\pi}_t\} + \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \mathrm{Var}\{\tilde{y}_t\} \end{aligned}$$

where $\lambda \equiv (1 - \beta\theta)(1 - \theta)\Theta/\theta$

Microfoundations of welfare weights

- Welfare weight on inflation variance

$$\omega_\pi \equiv \frac{\varepsilon}{\lambda} = \frac{\varepsilon\theta}{(1-\beta\theta)(1-\theta)\Theta} = \frac{\varepsilon\theta(1-\alpha+\alpha\varepsilon)}{(1-\beta\theta)(1-\theta)(1-\alpha)}$$

Increasing in elasticity of substitution ε (scales up loss from cross-sectional dispersion in prices), increasing in price stickiness θ

- Welfare weight on output gap variance

$$\omega_y \equiv \sigma + \frac{\varphi+\alpha}{1-\alpha}$$

Increasing in utility curvature parameters σ, φ, increasing in production curvature α

Evaluate policy rules

- Example: Taylor-type rule

$$\begin{aligned} i_t &= \rho + \phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t \\ &= \rho + \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t + v_t \end{aligned}$$

  where "shock" is $v_t \equiv \phi_y \hat{y}_t^n$, not a monetary policy shock

- Compute equilibrium outcomes, evaluate according to

$$L = \omega_\pi \text{Var}\{\hat{\pi}\} + \omega_y \text{Var}\{\tilde{y}\}$$

  for various settings of ϕ_π, ϕ_y etc. Report losses as *percentages of steady state consumption equivalents*

Welfare losses from alternative policy rules

Table 4.1 Evaluation of Simple Monetary Policy Rules

	Taylor Rule				Constant Money Growth	
ϕ_π	1.5	1.5	5	1.5	—	—
ϕ_y	0.125	0	0	1	—	—
$(\sigma_\zeta, \rho_\zeta)$	—	—	—	—	(0, 0)	(0.0063, 0.6)
$\sigma(\widetilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
welfare loss	0.30	0.08	0.002	1.92	0.08	0.38

Other parameters: $\beta = 0.99$, $\sigma = \varphi = 1$, $\alpha = 1/3$, $\varepsilon = 6$, $\theta = 2/3$.
Implies $\omega_y = 6$ and $\omega_\pi \approx 156$

Summary of optimal policy

- Eliminates relative price distortions caused by nominal rigidity
- Can be implemented with simple interest rate rules
- Sufficient condition for uniqueness of equilibrium $\phi_\pi > 1$
- Under optimal policy $i_t = r_t^n$ *in equilibrium*
- With "simple" (non-optimal) rules like Taylor rules, equilibrium volatility of inflation is lower the higher is ϕ_π

Next class

- Monetary/fiscal interactions in the new Keynesian model, part one

- Fiscal policy and multipliers

- Reading

  ◇ Woodford "Simple analytics of the government expenditure multiplier" *AEJ: Macroeconomics*, 2011

Available from the LMS