

## Advanced Macroeconomics Tutorial #8

**Productivity shocks in the basic new Keynesian model.** Consider a new Keynesian model where the output gap  $\hat{x}_t$  and inflation  $\hat{\pi}_t$  solve

$$\hat{x}_t = -\frac{1}{\sigma} (i_t - \mathbb{E}_t \{\hat{\pi}_{t+1}\} - r_t^n) + \mathbb{E}_t \{\hat{x}_{t+1}\} \quad (1)$$

and

$$\hat{\pi}_t = \kappa \hat{x}_t + \beta \mathbb{E}_t \{\hat{\pi}_{t+1}\} \quad (2)$$

and where monetary policy is given by the interest rate rule

$$i_t = \rho + \phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t$$

with  $\phi_\pi > 1$ . The natural level of output is proportional to productivity,  $\hat{y}_t^n = \psi_{yz} \hat{z}_t$  for some coefficient  $\psi_{yz} > 0$  and some exogenously given process for productivity  $\hat{z}_t$ . The production function is linear in labor so that  $\hat{y}_t = \hat{z}_t + \hat{l}_t$ .

- (a) Describe in words the ‘microfoundations’ of equations (1) and (2).
- (b) Suppose that productivity is given by an AR(1) process

$$\hat{z}_{t+1} = \rho_z \hat{z}_t + \varepsilon_{t+1}, \quad 0 \leq \rho_z < 1$$

where the innovations  $\varepsilon_t$  are IID normal with mean zero and variance  $\sigma_z^2$ . What process does this imply for the natural real rate  $r_t^n$ ?

- (c) Guess that the output gap and inflation are linear in the shock,  $\hat{x}_t = \psi_{xz} \hat{z}_t$  and  $\hat{\pi}_t = \psi_{\pi z} \hat{z}_t$  for two coefficients  $\psi_{xz}$  and  $\psi_{\pi z}$ . Use the method of undetermined coefficients to solve for  $\psi_{xz}$  and  $\psi_{\pi z}$ .
- (d) Explain how the output gap, output, employment, inflation, and the nominal and real interest rates respond to a productivity shock  $\hat{z}_t$ . Give intuition for your results.

Now suppose that productivity is given by an AR(1) process *in growth rates*

$$\Delta \hat{z}_{t+1} = (1 - \rho_\Delta) \bar{g} + \rho_\Delta \Delta \hat{z}_t + \varepsilon_{t+1}, \quad 0 \leq \rho_\Delta < 1$$

where  $\bar{g} \geq 0$  denotes the long-run growth rate of productivity.

- (e) Suppose  $\rho_\Delta = 0$ . What process does this imply for the natural real rate  $r_t^n$ ? Explain. How does this affect the responses of the output gap, output, employment, inflation, and the nominal and real interest rates to a productivity growth shock  $\Delta \hat{z}_t$ ?
- (f) Now suppose  $0 < \rho_\Delta < 1$ . How if at all do your answers to (e) change? Explain.
- (g) Now suppose the monetary policy can condition on the natural real rate

$$i_t = r_t^n + \phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t$$

How if at all do your answers to (e) change? Explain.