Advanced Macroeconomics

Knowledge accumulation and endogenous growth

Chris Edmond

1st Semester 2019

This class

- Romer *endogenous* growth model
  - R&D and returns to knowledge accumulation
  - implications for aggregate growth

Endogenous growth

- In Solow-Swan and Ramsey-Cass-Koopmans growth models, the source of long-run growth is *exogenous*, unexplained by the model

- Various ways to make long-run growth endogenous

  – human capital accumulation

  – knowledge accumulation, including learning-by-doing, etc

- Have many formal similarities

  – key is returns to scale to produced factors

  – gives something like an 'AK' growth model

Knowledge accumulation

Knowledge takes many forms, from pure mathematics to soft drink recipes. Knowledge is different from conventional private goods

(1) All forms of knowledge are *non-rival*, my knowledge of the Pythagorean theorem does not prevent you knowing it too

(2) But forms of knowledge vary in degree of *excludability*, depends on

- technical details of the knowledge (e.g., complexity)
- institutional settings (e.g., patent law)

Conventional private goods are both rival and excludable

Romer (1990) growth model

- Knowledge embedded in goods that are *imperfect substitutes*
- Developer of new idea has monopoly rights to use of idea
- Provides incentives for R&D activities, knowledge production
- Resources allocated to R&D determine aggregate growth rate
- *Equilibrium* allocation to R&D $<$ socially optimal allocation

Setup

- Continuous time $t \geq 0$
- Constant labor force $L > 0$
- No physical capital (no transitional dynamics)
- Two sectors: (i) goods production sector employing L_Y and (ii) R&D sector employing L_A. Key is allocation of labor

$$L_Y + L_A = L$$

Imperfect substitutes

- Knowledge embedded in *intermediate goods* $i \in [0, A]$, range of goods $A > 0$ endogenous

- Intermediate goods combined to produce composite final good

- In particular, composite final good is CES function of intermediates

$$Y = \left(\int_0^A y(i)^{\frac{\eta-1}{\eta}} \, di \right)^{\frac{\eta}{\eta-1}}, \qquad \eta > 1$$

  Perfect substitutes is the special case $\eta \to \infty$, Cobb-Douglas is the special case $\eta \to 1^+$, ($\eta < 1$ not permitted — we'll see why)

- For convenience, let

$$\phi \equiv \frac{\eta - 1}{\eta} \in (0, 1)$$

Imperfect substitutes

- Intermediates produced with labor one-for-one

$$y(i) = l(i), \qquad L_Y = \int_0^A l(i)\,di$$

- Suppose all intermediates use constant $l(i) = l$ labor

$$y = l, \qquad L_Y = Al$$

Then production of goods is

$$Y = \left(\int_0^A \left(\frac{L_Y}{A} \right)^{\phi} di \right)^{\frac{1}{\phi}} = A^{\frac{1-\phi}{\phi}} L_Y$$

Constant returns to L_Y, increasing in A.

Market structure

- Final good produced by competitive firms

- Final good producers buy intermediates at relative price $p(i)$ to maximize profits

$$Y - \int_0^A p(i)y(i)\,di \qquad \text{subject to} \qquad Y = \left(\int_0^A y(i)^{\phi}\,di\right)^{\frac{1}{\phi}}$$

  This implies a *demand curve* facing each intermediate

- Intermediate producers choose price $p(i)$ internalizing the effect on demand (i.e., recognizing their market power)

- This is *monopolistic competition* between the intermediates. Ethier (1982) version of Dixit-Stiglitz (1977)

Final good producers

- Choose $y(i)$ to maximize profits

$$\left(\int_0^A y(i)^\phi \, di\right)^{\frac{1}{\phi}} - \int_0^A p(i)y(i) \, di$$

- So for each $i \in [0, A]$ have the first order condition

$$y(i): \qquad \left(\int_0^A y(i)^\phi \, di\right)^{\frac{1-\phi}{\phi}} y(i)^{\phi-1} - p(i) = 0$$

which can be written

$$y(i) = p(i)^{\frac{1}{\phi-1}} Y = p(i)^{-\eta} Y$$

(i.e., with demand elasticity $\frac{1}{\phi-1} = -\eta < -1$)

Intermediate producers

- Choose $l(i)$ to maximize profits

$$\pi(i) = p(i)y(i) - wl(i)$$

subject to (i) their production function $y(i) = l(i)$ and (ii) the downward-sloping demand curve

$$y(i) = p(i)^{-\eta}\, Y$$

- Equivalently, choose $p(i)$ to maximize

$$\pi(i) = \left[p(i)^{1-\eta} - wp(i)^{-\eta}\right] Y$$

with solution

$$p(i) = \frac{\eta}{\eta - 1}\, w$$

(price is *markup* $\frac{\eta}{\eta-1} > 1$ over marginal cost)

Intermediate producers

- Implies intermediate profits proportional to size

$$\pi(i) = p(i)y(i) - wl(i)$$

$$= \left(\frac{\eta}{\eta - 1} - 1\right) wl(i)$$

$$= \frac{1}{\eta - 1} wl(i)$$

$$= \frac{1 - \phi}{\phi} wl(i)$$

Knowledge production

- Labor allocation

$$L_Y(t) + L_A(t) = L$$

- Production of new ideas linear in $L_A(t)$

$$\dot{A}(t) = B\, L_A(t)\, A(t), \qquad B > 0, \qquad A(0) > 0$$

so that $g_A(t) \equiv \dot{A}(t)/A(t) = B\, L_A(t)$ is the growth rate of the stock of knowledge $A(t)$

- Parameter $B > 0$ measures the 'productivity' in R&D sector (i.e., difficult to create new knowledge if B is small)

Representative household

- Maximizes

$$U = \int_0^\infty e^{-\rho t} \log c(t)\, dt, \qquad \rho > 0$$

subject to the intertemporal budget constraint

$$\int_0^\infty q(t)c(t)\, dt = x(0) + \int_0^\infty q(t)w(t)\, dt$$

where $x(0)$ denotes initial wealth per worker and $q(t)$ denotes the intertemporal price of consumption

$$q(t) \equiv \exp\big(-\int_0^t r(s)\, ds\big)$$

- Simple consumption Euler equation, with log utility

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho$$

Free entry into R&D

- Monopoly rights ('patent') on new idea last forever

- Present value of profits from idea i introduced at $t \geq 0$

$$\int_t^\infty \frac{q(\tau)}{q(t)} \pi(i,\tau)\, d\tau$$

where $\pi(i,\tau)$ denotes flow profits on dates $\tau \geq t$ and where $q(\tau)/q(t)$ discounts flow profits from τ to t

- Anyone can produce new idea by hiring $\frac{1}{BA(t)}$ labor at $w(t)$. Acts like a sunk cost, implies free entry condition

$$\int_t^\infty \frac{q(\tau)}{q(t)} \pi(i,\tau)\, d\tau \leq \frac{w(t)}{BA(t)}$$

Symmetric equilibrium

- Consumption per worker, from goods market clearing

$$c(t) = C(t)/L = Y(t)/L$$

- Production and employment per intermediate

$$y(i,t) = y(t) = l(t) = l(i,t)$$

- Implies aggregate labor in goods production

$$L_Y(t) = \int_0^{A(t)} l(i,t)\,di = A(t)l(t)$$

and aggregate quantity of goods produced

$$Y(t) = A(t)^{\frac{1-\phi}{\phi}} L_Y(t)$$

so that growth rate of goods produced is

$$g_Y(t) = \frac{1-\phi}{\phi} g_A(t) + g_{L_Y}(t)$$

Solving the model

- Guess growth rate g_A is constant
- Hence from production function for new ideas

$$g_A = BL_A$$

for some constant L_A to be determined

- Hence from labor market clearing

$$L_Y = L - L_A$$

is constant too, so that

$$g_Y = \frac{1-\phi}{\phi} g_A = \frac{1-\phi}{\phi} BL_A$$

Solving the model

- In such an equilibrium, employment per intermediate is

$$l(t) = \frac{L - L_A}{A(t)}$$

- Zero profits for competitive final goods producers

$$Y(t) = \int_0^{A(t)} p(t)y(t)\,di = A(t)p(t)l(t) = \frac{\eta}{\eta - 1}w(t)(L - L_A)$$

- Hence also have

$$g_Y = g_w$$

Solving the model

- Profits per intermediate

$$\pi(t) = \frac{1-\phi}{\phi} w(t)l(t) = \frac{1-\phi}{\phi}\frac{w(t)}{A(t)}(L - L_A)$$

with

$$g_\pi = g_w - g_A = g_Y - g_A = \left(\frac{1-\phi}{\phi} - 1\right) g_A = \frac{1-2\phi}{\phi} g_A$$

Profits grow $g_\pi > 0$ if $\phi < 1/2$ [relatively low substitution] but shrink $g_\pi < 0$ if $\phi > 1/2$ [relatively high substitution]

- From consumption Euler equation

$$g_c = r - \rho$$

hence from goods market clearing

$$g_c = g_C = g_Y = r - \rho$$

Present value of profits

- Intermediates' profits grow/shrink at rate g_π, implies

$$\pi(\tau) = \pi(t)e^{g_\pi(\tau - t)}, \qquad \tau \geq t$$

- Since constant r, intertemporal prices likewise have form

$$q(\tau) = q(t)e^{-r(\tau - t)}, \qquad \tau \geq t$$

- Hence the present value of profits from idea introduced at $t \geq 0$

$$\int_t^\infty \frac{q(\tau)}{q(t)}\pi(\tau)\,d\tau = \int_t^\infty e^{-r(\tau-t)}\pi(t)e^{g_\pi(\tau-t)}\,d\tau$$

$$= \pi(t)\int_t^\infty e^{-(r-g_\pi)(\tau-t)}\,d\tau$$

$$= \frac{\pi(t)}{r - g_\pi}$$

Equilibrium free entry condition

- From the expression for profits per intermediate

$$\pi(t) = \frac{1-\phi}{\phi}\frac{w(t)}{A(t)}(L - L_A)$$

- From the consumption Euler equation

$$r - g_\pi = \rho + g_Y - (g_Y - g_A) = \rho + g_A$$

- Hence we can write the free entry condition

$$\frac{1-\phi}{\phi}\frac{L - L_A}{\rho + g_A}\frac{w(t)}{A(t)} \leq \frac{1}{B}\frac{w(t)}{A(t)}$$

  Simplifying and using $g_A = BL_A$ then gives

$$\frac{1-\phi}{\phi}(L - L_A) \leq \frac{\rho}{B} + L_A$$

Equilibrium free entry condition

- Equilibrium labor employed in knowledge production is

$$L_A^* = \max\left[\,0\,,\ (1-\phi)L - \phi\frac{\rho}{B}\,\right]$$

so that the equilibrium growth rate is

$$g_A^* = BL_A^* = \max\left[\,0\,,\ (1-\phi)BL - \phi\rho\,\right]$$

with

$$g_Y^* = \frac{1-\phi}{\phi}g_A^* = g_C^* = g_w^*, \quad g_\pi^* = g_Y^* - g_A^*$$

and so on

Comparative statics

- Equilibrium growth rate g_Y^* determined by ρ, ϕ, B, L
  - higher ρ [more impatience] reduces g_Y^*
  - higher ϕ [intermediates closer substitutes] reduces $g_Y^* = \frac{1-\phi}{\phi} g_A^*$, both directly and through g_A^*
  - higher B [more productive R&D sector] increases g_Y^*
  - higher L [larger economy] increases g_Y^*
- Last implication is troubling. Do larger economies grow faster?
- If ρ high, ϕ high, or BL small, i.e., if

$$\rho > \frac{1-\phi}{\phi} BL$$

then $L_A^* = 0$ hence $g_Y^* = g_A^* = 0$ (since no other source of growth)

Lifetime utility

- Representative household

$$U = \int_0^\infty e^{-\rho t} \log c(t)\, dt$$

- Suppose $c(t) = e^{gt} c(0)$ for some g, then

$$U = \int_0^\infty e^{-\rho t} [\log c(0) + gt]\, dt = \frac{\log c(0)}{\rho} + \frac{g}{\rho^2}$$

which uses the familiar

$$\int_0^\infty e^{-\rho t}\, dt = \frac{1}{\rho}$$

and integrating by parts

$$\int_0^\infty e^{-\rho t} t\, dt = \frac{1}{\rho^2}$$

- Lifetime utility in current value units

$$\rho U = \log c(0) + \frac{g}{\rho}$$

Optimal allocation

- Suppose planner chooses L_A to maximize

$$\rho U = \log c(0) + \frac{g}{\rho}$$

subject to growth rate

$$g = \frac{1-\phi}{\phi} B L_A$$

and level of consumption per worker

$$c(0) = \frac{C(0)}{L} = A(0)^{\frac{1-\phi}{\phi}} \left(\frac{L - L_A}{L} \right)$$

- Planner's objective is then

$$\rho U = \log \left(\frac{L - L_A}{L} \right) + \frac{1-\phi}{\phi} \log A(0) + \frac{1-\phi}{\phi} \frac{B L_A}{\rho}$$

Optimal allocation

- Planner's solution

$$L_A^{\text{optimal}} = \max\left[0\,,\; L - \frac{\phi}{1-\phi}\,\frac{\rho}{B}\right]$$

- Decentralized outcome

$$L_A^{\text{equilibrium}} = \max\left[0\,,\; (1-\phi)L - \phi\,\frac{\rho}{B}\right]$$

- Hence planner allocates more to R&D

$$L_A^{\text{equilibrium}} = (1-\phi)L_A^{\text{optimal}} < L_A^{\text{optimal}}$$

R&D externalities

- Planner internalizes three externalities from R&D

  (i) *producer surplus* effect

  final goods producers obtain surplus from intermediates
  [+ pecuniary externality]

  (ii) *business-stealing* effect

  new goods erode profits of existing producers
  [– pecuniary externality]

  (iii) *pure R&D* effect

  innovators earn return on idea in goods production but not in knowledge production [+ non-pecuniary externality]

- Net effect is in general ambiguous but in this example net positive