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Advanced Macroeconomics
Tutorial #8: Solutions

Productivity shocks in the basic new Keynesian model. Consider a new Keynesian model where the output gap $\hat{x}_t$ and inflation $\hat{\pi}_t$ solve

$$\hat{x}_t = -\frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left\{\hat{\pi}_{t+1}\right\} - r_t^n\right) + \mathbb{E}_t\left\{\hat{x}_{t+1}\right\} \tag{1}$$

and

$$\hat{\pi}_t = \kappa\hat{x}_t + \beta\mathbb{E}_t\left\{\hat{\pi}_{t+1}\right\} \tag{2}$$

and where monetary policy is given by the interest rate rule

$$i_t = \rho + \phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t$$

with $\phi_\pi > 1$. The natural level of output is proportional to productivity, $\hat{y}_t^n = \psi_{yz}\hat{z}_t$ for some coefficient $\psi_{yz} > 0$ and some exogenously given process for productivity $\hat{z}_t$. The production function is linear in labor so that $\hat{y}_t = \hat{z}_t + \hat{l}_t$.

(a) Describe in words the 'microfoundations' of equations (1) and (2).

(b) Suppose that productivity is given by an AR(1) process

$$\hat{z}_{t+1} = \rho_z\hat{z}_t + \varepsilon_{t+1}, \qquad 0 \leq \rho_z < 1$$

where the innovations ε_t are IID normal with mean zero and variance σ_z^2. What process does this imply for the natural real rate r_t^n?

(c) Guess that the output gap and inflation are linear in the shock, $\hat{x}_t = \psi_{xz}\hat{z}_t$ and $\hat{\pi}_t = \psi_{\pi z}\hat{z}_t$ for two coefficients ψ_{xz} and $\psi_{\pi z}$. Use the method of undetermined coefficients to solve for ψ_{xz} and $\psi_{\pi z}$.

(d) Explain how the output gap, output, employment, inflation, and the nominal and real interest rates respond to a productivity shock $\hat{z}_t$. Give intuition for your results.

Now suppose that productivity is given by an AR(1) process *in growth rates*

$$\Delta\hat{z}_{t+1} = (1-\rho_\Delta)\bar{g} + \rho_\Delta\Delta\hat{z}_t + \varepsilon_{t+1}, \qquad 0 \leq \rho_\Delta < 1$$

where $\bar{g} \geq 0$ denotes the long-run growth rate of productivity.

(e) Suppose $\rho_\Delta = 0$. What process does this imply for the natural real rate r_t^n? Explain. How does this affect the responses of the output gap, output, employment, inflation, and the nominal and real interest rates to a productivity growth shock $\Delta\hat{z}_t$?

(f) Now suppose $0 < \rho_\Delta < 1$. How if at all do your answers to (e) change? Explain.

(g) Now suppose the monetary policy can condition on the natural real rate

$$i_t = r_t^n + \phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t$$

How if at all do your answers to (e) change? Explain.

SOLUTIONS:

(a) Equation (1) is known as the *dynamic IS curve* and is derived from a log-linearized consumption Euler equation along with with the assumption of goods market clearing $\hat{c}_t = \hat{y}_t$ and then expressed in terms of deviations from 'natural levels' — so that the Euler equation ends up relating the output gap $\hat{x}_t \equiv \hat{y}_t - \hat{y}_t^n$ to the deviation in the prevailing real interest rate $r_t \equiv i_t - \mathbb{E}_t\{\pi_{t+1}\}$ from the natural real rate $r_t^n \equiv \rho + \sigma \mathbb{E}_t\{\Delta \hat{y}_{t+1}^n\}$. The natural level of output is the level of output in the same model but with fully flexible prices. The natural real rate is the corresponding real interest rate implied by the consumption Euler equation when output equals its natural level. Equation (2) is known as the *new Keynesian Phillips curve* and is a log-linearized version of the optimal pricing behavior of a forward-looking firm that faces random opportunities to re-optimize its price ('Calvo pricing') with the real marginal cost of the firm rewritten in terms of the output gap.

(b) As noted above, the natural real rate is

$$r_t^n \equiv \rho + \sigma \mathbb{E}_t\{\Delta \hat{y}_{t+1}^n\}$$

Since we are given $\hat{y}_t^n = \psi_{yz} \hat{z}_t$ this implies

$$r_t^n = \rho + \sigma \psi_{yz} \mathbb{E}_t\{\Delta \hat{z}_{t+1}\}$$

and since $\hat{z}_t$ follows an AR(1) with persistence ρ_z we also have

$$\mathbb{E}_t\{\Delta \hat{z}_{t+1}\} = (\rho_z - 1)\hat{z}_t$$

Hence

$$r_t^n = \rho - \sigma \psi_{yz}(1 - \rho_z)\hat{z}_t$$

The natural real rate is here inversely related to the level of productivity because high current levels of productivity $\hat{z}_t > 0$ are associated with negative expected productivity growth $\mathbb{E}_t\{\Delta \hat{z}_{t+1}\} = (\rho_z - 1)\hat{z}_t < 0$ due to mean reversion. Since the natural real rate is a linear function of $\hat{z}_t$, it is also an AR(1) with the same persistence. Specifically,

$$r_{t+1}^n = \rho + \rho_z(r_t^n - \rho) - \sigma \psi_{yz}(1 - \rho_z)\varepsilon_{t+1}$$

so that the natural real rate has the same persistence ρ_z and innovations that are perfectly negatively correlated with the innovations to productivity.

(c) Substituting in the expressions for the natural real rate and the interest rate rule, we now have the system

$$\hat{x}_t = -\frac{1}{\sigma}\left(\phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t - \mathbb{E}_t\{\hat{\pi}_{t+1}\} - u_t\right) + \mathbb{E}_t\{\hat{x}_{t+1}\}$$

and

$$\hat{\pi}_t = \kappa \hat{x}_t + \beta \mathbb{E}_t \{\hat{\pi}_{t+1}\}$$

driven by a single shock

$$u_t \equiv \sigma \psi_{yz} \mathbb{E}_t\{\Delta \hat{z}_{t+1}\} = -\sigma \psi_{yz}(1-\rho_z)\hat{z}_t$$

Now as in Lecture 13 guess that the solutions are linear in u_t

$$\hat{x}_t = \psi_{xu}\, u_t$$

and

$$\hat{\pi}_t = \psi_{\pi u}\, u_t$$

for some $\psi_{xu}, \psi_{\pi u}$ to be determined. From the dynamic IS curve

$$\psi_{xu} = -\frac{1}{\sigma}\left(\phi_\pi \psi_{\pi u} + \phi_x \psi_{xu} - \psi_{\pi u}\rho_u - 1\right) + \psi_{xu}\rho_u$$

and from the new Keynesian Phillips curve

$$\psi_{\pi u} = \kappa \psi_{xu} + \beta \psi_{\pi u}\rho_u$$

Solving these two equations in the two unknown coefficients $\psi_{xu}, \psi_{\pi u}$ gives

$$\psi_{xu} = (1-\beta\rho_u)\,\Lambda_u$$

and

$$\psi_{\pi u} = \kappa\,\Lambda_u$$

with common component

$$\Lambda_u \equiv \frac{1}{(1-\beta\rho_u)(\sigma(1-\rho_u)+\phi_x) + \kappa(\phi_\pi - \rho_u)} > 0$$

($\Lambda_u > 0$ since $\phi_\pi > 1$). Now since the shock $u_t = -\sigma\psi_{yz}(1-\rho_z)\hat{z}_t$ is linear in the AR(1) process $\hat{z}_t$ it has the same serial correlation, $\rho_u = \rho_z$. Hence the coefficients we seek can be written

$$\psi_{xz} = -\sigma\psi_{yz}(1-\rho_z)(1-\beta\rho_z)\Lambda_z < 0$$

and

$$\psi_{\pi z} = -\sigma\psi_{yz}(1-\rho_z)\kappa\Lambda_z < 0$$

where

$$\Lambda_z \equiv \frac{1}{(1-\beta\rho_z)(\sigma(1-\rho_z)+\phi_x) + \kappa(\phi_\pi - \rho_z)} > 0$$

(d) In response to a positive productivity shock $\hat{z}_t > 0$, the natural level of output $\hat{y}_t^n = \psi_{yz}\hat{z}_t$ increases and the natural real rate $r_t^n = \rho - \sigma\psi_{yz}(1-\rho_z)\hat{z}_t$ falls. From our solution for ψ_{xz}, the output gap $\hat{x}_t$ falls on impact

$$\frac{\partial \hat{x}_t}{\partial \hat{z}_t} = \psi_{xz} = -\sigma\psi_{yz}(1-\rho_z)(1-\beta\rho_z)\Lambda_z < 0$$

The actual level of output is given by $\hat{y}_t = \hat{x}_t + \hat{y}_t^n$ so that

$$\frac{\partial \hat{y}_t}{\partial \hat{z}_t} = \frac{\partial \hat{x}_t}{\partial \hat{z}_t} + \frac{\partial \hat{y}_t^n}{\partial \hat{z}_t} = -\sigma\psi_{yz}(1-\rho_z)(1-\beta\rho_z)\Lambda_z + \psi_{yz}$$

which with a bit of algebra condenses to

$$\frac{\partial \hat{y}_t}{\partial \hat{z}_t} = \psi_{yz}\left[(1-\beta\rho_z)\phi_x + \kappa(\phi_\pi - \rho_z)\right]\Lambda_z > 0$$

In short, while an increase in productivity increases both the actual level of output $\hat{y}_t$ and the natural level of output $\hat{y}_t^n$, it increases the latter by a larger amount so that the output gap $\hat{x}_t = \hat{y}_t - \hat{y}_t^n$ falls. In this sense, there is more 'slack' in the economy (in the short run). As a consequence, inflation also falls on impact

$$\frac{\partial \hat{\pi}_t}{\partial \hat{z}_t} = \psi_{\pi z} = -\sigma\psi_{yz}(1-\rho_z)\kappa\Lambda_z < 0$$

From the production function employment is $\hat{l}_t = \hat{y}_t - \hat{z}_t$ so that

$$\frac{\partial \hat{l}_t}{\partial \hat{z}_t} = \psi_{yz}\left[(1-\beta\rho_z)\phi_x + \kappa(\phi_\pi - \rho_z)\right]\Lambda_z - 1$$

In general this is ambiguous, employment can rise or fall depending on parameters. To get a bit more sense of things, consider the special case of log utility $\sigma = 1$ which implies that the natural level of output fluctuates 1-for-1 with productivity (since the substitution and income effects on employment cancel) so that $\psi_{yz} = 1$. For this special case we get, after a thicket of algebra

$$\left.\frac{\partial \hat{l}_t}{\partial \hat{z}_t}\right|_{\sigma=1} = -\frac{(1-\beta\rho_z)(1-\rho_z)}{(1-\beta\rho_z)((1-\rho_z)+\phi_x)+\kappa(\phi_\pi-\rho_z)} < 0$$

In short, in this case employment would be constant under flexible prices but falls in response to a productivity shock when prices are sticky. Intuitively, this is because the productivity shock is labor-saving and the 'demand' for output does not increase 1-for-1 with the increase in 'supply'. A permanent increase in the level of productivity is associated with no change in employment while a transitory increase is associated with a relatively large fall in employment. To see this, notice that response of employment is monotonically increasing in the persistence of the shock ρ_z, from

$$\left.\frac{\partial \hat{l}_t}{\partial \hat{z}_t}\right|_{\sigma=1,\rho_z=0} = -\frac{1}{1+\phi_x+\kappa\phi_\pi} \in (-1,0)$$

to

$$\left.\frac{\partial \hat{l}_t}{\partial \hat{z}_t}\right|_{\sigma=1,\rho_z=1} = 0^-$$

so that persistent productivity shocks are associated with a smaller loss of employment. The response of the nominal interest rate is given by

$$\frac{\partial i_t}{\partial \hat{z}_t} = \phi_x\psi_{xz} + \phi_\pi\psi_{\pi z} < 0$$

since both the output gap and inflation fall on impact. In short, monetary policy partially accommodates the productivity shock. The response of the real interest rate is given by

$$\frac{\partial r_t}{\partial \hat{z}_t} = \frac{\partial i_t}{\partial \hat{z}_t} - \frac{\partial \mathbb{E}_t\{\pi_{t+1}\}}{\partial \hat{z}_t}$$

Since productivity has persistence ρ_z and there are no other shocks, the response of expected inflation is simply

$$\frac{\partial \mathbb{E}_t\{\pi_{t+1}\}}{\partial \hat{z}_t} = \psi_{\pi z}\rho_z$$

so that

$$\frac{\partial r_t}{\partial \hat{z}_t} = \phi_x\psi_{xz} + (\phi_\pi - \rho_z)\psi_{\pi z} < 0$$

since by assumption monetary policy is sufficiently reactive to inflation, $\phi_\pi > 1 > \rho_z$. Hence the real rate also falls on impact.

(e) In this case productivity is a random walk with drift $\bar{g} \geq 0$ and the natural real rate is a constant

$$r_t^n = \rho + \sigma\psi_{yz}\mathbb{E}_t\{\Delta\hat{z}_{t+1}\} = \rho + \sigma\psi_{yz}\bar{g}$$

independent of the current growth rate (there is no mean-reversion). Since in this model fluctuations in the natural real rate are the only source of fluctuations in the endogenous variables, this means the output gap and inflation are constant. Following the same steps as in part (c) above, we get the constant solutions

$$\hat{x} = \sigma\psi_{yz}\bar{g}\,\frac{(1-\beta)}{(1-\beta)\phi_x + \kappa(\phi_\pi - 1)} > 0$$

and

$$\hat{\pi} = \sigma\psi_{yz}\bar{g}\,\frac{\kappa}{(1-\beta)\phi_x + \kappa(\phi_\pi - 1)} > 0$$

so that there is a 'permanent boom' and a corresponding permanent level of inflation.

The actual level of output then fluctuates 1-for-1 with the natural level of output, $\hat{y}_t = \hat{x} + \hat{y}_t^n$ which then implies that employment is given by $\hat{l}_t = \hat{y}_t - \hat{z}_t = \hat{x} + \psi_{yz}\hat{z}_t - \hat{z}_t$ so that whether employment increases or decreases in response to productivity is purely determined by whether ψ_{yz} is greater or less than one, as in the underlying flexible price model. Since the output gap and inflation are constant, so are the nominal interest rate and the real interest rate. As we will see below, monetary policy can completely stabilize the economy here if instead of the naive intercept ρ in the interest rate rule it had used the actual constant natural real rate $r^n = \rho + \sigma\psi_{yz}\bar{g}$. If so, we would find that the constant output gap and inflation are constant at zero. The fact that there is a 'permanent boom' and inflation is here due to the fact that the interest rate rule is in a sense misspecified relative to the true disturbances hitting the economy.

(f) With $0 < \rho_\Delta < 1$, the natural real rate is not constant and is now given by

$$r_t^n = \rho + \sigma\psi_{yz}\big[\bar{g} + \rho_\Delta(\Delta\hat{z}_t - \bar{g})\big]$$

so that the natural real rate is high when productivity growth is above its long-run average $\bar{g}$. The analysis is similar to part (c) but to simplify the algebra, let's modify the problem a bit and suppose that the interest rate rule is

$$i_t = r^n + \phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t$$

where

$$r^n \equiv \rho + \sigma\psi_{yz}\bar{g}$$

denotes the natural real rate when productivity growth is at its long run average $\bar{g}$. Then the analysis is in fact the same as in part (c) above except that the shock driving the system is

$$u_t = \sigma\psi_{yz}\rho_\Delta(\Delta\hat{z}_t - \bar{g})$$

The equilibrium dynamics of the output gap and inflation are given by

$$\hat{x}_t = \psi_{x\Delta}\left(\Delta\hat{z}_t - \bar{g}\right)$$

and

$$\hat{\pi}_t = \psi_{\pi\Delta}\left(\Delta\hat{z}_t - \bar{g}\right)$$

with coefficients

$$\psi_{x\Delta} = +\sigma\psi_{yz}\rho_\Delta(1-\beta\rho_\Delta)\Lambda_\Delta > 0$$

and

$$\psi_{\pi\Delta} = +\sigma\psi_{yz}\rho_\Delta\kappa\Lambda_\Delta > 0$$

where

$$\Lambda_\Delta \equiv \frac{1}{(1-\beta\rho_\Delta)(\sigma(1-\rho_\Delta)+\phi_x)+\kappa(\phi_\pi-\rho_\Delta)} > 0$$

In short, high expected productivity *growth* $\Delta\hat{z}_t > \bar{g}$ is associated with a boom $\hat{x}_t > 0$ and inflation $\hat{\pi}_t > 0$. In response to high expected productivity growth, monetary policy tightens with the nominal interest rate rising by more than expected inflation so that the real rate also rises. The actual level of output is given by $\hat{y}_t = \hat{x}_t + \hat{y}_t^n = \psi_{x\Delta}(\Delta\hat{z}_t - \bar{g}) + \psi_{yz}\hat{z}_t$ and so is increasing on impact. The response of employment is ambiguous in the usual kind of way. Notice that the actual level of output is the sum of a stationary component $\hat{x}_t$ that is proportional to productivity growth plus a term that is proportional to the nonstationary level of productivity. In other words, output and the natural level of output are $I(1)$ processes while the output gap (and inflation and the interest rates) are $I(0)$ processes.

(g) If monetary policy can fully condition on the natural real rate r_t^n then, as alluded to in parts (e) and (f) above, policy can isolate the economy from shocks entirely. The only fluctuations in this model are due to the fact that the interest rate rule is in a sense 'misspecified'. With $i_t = r_t^n + \phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t$ we get the system

$$\hat{x}_t = -\frac{1}{\sigma}\left(\phi_\pi\hat{\pi}_t + \phi_x\hat{x}_t - \mathbb{E}_t\{\hat{\pi}_{t+1}\}\right) + \mathbb{E}_t\{\hat{x}_{t+1}\}$$

and

$$\hat{\pi}_t = \kappa\hat{x}_t + \beta\mathbb{E}_t\left\{\hat{\pi}_{t+1}\right\}$$

which is completely independent of any shocks (is independent of r_t^n) and is solved by the constants

$$\hat{x} = 0$$

and

$$\hat{\pi} = 0$$

Since the output gap is zero, the actual level of output equals the natural level of output, $\hat{y}_t = \hat{y}_t^n = \psi_{yz}\hat{z}_t$ which then implies that employment is given by $\hat{l}_t = \hat{y}_t - \hat{z}_t = (\psi_{yz} - 1)\hat{z}_t$ so that the sign of the employment response is again purely determined by whether ψ_{yz} is greater or less than one, as in the underlying flexible price model. Since the natural real rate fluctuates, unlike part (e) we do not have constant interest rates but since the output gap and inflation are constant, the equilibrium interest rates are simply

$$i_t = r_t^n = r_t = \rho + \sigma\psi_{yz}\rho_\Delta\Delta\hat{z}_t$$

Importantly, the coefficients of the interest rate rule do not appear in the equilibrium dynamics of the interest rate (or anywhere else). Nonetheless, as we will discuss at some length in class, the assumption that ϕ_π is sufficiently high is essential in ensuring that the constant solution $\hat{x} = \hat{\pi} = 0$ is in fact the only solution to this system.